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# *Problem Solving Method and Pre-service Mathematics Teachers Conceptual and Procedural Understanding of Linear Functions and Graphing*

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## *Abstract*

*This study aimed at investigating the effect of Problem Solving on conceptual and procedural understanding of pre-service mathematics teachers on the graphs of linear functions. The study employed mixed method approach. A pretest-posttest equivalent group design was used. Twenty four regular second year pre-service mathematics teachers participated in this study as Intervention Group( IG) and twenty four pre-service mathematics teachers participated as Comparison Group(CG). Pre-service teachers' conceptual and procedural understanding was diagnosed using Conceptual Understanding Diagnose Test (CUDT) Procedural Understanding Diagnose Test(PUDT) and interview. Cronbach's alpha values of the internal consistence are 0.747and 0.749 respectively for CUDT and PUDT. Data were analyzed using the paired samples t-test, the independent samples t-test for quantitative part and narrative was used for qualitative data. Findings of the study depicted that there was statistically significant difference between the pretest and posttest in conceptual and procedural understanding in IG, but there was no statistically significant difference between the pre-test and posttest results for CG. Similarly, results obtained from the independent samples t-test revealed that the two groups were almost equal in pre-test results, but the IG outscored the CG in the posttest. This means the IG developed more conceptual and procedural understanding on linear functions and graphing as compared to the CG. It further implies that instruction in graphs of linear functions needs to be supported with appropriate use of problem solving method in order to improve pre-service teachers' conceptual and procedural understanding on the graphs of Linear Functions.*

*Key words: Problem Solving method, Conceptual Understanding, Procedural Understanding,*

## *Introduction*

*Education can be made more effective when there is new techniques of teaching and learning (Iqbal, 2004)as cite in(Perveen, 2010) . Skinner (1984) states that the term "problem-solving" is defined as the frame work or pattern within which creative thinking and learning takes place cited in (Perveen,2010). Polya et al., (1945) defines problem-solving as the process used to solve a problem that does not have an obvious solution as cited in (Perveen, 2010). One way to teach students to problem solve is to teach the four-step processes developed by(Polya et al., 1945) :*

*understand the problem, devise a plan, carry out the plan, and look back. Several studies focus the change in knowledge and skill levels that occur with problem-solving techniques. problem solving is a deliberate and serious act that involves the use of higher order thinking and systematic planned steps for the acquisition set goals(Ali, 2010). Presenting the students with a problem, give them opportunity to take risks, to adopt new understandings, to apply knowledge, to work in context(Ali,2010). The individual, autonomous self-directed learning gives the freedom to the learner to decide individually and consciously on the learning strategy and on the time scale,the learner wants to follow(Ali,2010).*

*Conceptual knowledge denotes knowledge of particular networks, the elements of which can be concepts,rules and problems given in various representation forms(Haapasalo, 2003). Conceptual knowledge is the understanding of concepts in the minds of students(Surif et al., 2020). Conceptual knowledge is something that is rich in relationships and in which linking relations areas important as each piece of information itself (Hiebert & Lefevre, 1986; Hiebert & Wearne, 1986).*

*Procedural understanding denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representation forms(Haapasalo,2003). This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them(Haapasalo,2003). Procedural knowledge relies very much on computational skills and utilisation of procedures within different representation forms(Haapasalo,2003).*

*Mathematical competence rests on developing knowledge of concepts and procedures (Rittle-johnson et al., 2015).*

*Procedural knowlede often calls for automated and unconscious steps, whereas conceptual knowledge typically requires conscious thinking.*
*The knowledge of concepts and procedures is imperative for competence in mathematics(Zuya, 2017).Knowledge of subect matter is essential for mathematics tacher to be competent and effective in teaching because subject mattter knowledge is a combination of concepts and procedures(Zuya,2017).*
*Mathematical knowledge consists of both procedural and conceptual knowledge, and"linking conceptual and procedural knowledge would have many advantages for acquiring and using procedural knowledge" (Hiebert & Lefevre 1986). Perhaps it is Possible to possess one of them, but this is incomplete in the sense that one can have a good feel for mathematics without being able to perform calculations or one can be able to calculate answers without understanding their meaning.*

