



## **Pile Capacity Prediction Using Multivariate Regression Model**

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### **Abstract:**

This paper considers the use of multivariate regression model in predicting the axial capacity of concrete pile driven into layered soil. The model incorporates the possible factors that affect the bearing capacity of pile as input variables. The various input parameters are settlement, effective vertical stress, undrained shear strength of the clay layer, N-value of the sand layer and thicknesses of the soil layers. Predictions from the model were validated with the measured load test values while various statistical error criteria were utilised in ascertaining the predictive ability of the model. The mean absolute error, root mean square error and mean absolute error were obtained as 178.56, 231.23 and 9.34% respectively. These results proved the ability of the model to predict the capacity of the pile with an acceptable degree of accuracy. The output from the model also surpasses the results from the pile load test prediction methods and pile dynamic formulas that were considered.

*Keywords:* multivariate, region, layered soil, error criteria, axial capacity

### **1.0 Introduction**

Pile foundation is usually adopted as a support system for structures with high load intensities and for areas marked by weak ground conditions. The performance is influenced by a number of factors such as the condition of the subsoils and groundwater, pile size, depth of embedment, the method of installation, design method and the loads to which the pile is subjected. The bearing capacity of a pile foundation is determined by any of several approaches namely, pile load tests, dynamic analysis and static analysis using soil properties and in-situ test results.

The application of the different available methods in the determination of pile capacity has shown that they cannot provide consistent values of pile capacity. The outcome of the predictions varies even for the same case study site depending on the method applied, such as Horvitz et al. (1981), Dewaiker and Pallavi (2000), Titi and Abu-Farsakh (2004), Eslami et al. (2014), Mishra et al. (2019). It has been noticed that most of the methods were developed based on local information from specific regions with limited number of tests in designated locations. As such, they are associated with uncertainties and errors when applied to other soil conditions and environments (Alilahi and Adampira, 2015). This implies that a given pile prediction model cannot be considered generally applicable for all soil conditions.

In recent years, extensive progress in the use of soft computing techniques in civil engineering has resulted in the use several applications to solve practical geotechnical engineering problems. One of such is the regression-based modelling approach which, currently, is widely used in the development of models for pile capacity prediction. It is also utilised in the enhancement of the prediction accuracy and in determining the relationship between the input and output data in pile capacity prediction.

Several research works have reported the use of this approach in modelling pile capacity. Pal (2011) presented a generalised regression neural network to predict the bearing capacity of pile and the results were found to be in good agreement with the experimental results. Tarawneh and Imam (2014) used regression analysis and the artificial neural networks approach to evaluate H-pile, concrete pile and pipe pile setup. In using the regression approach, Alkroosh et al., (2015) considered the results of cone penetration test in predicting the bearing capacity of bored piles installed in sand and mixed soils. Shatnawi et al, (2019) developed a regression model for driven pile capacity prediction based on results of 162 dynamic load tests. Juwaied and Al-Zwainy (2017) applied the multiple linear regression model in predicting the capacity of piles from a

database of 63 historical cases in Baghdad. Pham et al; (2020) used classical multi-variant regression to predict capacity of driven piles and compared it with results from artificial neural network, random forest as well as bearing capacity based on traditional formulas. Determination of uplift capacity of model open-ended pile installed in cohesionless soil using the regression analysis was carried out by Abdul-Husain and Hamadi (2021).

In this study, regression analysis has been adopted in modelling the capacity of driven piles using many soil-pile variables as input parameters. The actual behaviour of the 300mm diameter concrete pile was validated with results of static pile load tests. The accuracy of the model in predicting the capacity of the piles was verified using statistical approaches such as second moment, mean absolute error, root mean square error, mean absolute percentage error and relative accuracy.

## 2.0 Data Collection

Data for the study were obtained from 3 locations within the Niger Delta region of Nigeria. The data included properties of the soils underlying the sites, pile load test results and pile driving records. The pile load test and pile driving records are for 11 cases of 300mm diameter reinforced concrete piles driven to depths of between 15.75m and 19.75m. In two of the study locations, static axial pile load tests were conducted on 300mm diameter test piles to about 150% to 200% of the design load, while at the remaining site the piles were tested to failure load which was taken as the load corresponding to a settlement value of 10% of the pile diameter. Kentledge method, based on ASTM D 1143-81, was adopted in the pile load test. Results of the tests were used to plot the pile load-settlement curves from which the ultimate capacity was obtained by several graphical and semi-empirical methods. The applied methods include the Tangent method, and those proposed by Brinch Hanson (1963), Shen (1980), Chin (1970), Decourt (1999), and Abd Elsamee (2012).

The blow counts from the pile driving exercise were used to obtain the pile set which, in turn, was combined with the hammer and pile characteristics to determine the pile capacity. Several pile dynamic formulae were considered including Navy-McKay (1966), Danish (1967) and Janbu (1967).

