Global Scientific JOURNALS

GSJ: Volume 9, Issue 7, July 2021, Online: ISSN 2320-9186
www.globalscientificjournal.com

Quantitative study of two incoherent waves interfering in a homogeneous medium.

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KEYWORDS: Band spectrum, carrier waves, Green's function, homogeneous medium, host parasite, incoherent waves and parasitic wave.

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ABSTRACT

Interference effect that occurs when two or more waves overlap or intersect is a common phenomenon in physical wave mechanics. Driving forces in anti-phase, which occurs when the two interfering phase angles are oppositely related, provide full destructive superposition and the minimum possible amplitude; while driving forces in phase, that is, when the two phase angles are equal, provides full constructive superposition and maximum possible amplitude. In this study, we used the Green's function technique to evaluate the behaviour of a 2D constitutive carrier wave propagating in a cylindrical pipe of a known length and diameter. A constitutive carrier wave in this wise, is a corrupt wave function, it is the resultant of the interference of a parasitic wave on a host wave. Also in this work, we showed quantitatively the method of determining the intrinsic characteristics of the constituents of a constitutive carrier wave which are initially not known. It is evident from this study, that the frequency and the band spectrum of the Green's function are higher than those of the general solution of the propagating constitutive carrier wave. The study also shows that the band spectrum of the constitutive carrier wave does not attenuate completely to zero due to the presence of some residual signal still present in it even when the parasitic wave would have completely eroded the active wave characteristics of the host wave. The point source driven potential of the Green's function is spherically symmetric about the source, and falls off smoothly with increasing distance from the source.

1.0 INTRODUCTION

If a wave is to travel through a medium such as water, air, steel, or a stretched string, it must cause the particles of that medium to oscillate as it passes [1]. For that to happen, the medium must possess both mass (so that there can be kinetic energy) and elasticity (so that there can be potential energy). Thus, the medium's mass and elasticity property determines how fast the wave can travel in the medium.

When a wave equation ψ and its partial derivatives never occur in any form other than that of the first degree, then the wave equation is said to be linear. Consequently, if ψ_1 and ψ_2 are any two solutions of the wave equation ψ, then $a_1\psi_1 + a_2\psi_2$ is also a solution, a_1 and a_2 being two arbitrary constants [2, 3]. This is an illustration of the principle of superposition, which states that, when all the relevant equations are linear we may superpose any number of individual solutions to form new functions which are themselves also solutions.

Interference effect that occurs when two or more waves overlap or intersect is a common phenomenon in physical wave mechanics. When waves interfere with each other, the amplitude of the resulting wave depends on the frequencies, relative phases and amplitudes of the interfering waves. The resultant amplitude can have any value between the differences and sum of the individual waves [4].

The principle of superposition of wave states that if any medium is disturbed simultaneously by a number of disturbances, then the instantaneous displacement will be given by the vector sum of the disturbance which would have been produced by the individual waves separately. Superposition helps in the handling of complicated wave motions. It is applicable to electromagnetic waves and elastic waves in a deformed medium provided Hooke's law is obeyed [5].

In this work, we studied the basic properties associated with a constitutive carrier wave CCW when propagating in a hollow pipe. The constitutive carrier wave is the resultant of the superposition of a parasitic wave on a host wave. A parasitic wave as the name implies, has the ability of destroying and transforming the intrinsic constituents of the host wave to its form after a sufficiently long time. It contains an inbuilt raising multiplier λ which is capable of increasing the intrinsic parameters of the parasitic wave to become equal to those of the host wave. Ultimately, once this equilibrium is achieved, then all the active components of the host wave would have been completely eroded and the constituted constitutive carrier wave ceases to exist [6].

Green's Theorem is another higher dimensional analogue of the fundamental theorem of calculus: it relates the line integral of a vector field around a plane curve to a double integral of "the derivative" of the vector field in the interior of the curve. It admits two different but completely equivalent formulations, a "flux" version for normal line integrals and a "circulation" version for tangent line integrals. We have already predicted the former version as an "integral form" of the fact that divergence is equal to flux density [7].

The Green's function is a tool to solve non-homogeneous linear equations. The method of Green's functions can be used to solve other equations, in 1D, 2D and 3D.The Green's function method can also be used to solve time-dependent problems, such as the wave equation and the heat equation [8]. Remarkably, a Green's function can be used for problems with inhomogeneous boundary conditions even though the Green's function itself satisfies homogeneous boundary conditions. It is evident that one very general way to solve inhomogeneous partial differential equations (PDEs) is to build a Green's function and write the solution as an integral equation [9].

The organization of this paper is as follows. In section 1, we discuss the nature of wave and interference. In section 2, we show the mathematical theory. The results emanating from this study is shown in section 3. The discussion of the results of our study is presented in section 4. Conclusion and suggestions for further work is discussed in section 5. The paper is finally brought to an end by a few lists of references and appendix.

1.1 Research methodology.

In this work, the constitutive carrier wave which is the resultant of the superposition of two incoherent waves is allowed to propagate in a narrow pipe containing air. We then used the Green's function technique to study the attenuation process of the constitutive carrier wave during the propagation.

2.0 Mathematical theory and the general wave equation.

Generally, the wave equation (WE) can be described by two basic equation given below.

$$\nabla^2\phi - \in\mu\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\in} \qquad (2.1)$$

$$\nabla^2 A - \in\mu\frac{\partial^2 A}{\partial t^2} = -\mu J \qquad (2.2)$$

where ∇^2 is a three dimensional (3D) Laplacian operator in Cartesian coordinate system, the scalar potential is given by ϕ, the vector potential is given by A, the charge density is ρ, the permittivity is ϵ, while the permeability is μ, and the current density is J, the permittivity and the permeability of air is $\in$ and μ respectively. It is very obvious that both wave equations have the same basic structure; hence in a free space we can write a single wave that would connect the two equations as follows.

$$\left(\nabla^2 - \in\mu\frac{\partial^2}{\partial t^2}\right)\varphi(x,t) = -f(x,t) \qquad (2.3)$$

Where $f(x,t)$ is a known source distribution having space – time functions. The solutions to (2.3) are superposable (since the equation is linear), so a Green's function method of solution is appropriate. The Green's function $G(x,t\,|\,x',t')$ is the potential generated by a point impulse located at position x' and applied at time

t' [13, 14]. Now to solve (2.3) we find the Green's function for the equation, that is, we replace φ by G and $f(x,t)$ by Dirac delta δ and obtain expression for Green's function as

$$\left(\nabla^2 - \in \mu \frac{\partial^2}{\partial t^2}\right) G(x,t \,|\, x',t') = -\,\delta(x-x')\,\delta(t-t') \qquad (2.4)$$