*Heibert and Wearne (1996) claim that mathematical incompetence often is due to absence of connection between conceptual and procedural knowledge. It is hard to operate on functions without knowing the concept of function, but it is also unlikely that one is able to put meaning into functions without being able to operate on them.*

## *Statement of the Problem*

*Functions play a crucial role throughout the mathematics curriculum. Students" earliest experiences with functions typically involve the study of linear relationships, building a foundation on which more advanced functional relationships are built (Nagle & Moore-Russo, 2013; NGA Center & CCSSO, 2010) and students need a comprehensive knowledge of a procedure, along with an ability to make critical judgments about which procedure is appropriate for use in a particular situation (Star, 2005) . Studies have found both U.S. and international students have a minimal understanding of slope (Greens, Chang, & Ben-Chaim, 2007) and experience various conceptual difficulties (Hattikudur, Prather, Asquith, Knuth, Nathan & Alibali, 2011; Lobato & Siebert, 2002; Simon & Blume, 1994; Stump, 2001a; Stump, 2001b; Teuscher & Reys 2010; Zaslavsky, Sela, & Leron, 2002) as cited in (Cho et al., 2017).Similarilty,pre-service mathematics teachers need to developm both conceotual and procedural understanding graphs of linear functions which is a basis foradvanced courses such as calculus and algebra.Moreove;it is relevant for primary school mathematics they are goning to teach. Mathematics teachers'understanding of quadratic functions is critical for student success in mathematics and there appears to be agreement that for many high school students, solving and understanding quadratic functions can be conceptually challenging because of the need to make connections between various representations of the function as well as connections between the various forms in which the quadratic equation can be expressed as. (Didis, Bas, & Erbas, 2011; Kilic, 2009) as cited in (Ubah & Bansilal, 2018).*

*Eventhough varios studies were conducted on pre-service understanding of functions ,the researcher believe that there is still a gap in considering the relation of the problem solving and pre-service teachers conceptual and procedural understanding.Therefore ,this research was conducted to investigate the instructiona use of problem solving method on per-service teachers's conceptual and procedural understanding in graphs of linear functions,specifically to answer the following basic questions:*

1. *What is the effect of Problem Solving Method on pre-service mathematics teachers conceptual understanding of graphs of linear functions?*
2. *What is the effect of Problem Solving Method on pre-service mathematics teachers procedural understanding of graphs of linear functions?*
3. *What relation is there between conceptual understanding and procedural understanding?*

### *Significance of the study*

*This section provides information to the reader on how the study contributes and this study may add to the literature as reference materials for further related research work. Specifically, the main objectives of this study was to examine the effect of problem solving method on mathematics pre-service teachers conceptual and procedural understanding on the graphs of linear functions . Beside this, the following are among the main contributions of the study:*

1. *Provide a valuable information on the effect of problem solving method on conceptual and procedural understanding of the graphs of linear functions;*

2. *Pre-service mathematics teachers grasp conceptual and procedural understanding on sketching graphs of linear functions;*
3. *It encourages other researchers who want to conduct further study in this area.*

## *Methodology*

### *Design of the Study*

*The pretest-posttest equivalent group experimental design was used. Students were randomly assigned as intervention group and comparison group based on their pre-test results. This was chosen because it was possible to assign the groups in the college. The pretest-posttest design was required for the intervention group to know the effect of the intervention. The pretest is required for both groups before the intervention in order to have some idea on how similar or different the two groups are. Similarly, the posttest is required to know the effect of the intervention. There were one intervention and one comparison groups. The intervention group received the instruction using problem solving method and the comparison group received only the traditional method of instruction. Treatment took four weeks on the concept of graphs of linear functions by giving instruction for comparison groups using traditional method and intervention groups using problem solving method .*