Information obtained from the site investigation revealed that the sites are underlain by sandy clay of firm – stiff consistency occupying the top horizon. The thickness of the soil varies between 6m and 15.0m. This clayey layer overlies sand deposits of medium – dense relative density, extending to great depths. All the piles terminated within the sand layers.

## 3.0 Development of Regression Model

Mathematical model for pile foundation performance for the study sites was developed using regression analysis.

The regression model is expressed in the form:

$$y = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i$$

Where:

$y$  = Pile foundation performance for this study.

$i = 1, 2, 3, \dots, n$

$x_i$  is the response that corresponds to the levels of the explanatory variables  $x_1, x_2, \dots, x_p$  at the  $i^{\text{th}}$  observation.

$\beta_0, \beta_1, \beta_2, \dots, \beta_p$  = Coefficient in the linear relationship.

$\varepsilon_1, \varepsilon_2, \dots, \varepsilon_i$  = Errors that create scatter around the linear relationship at each of the  $i = 1$  to  $n$  observations.

In developing the model for predicting the capacity of piles for the study sites factors such as the pile geometry and the properties of the soils were considered. In addition, the relationships

between several independent variables and the pile capacity were studied. These variables include settlement, thickness of the clay layer ( $L_1$ ), thickness of the sand layer ( $L_2$ ), average undrained shear strength of the clay layer ( $c_u$ ), the effective vertical stress at the pile tip ( $\sigma'_v$ ), the average value of SPR of the sand layer along the pile shaft ( $N_s$ ) and the average value of SPR at depth of  $4D$  below the pile tip ( $N_t$ ). The ultimate pile capacity ( $Q_u$ ) is the dependent variable in the study. The dataset used in this study is presented in Table 1, along with the statistical information of the variables. Results of the application of the pile load test and dynamic formula prediction methods in predicting the capacity of the piles are presented in Table 2. The average of the pile capacity from these methods was used in the development of the model. The settlement values were derived from the Brinch Hanson (1963) method. Owing to the limited size of the dataset, all the data values were used in the training of the model. Presented in Figure 1.0 is the schematic diagram of the soil profile and pile geometry used for the development of the model.

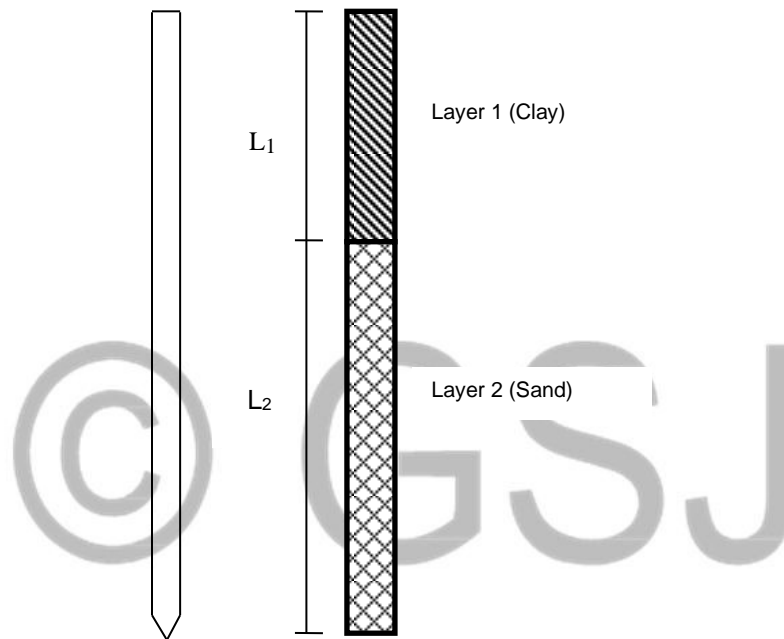


Figure 1.0: Schematic diagram for soil profile and pile geometry.

Table 1.0: Data used in the development of model

Site Location	Pile ref.	Dependent variable	Independent variables						
		$Q_u$ (kN)	$\delta$ (m)	$L_1$ (m)	$L_2$ (m)	$c_u$ (kPa)	$\sigma'_v$ (kN/m <sup>2</sup> )	$N_s$	$N_t$
1	PL1A	1681	0.0173	6	13	75	175.52	23	39
	PL1B	1195	0.0356	8	10.25	50	168.02	17	31
	PL1C	1318	0.0337	8.5	9.5	60	165.52	15	36
	PL1D	1357	0.0863	8.5	10.75	62	178.02	15	36
2	PL2A	2101	0.173	13.5	5.75	87	211.3	35	43
	PL2B	1740	0.116	14.5	5	84	213.8	27	42
	PL2C	1589	0.0237	15	3.75	70	206.3	19	40
	PL2D	1591	0.0713	15	4	78	208.8	22	40
3	PL3A	1702	0.105	7	12.75	75	191.23	20	39
	PL3B	1436	0.07	8	7.75	60	151.23	17	35
	PL3C	1731	0.048	7	12.75	75	191.23	18	38
Min		1195	0.017	6.0	3.8	50	151	15	31
Max		2101	0.173	15.0	13.0	87	214	35	43
Range		906	0.156	9.0	9.3	37	63	20	12
STD		251.38	0.047	3.59	3.58	11.33	21.22	5.95	3.419
COV		0.28	0.302	0.40	0.39	0.31	0.34	0.30	0.285