Hence, (2.4) is the Green's function for one dimensional (1D) space. However, the Laplacian in 3D Cartesian space is given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (2.5)$$

Suppose we confine the motion to two coordinates x and y axes, that is, we make the motion to be constant with respect to one of the axes, say z - axis, then the Laplacian becomes

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad (2.6)$$

The variation in the Laplacian will also lead to a variation in the Green's function. Green's function depends both on a linear operator and boundary conditions. As a result, if the problem domain changes, a different Green's function must be found. Accordingly, we get

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \in \mu \frac{\partial^2}{\partial t^2}\right) G(x,y,t \,|\, x',y',t') = -\,\delta(x-x')\delta(y-y')\delta(t-t') \qquad (2.7)$$

The Dirac delta δ function in (2.7) thus represents two coordinate systems. It is defined accordingly as

$$\delta(x-x')\delta(y-y')\delta(t-t') = \frac{1}{(2\pi)^4}\int d^3k \int d\omega\, e^{i(k_x - k_x'\lambda)(x-x')} e^{j(k_y - k_y'\lambda)(y-y')} \times$$

$$e^{-i\,[(\omega_x - \omega_x'\lambda)(t-t') - E(t)]} \times e^{-j\,[(\omega_y - \omega_y'\lambda)(t-t') - E(t)]} \qquad (2.8)$$

In differential equations, there is a great need to define objects that arise as limits of functions and behave like functions under integration but are not, properly speaking, functions themselves. These objects are sometimes called generalized functions or distributions. The most basic one of these is the so-called delta δ-function. Since we are dealing with dynamic variable coordinates, the source function is normally represented by the delta function. However, the last exponential function in the integrand does not depend on any coordinate. As a result, it can be contracted by setting the direction $j = i$, then the result yields

$$\delta(x-x')\delta(y-y')\delta(t-t') = \frac{1}{(2\pi)^4}\int d^3k \int d\omega\, e^{i(k_x - k_x'\lambda)(x-x')} e^{j(k_y - k_y'\lambda)(y-y')} \times e^{-2i\,[(\omega_x - \omega_x'\lambda)(t-t') - E(t)]} \qquad (2.9)$$

The Green's function is related to the Dirac delta function. Both of them must be 2D with x and y components.

$$G(x,y,t \,|\, x'.y',t') = \frac{1}{(2\pi)^4}\int d^3k \int d\omega\, g(k,\omega)\, e^{i(k_x - k_x'\lambda)(x-x')} e^{j(k_y - k_y'\lambda)(y-y')} \times e^{-2i\,[(\omega_x - \omega_x'\lambda)(t-t') - E(t)]} \qquad (2.10)$$

where $g(k,\omega)$ is the Fourier component or the scattering amplitude. Substitution of (2.10) into (2.7) and equate the result of the substitution to (2.8) then the result is

$$\left\{\left(i\,(k_x - k_x'\lambda)\right)^2 + \left(j\,(k_y - k_y'\lambda)\right)^2 - \in \mu \left(-2i\left((\omega - \omega'\lambda) - z(t)\right)^2\right)\right\} \times g(k,\omega) = -1 \qquad (2.11)$$

$$g(k,\omega) = \frac{1}{\left\{(k_x - k_x'\lambda)^2 + (k_y - k_y'\lambda)^2 - 4 \in \mu \left((\omega - \omega'\lambda) - z(t)\right)^2\right\}} \qquad (2.12)$$

If the wave number mode is the same irrespective of the coordinate axes, that is, $k_y = k_x$ and $j = i$, then

$$g(k,\omega) = \frac{1}{\left\{2(k-k'\lambda)^2 - 4\in\mu\left((\omega-\omega'\lambda)-z(t)\right)^2\right\}} \qquad (2.13)$$

$$G(x,y,t\left|x'.y',t'\right.) = \frac{1}{(2\pi)^4}\int d^3k\int d\omega \frac{e^{2i(k-k'\lambda)|x-x'|}e^{-2i\left[(\omega-\omega'\lambda)(t-t')-E(t)\right]}}{2(k-k'\lambda)^2 - 4\in\mu\left((\omega-\omega'\lambda)-z(t)\right)^2} \qquad (2.14)$$

$$G(x,y,t\left|x'.y',t'\right.) = \frac{1}{(2\pi)^4}\int d^3k\int d\omega \frac{e^{2i\left[(k-k'\lambda)|x-x'|-((\omega-\omega'\lambda)(t-t')-E(t))\right]}}{2(k-k'\lambda)^2 - 4\in\mu\left((\omega-\omega'\lambda)-z(t)\right)^2} \qquad (2.15)$$

2.1 Evaluation of the retarded distance and retarded time.

Accordingly, because of the uniform nature of (2.15) the poles can be found when we factorize it as follows.

$$2(k-k'\lambda)^2 - 4\in\mu\left((\omega-\omega'\lambda)-z(t)\right)^2 =$$

$$2\left((k-k'\lambda)+\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right)\left((k-k'\lambda)-\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right) \qquad (2.16)$$

$$(k-k'\lambda) = \sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right) \qquad (2.17)$$

The integral (2.15) vanishes unless the exponential power in the numerator of the integrand is equal to zero.

$$2i\left[(k-k'\lambda)\left|x-x'\right|-(\omega-\omega'\lambda)(t-t')-E(t)\right] = 0 \qquad (2.18)$$

$$\left[\left(\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right)\left|x-x'\right|-(\omega-\omega'\lambda)(t-t')-E(t)\right] = 0 \qquad (2.19)$$

$$\left|x-x'\right| = \frac{(\omega-\omega'\lambda)(t-t')-E(t)}{\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)} \qquad (2.20)$$

Also, from equation (2.19) we can equally factor out the difference in the time component. That is we have

$$(t-t') = \frac{\left(\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right)\left|x-x'\right|+E(t)}{(\omega-\omega'\lambda)} \qquad (2.21)$$

$$x' = x - \frac{(\omega-\omega'\lambda)(t-t')-E(t)}{\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)} = 0 \qquad (2.22)$$

$$t' = t - \frac{\left(\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right)\left|x-x'\right|+E(t)}{(\omega-\omega'\lambda)} \qquad (2.23)$$