### *Method of the Study*

*This study employed both qualitative and quantitative mixed research methods since it is data were collected using quantitative and qualitative tools. For the quantitative the study, the pre-test-posttest experimental research method was used .To this end, data were collected from CUDT and PUD, multistep questions prepared by teacher to measure conceptual and procedural understanding on which rubric measure were applied. Both of them involved numerical data. These data were recorded; analyzed and interpreted using numerical data. To help generate meaning, qualitative method was also employed in which data were collected from interview both in pre-test and in posttest. Finally, to generate meaning on pre-service teachers' conceptual and procedural understanding, data collected from the quantitative and qualitative were triangulated.*

### *Participants of the Study*

*There was one mathematics second year pre-service section in Fiche College of teaches Education in 2018 academic year. The researcher classified pre-service teachers in to two equivalent groups based on their pre-test results on conceptual and procedural understanding. There were 24 pre-service teachers in the intervention group and 24 pre-service teachers in the comparison group with a total of 48 pre-service teachers.*

***Instruments***

## *CUDT and PUDT Items*

*A CUDT items consisting of 6 questions were administered both before and after the intervention in order to diagnose the magnitude of conceptual understanding of pre-service teachers. Similarly, PUDT items were applied to diagnose the magnitude of procedural understanding of pre-service teachers. Pre-service teachers' conceptual understanding was measured based on Polya's stages: understanding the problem, devising a plan, carrying out a plan and looking back. Each Polya's(1971) stage is accompanied with four scale lengths of rubric measure (0, 1, 2, & 3). The point 0 was given if pre-service teachers can give no response, the point 1 was given if pre-service teachers respond but all inappropriate, the point 2 was given if pre-service teachers respond but miss most part and the point 3 was given if pre-service teachers respond complete and appropriate answers to the given problem.*

## *Interview*

*Interview was made with students' on conceptual and procedural understanding on the graph of linear functions before and after the intervention in order to in order to generate meaning and make triangulation with quantitative data.*

***Procedure of Data Collection***

*The teacher made test consisting of conceptual and procedural understanding questions involving graphs of linear functions was administered as a pretest. Based on Polya's(1971) model rubric measure were applied to measure conceptual understanding and procedural understanding . Activities were planned before intervention was made. Orientation was given to pre-service teachers prior to intervention. The intervention was conducted for four weeks during block periods. Problem solving method was used as teaching methods during intervention to address pre-service teachers' conceptual and procedural understanding of graphs of linear functions. Practical activities involving conceptual and procedural understanding were prepared. Pre-service teachers were given opportunities to solve problems on their own. Finally, after the intervention took place, posttest teacher made test consisting of conceptual and procedural questions equivalent with the pretest was employed.*

***Data analysis***

1. *Data gathered from pretest and post-test were analyzed using SPSS version 20. The paired samples t-test was used to analyze if there is statistically significant difference between pretest and posttest results within the same group. Similarly, the independent samples t-test was applied to compare if there is statistically significant difference between the two groups both in the pre-test and in the posttest.*
2. *Data gathered from interview were themed and analyzed narratively.*
3. *Results obtained from quantitative and qualitative tools were triangulated.*

## *Results and Discussion*

### *Magnitude of pre-service teachers' conceptual and procedural understanding*

*In order to know the magnitude of Pre-service teachers conceptual and procedural understanding data were collected from pretest and post- test using teacher made test items and the results were presented in the following table.*

***Table 1.****Conceptual Understanding and Procedural understanding pretest-posttest comparison for the Intervention and comparison Groups*

| *Variable* | | | *Pre-test* | | *Post-test* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Groups* | *N* | *M* | *SD* | *M* | *SD* | *t* | *p* |
| *Conceptual understanding* | *IG* | *24* | *1.295* | *.34561* | *2.185* | *.24767* | *-9.857* | *0.00* |
| | *CG* | *24* | *1.291* | *.31* | *1.583* | *.29* | *-.691* | *.512* |