Table 2.0: Estimation of Ultimate Capacity from Predictive methods

Site Location	Pile ref.	Ultimate bearing Capacity (kN)								
		Brinch Hanson	Shen	Chin	Decourt	Tangent	Abd Elsamee	Navy-McKay	Danish	Janbu
1	PL1A	1042.6	999	3333	3055	860	2530	1031	1223	1051
	PL1B	1813.7	1000	1667	1660	850	1062	767	996	938
	PL1C	1863	1000	1667	1686	820	977	1307	1408	1134
	PL1D	2331	890	2000	2111.9	870	1049	876	1101	986
2	PL2A	3296.9	1200	3333	3127	1250	2140	1821	1690	1049
	PL2B	2008.1	1300	3333	3097.5	1180	1030.8	1256	1383	1076
	PL2C	1111.4	990	3333	2966.3	820	1075	1417	1479	1109
	PL2D	2564	980	1667	1830	850	1337.6	2226	1861	1002
3	PL3A	2112.9	1500	2500	2428.8	1550	1610.9	1202	1349	1063
	PL3B	1725	1400	2000	1878.8	1400	1276	981	1156	1107
	PL3C	2094.3	1800	2500	2605.9	1700	1221.3	1227	1365	1065

The resulting pile capacity prediction model developed from Regression Analysis in this study based on above considerations is expressed as follows:

$$Q_u = 38330 - 65966\delta - 9248L_1 + 4060L_2 + 170.3c_u + 63.30\sigma'_v - 535.1N_s - 417.2N_t + 598874\delta^2 + 415.3L_1^2 - 255.8L_2^2$$

where  $\delta$  = settlement;  $L_1, L_2$  = thicknesses of clay and sand layers respectively;  $c_u$  = undrained shear strength of the Clay layer;  $\sigma'_v$  = vertical effective stress at pile tip;  $N_s$  = average N-value along pile shaft;  $N_t$  = average N-value at depth of 4D below pile tip.

#### 4.0 Performance Evaluation of the Model

The accuracy of pile capacity model can be evaluated using statistical approaches. The error and difference between the actual pile capacity (measured load from site 3 location) and the predicted capacity from the model were ascertained based on second moments (geometric mean, standard deviation and coefficient of variation), mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage of error (MAPE) and average accuracy (AA). A mean value that is nearly 1.0 and a coefficient of variation that is asymptotic to zero indicate good prediction (Awa-Allah, 2018). Also, the lower the values of the MAE and RMSE, the higher the accuracy of the model in predicting the pile capacity (Shatnawi et al., 2019; Pham et al; 2020). The expressions for the error criteria are presented below:

$$\text{Geometric mean, } \mu = \frac{1}{n} \sum_{i=1}^n \left( \frac{Q_p}{Q_m} \right) \quad 1$$

$$\text{Standard deviation, } STD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n \left( \left( \frac{Q_p}{Q_m} \right) - \mu \right)^2} \quad 2$$

$$\text{Coefficient of variation, } COV = \frac{STD}{\mu} \quad 3$$

$$MAE = \frac{\sum_{i=1}^n (Q_p - Q_m)}{n} \quad 4$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (Q_m - Q_p)^2} \quad 5$$

$$MAPE = \frac{\sum_{i=1}^n \left[ \frac{Q_m - Q_p}{Q_m} \right] * 100\%}{n} \quad 6$$

$$AA\% = 100\% - MAPE \quad 7$$

where:

$Q_p$  = Predicted Pile Capacity (from developed model).

$Q_m$  = Measured Pile Capacity from pile load Test.

$n$  = total number of data used for the development of the model.

#### 5.0 Results and Discussion

Results of the analysis of variance showed that there is strong relationship among the variables in the model as revealed by the coefficient of determination,  $R^2$ , which was found to be 100%.

The summary of the prediction capability of the model is presented in Table 3. The uncertainties about the model were examined using the model factor which is the ratio of the measured pile capacity to the model-predicted capacity. It is observed that the mean value is close to 1.0, indicating that the model has a good predictability. The respective standard deviation and coefficient of variation values of 0.107 and 0.097 imply that the difference between the measured and predicted pile capacities is small. The results of the other performance measures gave a mean absolute error, MAE value of 178.56, while the root mean square error, RMSE is 231.23. The average accuracy percentage, AA was found to be 90.7% while the average difference between the actual value and the predicted is approximately 9.3%.