Equation (2.23) simply means that the causal behaviour or the effect associated with a wave at the point x and time t is due to a disturbance which originated at an earlier or retarded time t'. The reader should note that $\sqrt{\in\mu}\left|x-x'\right|$ is a time component. Now using (2.16) in the denominator of (2.15) we get

$$G(x,y,t\left|x'.y',t'\right.) = \frac{1}{2(2\pi)^4}\int d^3k\int d\omega e^{2i\left[(k-k'\lambda)|x-x'|-((\omega-\omega'\lambda)(t-t')-E(t))\right]}\times$$

$$\frac{1}{\left((k-k'\lambda)+\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right)\left((k-k'\lambda)-\sqrt{2\in\mu}\left((\omega-\omega'\lambda)-z(t)\right)\right)} \qquad (2.24)$$

2.2 Evaluation of the Green's function by contour integration.

Equation (2.24) can be solved by contour integration. We can also determine the validity of the Green's function $G(x,y,t|x'.y',t')$ by observing the poles of the equation. Now due to the quadratic nature of the denominator of (2.24), there are two possible poles say $f(z_1)$ and $f(z_2)$ in the integrand, that is,

$$f(z_1) = f(\omega-\omega'\lambda) = -\left(\frac{(k-k'\lambda) + z(t)\sqrt{2\in\mu}}{\sqrt{2\in\mu}}\right) = -W \qquad (2.25)$$

$$f(z_2) = f(\omega-\omega'\lambda) = +\left(\frac{(k-k'\lambda) + z(t)\sqrt{2\in\mu}}{\sqrt{2\in\mu}}\right) = +W \quad (2.26)$$

Thus, if we carry out a contour integration along the part of the upper and the lower half planes, then in either case, the residue of each pole would contribute to the integral. While residue $f(z_1)$ contributes to the integral in the left lower quarter plane, $f(z_2)$ contribute to the integral in the right upper quarter plane. The residue theorem is the single biggest tool we have for evaluating (real-valued) integrals and series. It is the complex version of the weak path-independence property of irrotational vector fields. Thus, the residue of $f(z_1)$ and $f(z_2)$ at the poles is

$$\text{Residue of } f(z_1) = -\frac{e^{2i\left[(k-k'\lambda)|x-x'| - \left(-W(t-t')-E(t)\right)\right]}}{2W} \qquad (2.27)$$

$$\text{Residue of } f(z_2) = \frac{e^{2i\left[(k-k'\lambda)|x-x'| - \left(W(t-t')-E(t)\right)\right]}}{2W} \qquad (2.28)$$

If we define the sum of the residues as $f(z_1) + f(z_2) = f(z)$ then

$$f(z) = \frac{e^{2i\left[(k-k'\lambda)|x-x'| - \left(W(t-t')-E(t)\right)\right]} - e^{2i\left[(k-k'\lambda)|x-x'| + \left(W(t-t')+E(t)\right)\right]}}{2W} \quad (2.29)$$

$$f(z) = \frac{e^{2i\left[(k-k'\lambda)|x-x'|\right]}e^{-2i\left(W(t-t')-E(t)\right)} - e^{2i\left[(k-k'\lambda)|x-x'|\right]}e^{+2i\left(W(t-t')+E(t)\right)}}{2W} \quad (2.30)$$

$$f(z) = \frac{e^{2i\left[(k-k'\lambda)|x-x'|\right]}}{2W}\left(e^{-2i\left(W(t-t')-E(t)\right)} - e^{+2i\left(W(t-t')+E(t)\right)}\right) \quad (2.31)$$

$$f(z) = \frac{e^{2i\left[(k-k'\lambda)|x-x'|\right]}e^{2iE(t)}}{2W}\left(e^{-2i\left(W(t-t')\right)} - e^{+2i\left(W(t-t')\right)}\right) \quad (2.32)$$

$$f(z) = -\frac{e^{2i\left[(k-k'\lambda)|x-x'| + E(t)\right]}}{2W}\left(e^{+2i\left(W(t-t')\right)} - e^{-2i\left(W(t-t')\right)}\right) \quad (2.33)$$

We can use the identity given below for further simplification of the sum of the residues. $e^{i\theta} = \cos\theta + i\sin\theta$ and also in combination with $e^{-i\theta} = \cos\theta - i\sin\theta$, hence, $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$, so that upon addition we get $e^{2i\theta} - e^{-2i\theta} = 4i\sin\theta$. As a result

$$\left(e^{+2i\left(W(t-t')\right)} - e^{-2i\left(W(t-t')\right)}\right) = 4i\sin\left(W(t-t')\right) \quad (2.34)$$

$$f(z) = -i\,\frac{4\sin\left(W(t-t')\right)e^{2i\left[(k-k'\lambda)|x-x'| + E(t)\right]}}{2W} \quad (2.35)$$

Hence by Cauchy's Residue theorem, the integral (2.15) becomes

$$G(x,y,t|x'.y',t') = \frac{1}{2(2\pi)^4}\left(2\pi i \times \textit{sum of the residues } f(z)\right) \qquad (2.36)$$

$$G(x,y,t\,|\,x'.y',t') = \frac{2\pi i}{2(2\pi)^4}\left(-\ i\ \frac{4\sin\left(W(t-t')\right)e^{2\,i\,\left[(k-k'\lambda)\,|x-x'|\ +\ E(t)\right]}}{2W}\right) \quad (2.37)$$

$$G(x,y,t\,|\,x'.y',t') = \frac{1}{(2\pi)^3}\left(\frac{\sin\left(W(t-t')\right)e^{2\,i\,\left[(k-k'\lambda)\,|x-x'|\ +\ E(t)\right]}}{W}\right) \quad (2.38)$$

The exponential function in the numerator of equation (2.38) can further be simplified as follows:

$$e^{2i\left[\left((k-k'\lambda)\,|x-x'|-E(t)\right)\right]} = \cos 2\left((k-k'\lambda)\,|x-x'|-E(t)\right)+i\sin 2\left((k-k'\lambda)\,|x-x'|-E(t)\right) \quad (2.39)$$