*The paired samples t-test was used to test the differences between the pretest and posttest of targeted group with respect to conceptual understanding. Results in table 1 above indicates that the means and standard deviations were M=1.295, SD=.34561 in the pretest and M=2.1850, SD=.24767 in the posttest. This shows that the differences from pretest to posttest for the targeted group were statistically significant for conceptual understanding, which implies the intervention group performed better in conceptual understanding on linear functions and their graphs as compared to the comparison group.*

***Table 2.*** *Procedural knowledge pretest-posttest comparison for the Intervention and comparison Groups*

| *Variable* | | | *Pre-test* | | *Post-test* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Groups* | *N* | *M* | *SD* | *M* | *SD* | *t* | *p* |
| *Procedural Knowledge* | *IG* | *24* | *1.655* | *.3031* | *2.4* | *.23396* | *-10.783* | *0.00* |
| | *CG* | *24* | *1.708* | *.64* | *1.83* | *.66* | *-.919* | *.388* |

*Similarly, the paired samples t-test in table 2 revealed that the means and standard deviations were M=1.295, SD=.34561 in the pretest and M=2.1850, SD=.24767 in the posttest for intervention group. This shows that the differences from pretest to posttest for the targeted group were statistically significant for conceptual understanding, which implies the intervention group performed better in procedural understanding on linear functions and their graphs as compared to the comparison group.*

***Table 3.*** *Comparison between the Intervention and comparison groups for Conceptual understanding pretest-posttest*

| *Variables* | *Measure* | *Groups* | *N* | *M* | *SD* | *df* | *t* | *P* |
|---|---|---|---|---|---|---|---|---|
| | | *IG* | *24* | *1.295* | *.32* | *46* | *.734* | *.465* |
| | *Pretest* | *CG* | *24* | *1.291* | *.35* | | | |
| | | *IG* | *24* | *2.185* | *.38* | *46* | *2.425* | *.017* |
| *Conceptual Knowledge* | *Posttest* | *CG* | *24* | *1.583* | *.37* | | | |

*The independent samples t-test in table 3 depicted that results for were M=1.295, SD=.32 for experimental group and M=1.291, SD=.35 for comparison group in conceptual knowledge. There is no significant difference between the groups in the pretest for conceptual knowledge . The post-test results showed that M=2.4, SD=.38 for intervention group and M=1.83, SD=.37 for comparison group in conceptual knowledge. This means pre-service teachers who received instruction using problem solving method develop better conceptual understanding in the post test as compared to the comparison group.*

***Table 4.*** *Comparison between the Intervention and comparison groups for Procedural understanding of pretest-posttest*

| *Variable* | *Measure* | *Groups* | *N* | *M* | *SD* | *df* | *t* | *P* |
|---|---|---|---|---|---|---|---|---|
| | | *IG* | *25* | *1.655* | *.71* | *47* | *-.825* | *.412* |
| | *Pretest* | *CG* | *24* | *1.708* | *.72* | | | |
| | | *IG* | *25* | *2.4* | *1.04* | *47* | *4.188* | *.00* |
| *Procedural understanding* | *Posttest* | *CG* | *24* | *1.83* | *.707* | | | |

*Similarly, M=1.655, SD=.71 for Intervention group and M=1.708, SD=.72 for comparison group in the pretest for procedural understanding. There was no significant difference between the groups in the pretest for procedural knowledge. The post-test results showed that M=2.4, SD=1.04 for intervention group and M=1.83, SD=.707 for comparison group for procedural knowledge. This shows that there was statistically significant difference between the intervention and comparison groups in procedural knowledge.*

*Figure 1.shows a partial view of a pre-service teacher work during pretest. The candidate was asked to determine whether the slope of a given line negative or positive based on the given graph. The candidate has missed the concept of a decreasing and increasing functions where we can easily determine the slope of a given line to be negative or positive.*