A comparison of the actual and predicted values from the different methods is presented in Figure 2. The results from the application of the pile load test prediction methods indicated that the Chin (1970), Brinch Hanson (1963) and Decourt (1999) methods over-predicted the pile capacity while the Tangent, Shen (1980) and Abd Elsamee (2012) methods under predicted the pile capacity. On the other hand, all the pile dynamic formulae under-predicted the pile capacity. The predictions from the regression analysis model are lower but closer to the measured values, suggesting that it is conservative and more accurate than the predictive methods considered in the study location.

Table 3 Prediction Capability of the Model and other Prediction Model

Criteria	Study model	Test Load Interpretative methods						Dynamic formula methods		
		Brinch Hanson	Shen	Chin	Decourt	Tangent	Abd Elsamee	Navy-McKay	Danish	Janbu
$\mu$	1.10	0.912	1.147	0.773	0.785	1.158	1.343	1.586	1.394	1.672
Std	0.107	0.10	0.017	0.083	0.077	0.071	0.341	0.154	0.139	0.271
COV	0.09	0.109	0.014	0.107	0.099	0.061	0.254	0.097	0.099	0.162
MAE	178.56	181.2	233.33	533.3	504.5	250	430	663.3	510	721.6
RMSE	231.23	249.09	238.04	565.68	536.9	272.3	543.12	681.49	535.58	757.19
MAPE (%)	9.34	10.7	12.85	30.36	28.12	13.45	22.11	36.5	27.79	39.18
AA (%)	90.7	89.20	87.14	69.63	71.87	86.54	77.55	63.48	72.20	60.80

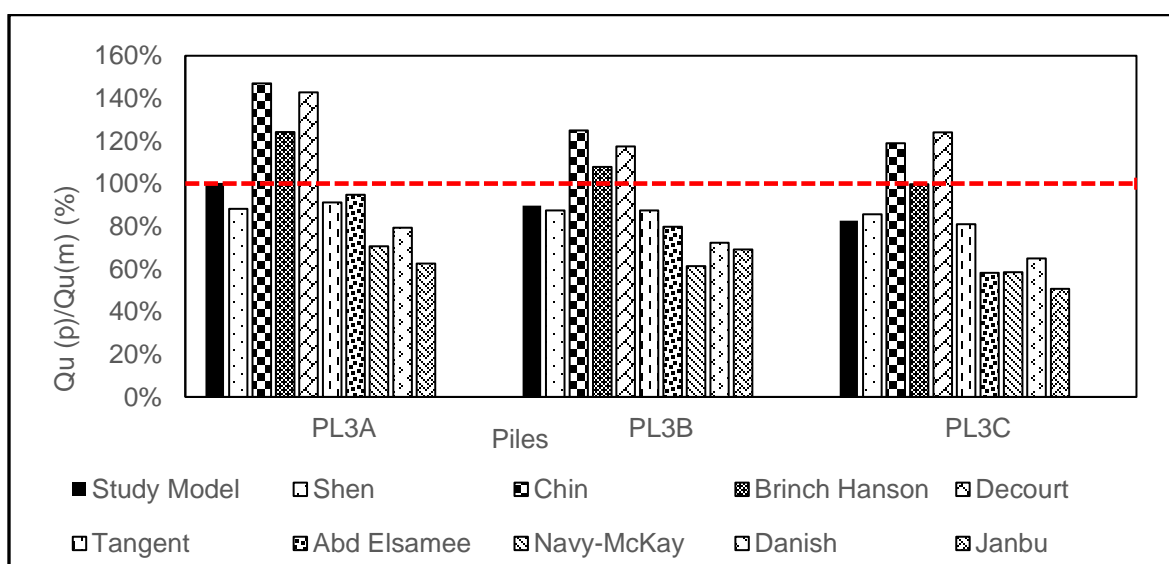


Figure 2: Comparison of pile capacity prediction methods

## 6.0 Conclusion

The use of regression analysis for the development of a model for predicting the capacity of driven pile has been demonstrated in the study. Settlement, thicknesses of the soil layers, effective vertical stress, undrained shear strength and standard penetration Resistance (N-value) were used as the input values. The performance of the model was verified using second moment, mean absolute error, root mean square error, mean absolute percentage error and relative accuracy analyses. Results from the model were further used to compare with predictions from some pile load test methods and pile dynamic formulae and the results showed that the model gave better accuracy for the study locations. When compared to the failure load, the model was found to be conservative with relative accuracy of 90.7%. The results signified that the model can be adopted for preliminary design in areas with similar subsurface conditions.

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