When (2.39) is substituted into (2.37) and the magnitude or the absolute value of the resulting equation is taken due to the presence of the imaginary function we get after simplification

$$G(x,y,t\,|\,x'.y',t') = \frac{1}{(2\pi)^3 W}\left(\sin\left(W(t-t')\right)\cos 2\left((k-k'\lambda)\,|x-x'|\ +E(t)\right)\ +\right.$$

$$\left. i\sin 2(k-k'\lambda)\ |x-x'|\ +E(t)\ )\ \right) \quad (2.40)$$

To further reduce (2.40) we recall that $\cos 2A = 1-2\sin^2 A$ and $\sin 2A = 2\sin A\cos A$ so that we can write

$$G(x,y,t\,|\,x'.y',t') = \frac{1}{(2\pi)^3 W}\sin\left(W(t-t')\right)\left\{1-2\sin^2\left[(k-k'\lambda)\,|x-x'|\ +E(t)\right]+\right.$$

$$\left. 2i\sin\left((k-k'\lambda)\,|x-x'|\ +E(t)\right)\cos\left((k-k'\lambda)\,|x-x'|\ +E(t)\right)\right\} \quad (2.41)$$

$$G(x,y,t\,|\,x'.y',t') = \frac{1}{(2\pi)^3 W}\left\{\sin\left(W(t-t')\right)-2\sin\left(W(t-t')\right)\sin^2\left((k-k'\lambda)\,|x-x'|\ +E(t)\right)+\right.$$

$$\left. 2i\sin\left(W(t-t')\right)\sin\left((k-k'\lambda)\,|x-x'|+E(t)\right)\cos\left((k-k'\lambda)\,|x-x'|\ +E(t)\right)\right\} \quad (2.42)$$

Thus, the Green's function is made up of the real part and imaginary part. The real and the imaginary part of the Green's function are respectively given by

$$G(x,y,t\,|\,x'.y',t') = \frac{1}{(2\pi)^3 W}\left(\sin\left(W(t-t')\right)-2\sin\left(W(t-t')\right)\sin^2\left((k-k'\lambda)\,|x-x'|\ +E(t)\right)\right) \quad (2.43)$$

$$G(x,y,t\,|\,x'.y',t') = \frac{2}{(2\pi)^3 W}\left(\sin\left(W(t-t')\right)\sin\left((k-k'\lambda)\,|x-x'|+E(t)\right)\cos\left((k-k'\lambda)\,|x-x'|+E(t)\right)\right) \quad (2.44)$$

The Green's function in this wise is time dependent and the unit is in seconds (*s*). However, for the purpose of the present study, we shall only concern our work with the real component of the Green's function.

2.3 General solution of the wave equation with respect to the constitutive carrier wave (CCW).

It follows that the potential generated by $\Psi(r,t)$ can be written as the weighted sum of point impulse driven potentials. Hence, the general solution of the wave equation (3) is

$$\Psi(r,t) = \int |\psi(x,y)|\,G(r,t\,|\,r',t')\,dr'dt' \quad (2.45)$$

$$\Psi(x,y;t)=\int\left|\psi(x,y)\right|G(x,y,t\left|x',y',t'\right)dx'dy'dt' \qquad (2.46)$$

If such a representation exists, the kernel of this integral operator $G(x,y,t\left|x',y',t'\right)$ is called the Green's function. Hence we think of $\Psi(x,y;t)$ as the response at x and y to the influence given by a source function $\psi(x,y)$. For example, if the problem involved elasticity, $\Psi(x,y;t)$ might be the displacement caused by an external force $f(x,t)$. If this were an equation describing heat flow, $\Psi(x,y;t)$ is the temperature arising from a heat source described by $f(x,t)$. The integral can be thought of as the sum over influences created by sources at each value of x' and y'. Now suppose we assume the source distribution function $\psi(x,y)$ to be the constitutive carrier wave which comprises of the host wave (a, ω, ε, k) characteristics and those of the parasitic wave (b, ω', ε' k'). The CCW applied in this study was developed by Enaibe [9] and is given by the equation

$$\psi(x,y) = \left\{(a^2-b^2\lambda^2)-2\left(a-b\lambda\right)^2\cos\left((\omega-\omega'\lambda)\,t-(\varepsilon-\varepsilon'\lambda\right)\right\}^{\frac{1}{2}}\times\cos\left(\vec{k}_C.\vec{r}-(\omega-\omega'\lambda)t-E(t)\right) (2.47)$$

$$E(t)=\tan^{-1}\left(\frac{a\sin\varepsilon+b\lambda\sin\left(\varepsilon'\lambda-(\omega-\omega'\lambda)t\right)}{a\cos\varepsilon+b\lambda\cos\left(\varepsilon'\lambda-(\omega-\omega'\lambda)t\right)}\right) (2.48)$$

where all the symbols retain their usual meaning. The CCW is two dimensional in character since it is a transverse wave, the position vector of the particle in motion is represented as $\vec{r}=r(\cos\theta i+\sin\theta j)$ and hence the motion is constant with respect to z-axis. The combined spatial frequency of the CCW is $\vec{k}_C=(k-k'\lambda)i+(k-k'\lambda)j$. Then, $\vec{k}_C.\vec{r}=r\,(k-k'\lambda)\left(\cos\theta+\sin\theta\right)$ is the coordinate of 2D position vectors and $\theta=\pi-(\varepsilon-\varepsilon'\lambda)$, the total phase angle of the CCW is represented by $E(t)$. By definition: $(\omega-\omega'\lambda)$ is the modulation angular frequency, the modulation propagation constant is $(k-k'\lambda)$, the phase difference δ between the two interfering waves is $(\varepsilon-\varepsilon'\lambda)$, and of course we have the interference term $2\left(a-b\lambda\right)^2\cos\left((\omega-\omega'\lambda)t-(\varepsilon-\varepsilon'\lambda\right)$, while waves out of phase interfere destructively according to $\left(a-b\lambda\right)^2$, however, waves in-phase interfere constructively according to $\left(a+b\lambda\right)^2$.

2.4 The calculus of the total phase angle $E(t)$ of the constitutive carrier wave.

Let us now determine the variation of the total phase angle with respect to time t. After a lengthy algebra (2.48) simplifies to

$$\frac{dE(t)}{dt}=-Z(t) \quad (2.49)$$

where $Z(t)$ is the characteristic angular velocity of the constitutive carrier wave and is given by

$$Z(t) = (\omega-\omega'\lambda)\left(\frac{b^2\lambda^2-ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(\omega-\omega'\lambda)t\right)}{a^2+b^2\lambda^2-2ab\lambda\cos\left((\varepsilon+\varepsilon'\lambda)-(\omega-\omega'\lambda)t\right)}\right) \qquad (2.50)$$

The dimension of $Z(t)$ is *rad./s* .Now for the purpose of clarity, we can set

$$Q=\sqrt{(a^2-b^2\lambda^2)-2\left(a-b\lambda\right)^2\cos\left((\omega-\omega'\lambda)t-(\varepsilon-\varepsilon'\lambda\right)} \qquad (2.51)$$

$$\Psi(r,t)=\int\frac{1}{(2\pi)^3W}\left(\sin\left(W(t-t')\right)-2\sin\left(W(t-t')\right)\sin^2\left((k-k'\lambda)\left|x-x'\right|+E(t)\right)\right)\times$$

$$Q\cos\left(\vec{k}_C.\vec{r}-(\omega-\omega'\lambda)\,t-E(t)\right)dx'dt' \qquad (2.52)$$