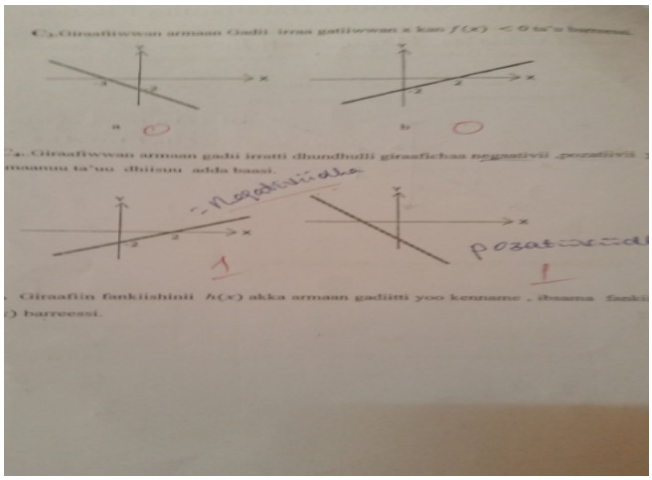
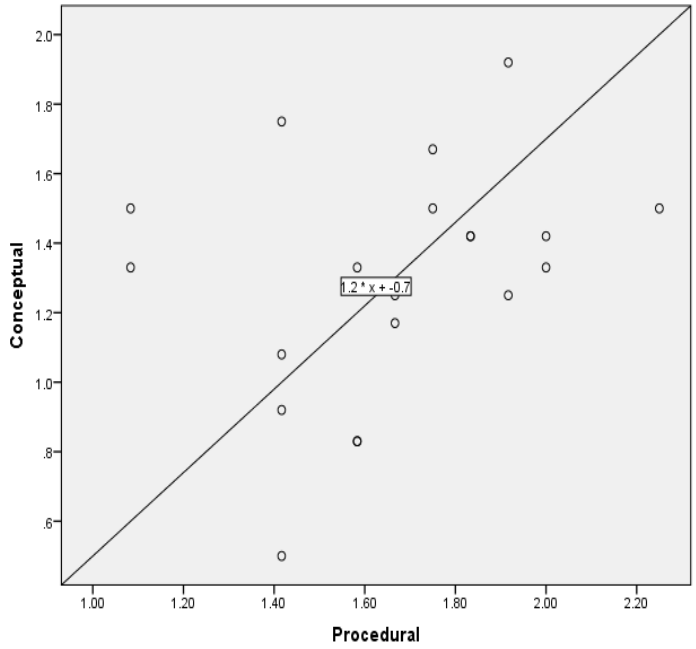


*Fig. 1 Partial view of student work*




*The above figure shows that there is a positive correlation between students conceptual and procedural understanding. This can be shown by the linear equation* $y = 1.2x + -0.7$ *This shows students who performed well in conceptual understanding also performed well in procedural understanding*

***Figure 2.*** *The Correlation between Scores in Procedural & Conceptual Understanding*



## *Conclusions*

*Based on the statistical analysis and findings of the study, the following conclusions were made:*

1. *Results obtained from CUDT items depicted that there were statistically significant different between the IG and the CG in conceptual understanding. This means the employed of problem solving method on the IG significantly increase the conceptual understanding of IG as compared to CG and there was statistically significant change in conceptual understanding for the CG.*
2. *Results obtained from PUDT reveals that there was statistically significant difference between in the post-test result between the means of the two groups. This means the intervention of problem solving method significantly improve the procedural understanding of the IG, but the CG, which received the traditional teaching method, did not show statistically significant change in means of procedural understanding in sketching graphs of linear functions.*
3. *The methods of teaching play an important role in conceptual and procedural understanding on mathematics. The art of problem solving is the heart and essence of mathematics. Therefore, it was necessary to conduct a study for improving conceptual and procedural understanding of graphs of linear functions of pre-service mathematics teachers using problem-solving method, as it is a vehicle for learning new mathematical ideas and skills.*

***Contributions of the Study***

- *Knowing the relationship between conceptual understanding and procedural understanding is important. but, whether the relationship is bi-directional or one way direction needs further study;*
- *This study contributes to pedagogical approaches for improved conceptual and procedural understanding of mathematical concepts;*
- *Finally, the study may serve as a reference and encourage other researchers who want to conduct further study in this area.*

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