But according to (2.22) and (2.23); $dx'=dt'=1$, as a result,

$$\Psi(r,t)=\frac{1}{(2\pi)^3W}\left(\sin\left(W(t-t')\right)-2\sin\left(W(t-t')\right)\sin^2\left((k-k'\lambda)\left|x-x'\right|+E(t)\right)\right)\times$$

$$Q \cos\left(\vec{k}_C.\vec{r} - (\omega - \omega'\lambda)\, t - E(t)\right) \quad (2.53)$$

Now in equation (2.53) we can simply replace $x \to |x - x'|$ and $t \to |t - t'|$ which is just the distance covered and the time taken by the constitutive carrier wave as it propagates in a uniform pipe.

$$\Psi\,(r,t) = \frac{1}{(2\pi)^3 W}\left(\sin\left(W t\right) - 2\sin\left(W t\right)\sin^2\left((k - k'\lambda)\, x + E(t)\right)\right) \times$$

$$\left\{(a^2 - b^2\lambda^2) - 2\left(a - b\lambda\right)^2 \cos\left((\omega - \omega'\lambda)\, t - (\varepsilon - \varepsilon'\lambda\right)\right\}^{\frac{1}{2}} \times \cos\left(\vec{k}_C.\vec{r} - (\omega - \omega'\lambda)t - E(t)\right) \quad (2.54)$$

The reader should know that the motion under study is two dimensional 2D. The fact that we constrained the motion to $x-$ axis only does not mean the $y-$axis is not implied. Note that it is the absolute values of the constitutive carrier wave $\psi(x, y)$ that we used in our computation.

2.5 Determination of the host wave parameters ($a, \omega, \varepsilon, k$).

In this section, we shall develop methods of obtaining the wave characteristics of the host wave which were initially not known from the constitutive carrier wave equation. This is a very crucial stage of the study since there was no initial knowledge or information about the values of the host wave characteristics and those of the parasitic wave contained in the propagating constitutive carrier wave. The propagating constitutive carrier wave given by (2.47) can only have a maximum value provided the spatial oscillating phase is equal to one. Consequently, we can separate the CCW into two distinct parts; the non-stationary amplitude A and the oscillating phase angle ϕ, that is

$$A = \left\{\left(a^2 - b^2\lambda^2\right) - 2\left(a - b\lambda\right)^2 \cos\left(\left(\omega - \omega'\lambda\right)t - \left(\varepsilon - \varepsilon'\lambda\right)\right)\right\}^{\frac{1}{2}} \quad (2.55)$$

$$\phi = \cos\left((k - k'\lambda)\, r\, (\cos\theta + \sin\theta) - (\omega - \omega'\lambda)t - E(t)\right) \quad (2.56)$$

With the application of the boundary conditions; time $t = 0$, $\lambda = 0$ and $A = a$, then

$$A = \left\{a^2 - 2a^2 \cos\left(-\varepsilon\right)\right\}^{\frac{1}{2}} = a\left\{1 - 2\cos\left(\varepsilon\right)\right\}^{\frac{1}{2}} \quad (2.57)$$

$$\left\{1 - 2\cos\left(\varepsilon\right)\right\}^{1/2} = 1 \Rightarrow \varepsilon = \cos^{-1}(0) = 90^0\,(1.5708\ rad.) \quad (2.58)$$

Any slight variation in the combined amplitude A of the constitutive carrier wave due to displacement with time $t = t + \delta t$ would invariably produce a negligible effect in the amplitude a of the host wave and under this situation $\lambda \approx 0$. Hence we can write

$$\lim_{\delta t \to 0}\left\{A + \frac{\delta A}{\delta t}\right\} = a \quad (2.59)$$

$$\lim_{\delta t \to 0}\left\{\left(a^2 - 2a^2 \cos\left(\omega\,(t + \delta t) - \varepsilon\right)\right)^{1/2} + \frac{n a^2 \sin\left(\omega\,(t + \delta t) - \varepsilon\right)}{\left(a^2 - 2a^2\cos\left(\omega(t + \delta t) - \varepsilon\right)\right)^{1/2}}\right\} = a \quad (2.60)$$

$$1 - 2\cos\left(\omega t - \varepsilon\right) + \omega\sin(\omega t - \varepsilon) = \left(1 - 2\cos\left(\omega t - \varepsilon\right)\right)^{1/2} \quad (2.61)$$

At this point of our work, it may not be easy to produce a solution to the problem; this is due to the mixed sinusoidal wave functions. However, to get out of this complication we have implemented a special approximation technique [10] to minimize the right hand side of (2.61). This approximation states that

$$\left(1 + \xi f(\phi)\right)^{\pm n} = \frac{d}{d\phi}\left(1 + n\xi f(\phi) + \frac{n(n-1)}{2!}\left(\xi f(\phi)\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\xi f(\phi)\right)^3 + \ldots\right) \quad (2.62)$$

The general background of this approximation is the differentiation of the resulting binomial expansion of a given variable function. This approximation has the advantage of converging functions easily and also it produces minimum applicable value of result. Consequently, (2.61) becomes

$$1-2\cos(\omega t-\varepsilon)+\omega\sin(\omega t-\varepsilon)=\omega\sin(\omega t-\varepsilon) \qquad (2.63)$$

$$\omega t-\varepsilon=\cos^{-1}(0.5)=60^0=1.0472\,rad.\Rightarrow\omega t=2.6182rad.\ \Rightarrow\omega=2.6182rad./s \qquad (2.75)$$

In (2.56), we use the boundary conditions; that for time independent state or stationary state we have that

$\delta t=0,\lambda\approx 0,\ \theta=\pi-(\varepsilon-\varepsilon'\lambda)=\pi-\varepsilon=3.142-1.5708=1.5712rad,\ E=\varepsilon=1.5708rad$, hence we get

$$\lim_{\delta t\to 0}\cos\left\{(k-k'\lambda)\,r\,\cos\theta+(k-k'\lambda)\,r\sin\theta-(\omega-\omega\lambda)\,(t+t\,\delta t)-E(t)\right\}=1 \qquad (2.64)$$

$$\left(k\,r\,(\cos\theta+r\sin\theta)-\omega t-\varepsilon\right)=0\ (\text{since, } \cos^{-1}1=0\ ) \qquad (2.65)$$

$$\left(k\,r\,(0.9996)-2.6182-1.5708\right)=0\Rightarrow k\,r=4.1907rad\Rightarrow k=4.1907rad/m \qquad (2.65)$$

The change in the resultant amplitude A of the CCW is proportional to the frequency of oscillation of the spatial oscillating phase ϕ multiplied by the product of the variation with time t of the inverse of the oscillating phase with respect to the radial distance r, and the variation with respect to the wave number $(k-k'\lambda)$. This condition would make us write (2.55) and (2.56) separately as

$$\frac{dA}{dt}=\frac{(\omega-\omega'\lambda)(a-b\lambda)^2\sin\left((\omega-\omega'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)}{\left((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left((\omega-\omega'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)^{1/2}} \qquad (2.66)$$

$$\frac{d\phi}{dr}=-(k-k'\lambda)\,(\cos\theta+\sin\theta)\sin\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta)-(\omega-\omega'\lambda)\,t-E(t)\right) \qquad (2.67)$$

$$\frac{d\phi}{dt}=\left((\omega-\omega'\lambda)+Z(t)\right)\sin\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta)-(\omega-\omega'\lambda)\,t-E(t)\right) \qquad (2.68)$$

$$\frac{d\phi}{d(k-k'\lambda)}=\left(-r\,(\cos\theta+\sin\theta)-E(t)\right)\sin\left((k-k'\lambda)\,r\,(\cos\theta+\sin\theta)-(\omega-\omega'\lambda)t-E(t)\right) \qquad (2.69)$$

$$\frac{dA}{dt}=\left(\frac{1}{2\pi}\frac{d\phi}{dt}\right)\left(\frac{1}{r}\frac{dr}{d\phi}\right)\left(\frac{d\phi}{d(k-k'\lambda)}\right)=f\,l \qquad (2.70)$$

The first term in the parenthesis of (2.70) is the frequency dependent term, while the combination of the rest two terms in the parenthesis represents the angular length or simply the length of an arc covered by the spatial oscillating phase. Note that the second term in the right hand side of (2.70) is the inverse of (2.67).

$$A=f\,l\,t \qquad (2.71)$$

Equation (2.71) can be interpreted to mean that the time rate of change of the resultant amplitude is equal to the frequency f of the spatial oscillating phase multiplied by the length l of the arc covered by the oscillating phase. Under this circumstance, we refer to A as the instantaneous amplitude of oscillation. With the usual implementation of the boundary conditions that at $t=0,\lambda=0,\ \theta=\pi-(\varepsilon-\varepsilon'\lambda)=\pi-\varepsilon=3.142-1.5708=1.5712rad$, $E(t)=\varepsilon=1.5708rad$, $dA/dt=a$, then the amplitude

$$a=-\left(\frac{1}{2\pi}\right)\left(\frac{(\cos\theta+\sin\theta)-\varepsilon}{k\sin\varepsilon\,(\cos\theta+\sin\theta)}\right)=0.0217m \qquad (2.72)$$

Note that $\cos(-\varepsilon)=\cos\varepsilon$ (even and symmetric function) and $\sin(-\varepsilon)=-\sin\varepsilon$ (odd and screw symmetric function). Hence generally we get

$$a=0.0217m,\ \omega=2.6182rad/s,\ \varepsilon=1.5708rad,\text{ and } k=4.1907rad/m \qquad (2.73)$$

2.6 Determination of the parasitic wave parameters (b , ω' , ε' , k')

Let us now determine the basic parameters of the parasitic wave which were initially not known before the interference from the derived values of the resident 'host wave' using the below method. The gradual depletion in the physical parameters of the system under study would mean that after a sufficiently long period of time all the active constituents of the resident host wave would have been completely attenuated by the destructive influence of the parasitic wave. On the basis of these arguments, we can now write as follows.

$$\left.\begin{array}{l} a-b\lambda=0\Rightarrow 0.0217=b\lambda \\ \omega-\omega'\lambda=0\Rightarrow 2.6182=\omega'\lambda \\ \varepsilon-\varepsilon'\lambda=0\Rightarrow 1.5708=\varepsilon'\lambda \\ k-k'\lambda=0\Rightarrow 4.1907=k'\lambda \end{array}\right\} \qquad (2.74)$$

Upon dividing the sets of relations in (2.74) with one another with the view to eliminate λ we get

$$\left.\begin{array}{l} 0.008288\omega'=b \\ 0.013820\varepsilon'=b \\ 0.005178k'=b \\ 1.6668\varepsilon'=\omega' \\ 0.6248k'=\omega' \\ 0.3748k'=\varepsilon' \end{array}\right\} (2.75)$$

However, there are several possible values that each parameter would take according to (2.75). But for a gradual and low varying decay process, that is for a slow depletion in the constituent of the host parameters we choose the least values of the parasitic parameters. Thus, a more realistic and applicable relation which we have selected in this study, is when we equate the first two relations of (2.75) to one another. That is, $0.01382\ \varepsilon'=0.005178k'$. Based on simple ratio we eventually get

$$\omega'=0.008631\,rad/s\,,\quad k'=0.01382\,rad/m\,,\quad \varepsilon'=0.005178\,rad\,,\quad b=0.00007153\,m \ (2.76)$$

When we substitute any of the values given by (2.76) into (2.74) we get a corresponding value of raising multiplier lambda $\lambda=303$. Hence the interval of the multiplier is $0\ \le\ \lambda\ \le\ 303$.

2.7 Determination of the attenuation or decay constant (η) of the constitutive carrier wave CCW.

Attenuation is a decay process. It brings about a gradual reduction and weakening in the initial strength of the basic parameters of a given physical system. In this study, the parameters are the amplitude (a), phase angle (ε), angular frequency (ω) and the spatial frequency (k). The dimension of the attenuation constant (η) is determined by the system under study. However, in this work, the attenuation constant is the relative rate of fractional change (FC) in the basic parameters of the constitutive carrier wave. There are 4 (four) attenuating parameters present in the constitutive carrier wave. Now, if a , ω , ε , k represent the initial basic parameters of the host wave that is present in the constitutive carrier wave and $a-b\lambda$, $\omega-\omega'\lambda$, $\varepsilon-\varepsilon'\lambda$, $k-k'\lambda$ represent the basic parameters of the host wave that survives after a given time. Then, the FC is

$$\sigma=\frac{1}{4}\times\left(\left(\frac{a-b\lambda}{a}\right)+\left(\frac{\varepsilon-\varepsilon'\lambda}{\varepsilon}\right)+\left(\frac{\omega-\omega'\lambda}{\omega}\right)+\left(\frac{k-k'\lambda}{k}\right)\right) \qquad (2.77)$$

$$\eta=\frac{FC\Big|_{\lambda=i}-FC\Big|_{\lambda=i+1}}{unit\ \ time\ (s)}\ =\ \frac{\sigma_i-\sigma_{i+1}}{unit\ \ time\ (s)} \qquad (2.78)$$

The dimension is *per second* (s^{-1}). Thus (2.78) gives $\eta=0.003297\,s^{-1}$ for all values of the multiplier $\lambda(i=0,\ 1,\ 2,\ldots,\ 303\,)$.

2.8 Determination of the attenuation time (t) of the constitutive carrier wave CCW.

The maximum time the constitutive carrier wave lasted as a function of the raising multiplier λ can be calculated from the attenuation equation. The reader should note that we have adopted a slowly varying regular interval for the raising multiplier since this would help to delineate clearly the parameter space that is accessible to our model. The attenuation time equation is given below.

$$\sigma = e^{-(2\eta t)/\lambda} \qquad (2.79)$$

$$t = -\left(\frac{\lambda}{2\eta}\right)\ln\sigma \qquad (2.80)$$

The equation is statistical and not a deterministic law. It gives the expected basic parameters of the host wave that survives after time t. Clearly, we used (2.80) to calculate the values of the decay time as a function of the raising multiplier λ (0, 1, 2, . . . , 303).

3.0 PRESENTATION OF RESULTS

Table 2.1: shows the calculated values of the characteristics of the constitutive carrier wave.

S/N	Physical Quantity	Symbol	Value	Unit
1	Amplitude of the host wave	a	0.0217	m
2	Angular frequency of the host wave	ω	2.6182	rad/s
3	Phase angle of the host wave	ε	1.5708	$radian$
4	Spatial frequency of the host wave	k	4.1907	rad/m
5	Amplitude of the parasitic wave	b	0.00007153	m
6	Angular frequency of the parasitic wave	ω'	0.008631	rad/s
7	Phase angle of the parasitic wave	ε'	0.005178	$radian$
8	Spatial frequency of the parasitic wave	k'	0.01382	rad/m
9	Raising multiplier	λ	0, 1, 2, … ,303	--

Table 2.2: shows calculated values of some of the parameters and some constants required for the work.

S/N	Physical Quantity	Symbol	Value	Unit
1	Attenuation constant	η	0.003297	s^{-1}
2	Radius of the pipe	r	0.03	m
3	Maximum attenuation time corresponding to the maximum multiplier	t	3.4112	s
4	Sum of the total time that the constitutive carrier wave lasted as a function of the multiplier	$\sum_{i=0}^{N} t_i \ (t_0 = 0)$	114.2805	s
5	Sum of the total distance covered by the constitutive carrier wave as a function of the multiplier	$\sum_{i=0}^{3030} x_i \ (x_0 = 0)$	460560 x = 0, 10, 20, … 3030	m
6	Permittivity of air	$\in$	8.85 x 10^{-12}	$C^2N^{-1}m^{-2}$
7	Permeability of air	μ	1.2566 x 10^{-6}	H/m
8	The product of Permittivity and Permeability of air	$\in\mu$	1.11209 x 10^{-17}	s^2/m^2

The relevant results obtained which is given by the equations (2.51), (2.59) and (2.63), respectively are shown graphically below. Note that the total time covered is calculated from the relation $t = \sum_{i=0}^{N} t_i = 114.2805$ seconds, where N is the terminal point under consideration, note that the time taken is not linearly proportional. Also the total distance covered is calculated from the relation $x = \sum_{i=0}^{3030} x_i = 460560$ m, where we have assume in this study that the total arbitrary distance covered is calculated form the relation x = 0, 10, 20, 30, . . ., 3030m.

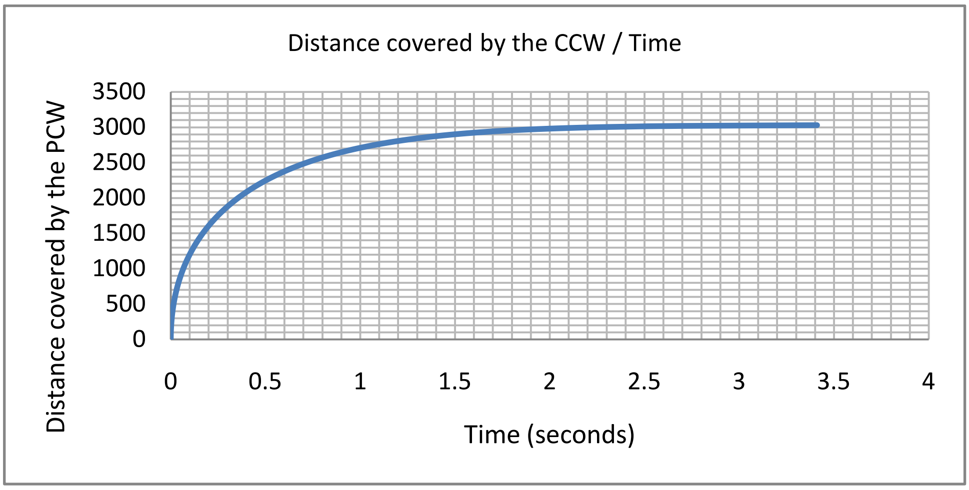

Fig. 3.1: Shows the spectrum of the distance covered by the propagating constitutive carrier wave as a function of time and multiplier ($0 \leq \lambda \leq 303$). Note that the distance covered is arbitrary and it is in the interval of $x = 0, 10, 20, \ldots, 3030$. The spectrum shows saturation beyond 1.5 seconds. The figure represents (2.20) when $x \to |x - x'|$.

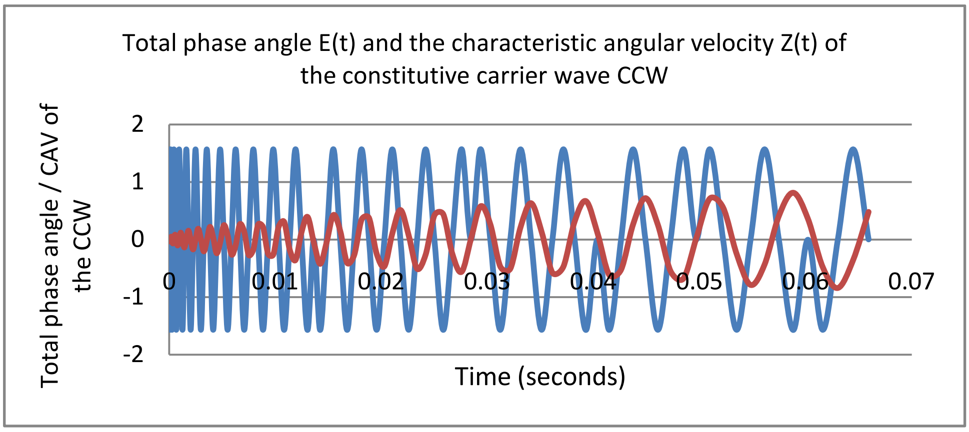

Fig. 3.2: Shows the spectrum of the total phase angle $E(t)$ and the characteristic angular velocity $Z(t)$ CAV of the resultant constitutive carrier wave as function of time and multiplier ($0 \leq \lambda \leq 100$) and $0 \leq \beta \leq 100$. The blue colour represents the $E(t)$ while the brown colour represent $Z(t)$. The figure represents (2.48) and (2.50) respectively.

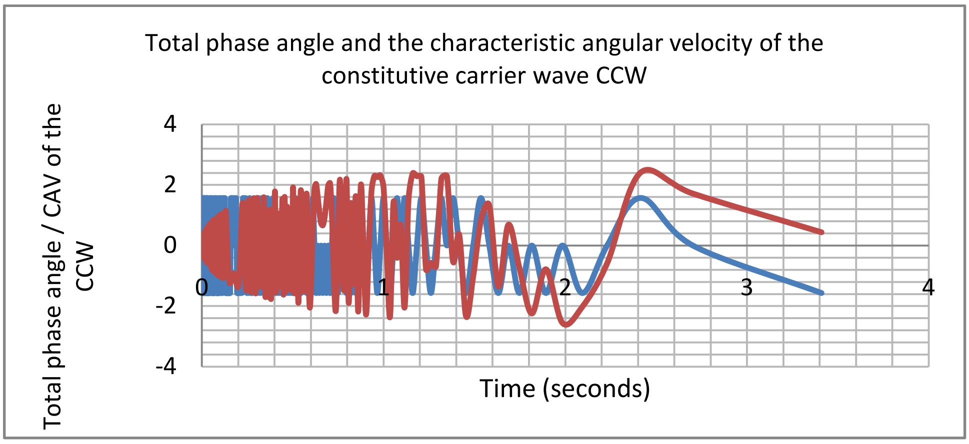

Fig. 3.3: Shows the spectrum of the total phase angle $E(t)$ and the characteristic angular velocity $Z(t)$ of the resultant constitutive carrier as a function of time and multiplier ($0 \leq \lambda \leq 303$) and $0 \leq \beta \leq 303$. The blue colour represents the $E(t)$ while the brown colour represent $Z(t)$. The figure represents (2.48) and (2.50) respectively.

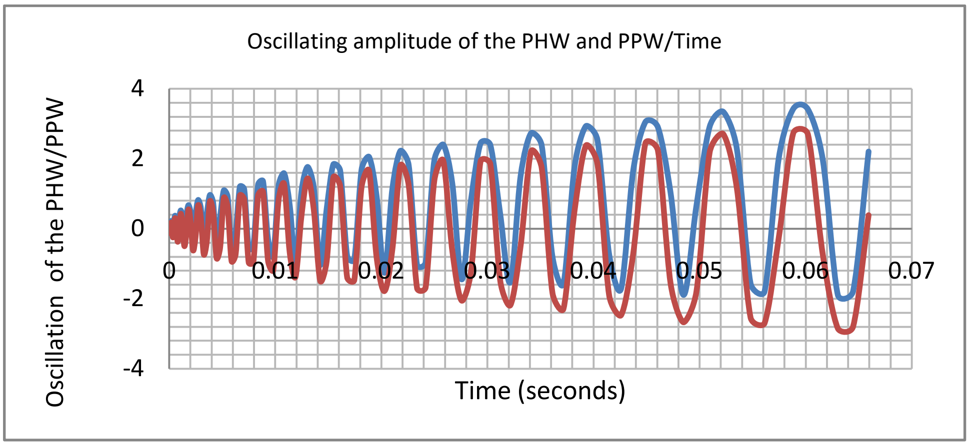

Fig. 3.4: Shows the spectrum of the blue colour shows the oscillating of the propagating host wave (PHW) as a function of only the multiplier $\beta\,(0 \le \beta \le 100)$ when the parasitic multiplier $\lambda = 0$. The spectrum of the brown colour shows the amplitude of the propagating parasitic wave (PPW) in the interval $\lambda\,(0 \le \lambda \le 100)$ when the host multiplier $\beta = 0$ .Please see appendix for details. The figure represents (A.13) and (A.15) when the oscillating phase is removed.

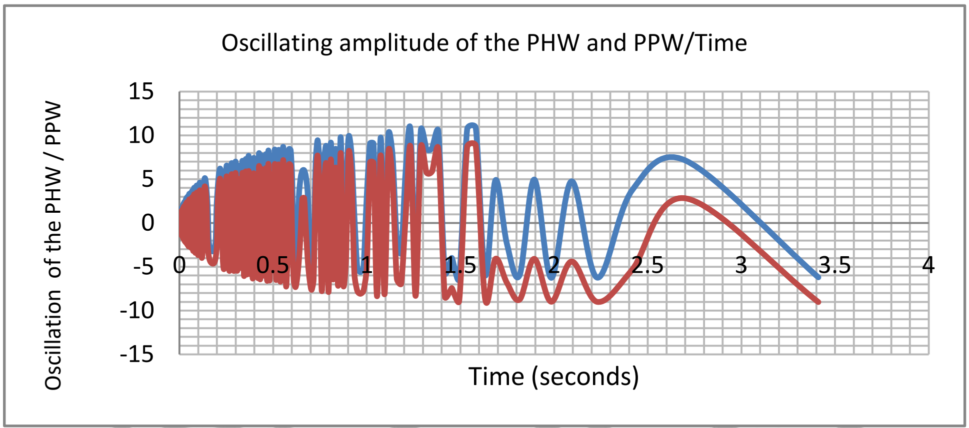

Fig. 3.5: The spectrum of the blue colour shows only the oscillating amplitude of the propagating host wave (PHW) as a function of only the raising multiplier $\beta\,(0 \le \beta \le 303)$ when the parasitic multiplier $\lambda = 0$. The spectrum of the brown colour shows only the oscillating amplitude of the propagating parasitic wave PPW in the interval λ $(0 \le \lambda \le 303)$ when the host multiplier $\beta = 0$ .The figure represents (A.13) and (A.15) when the oscillating phase is removed.

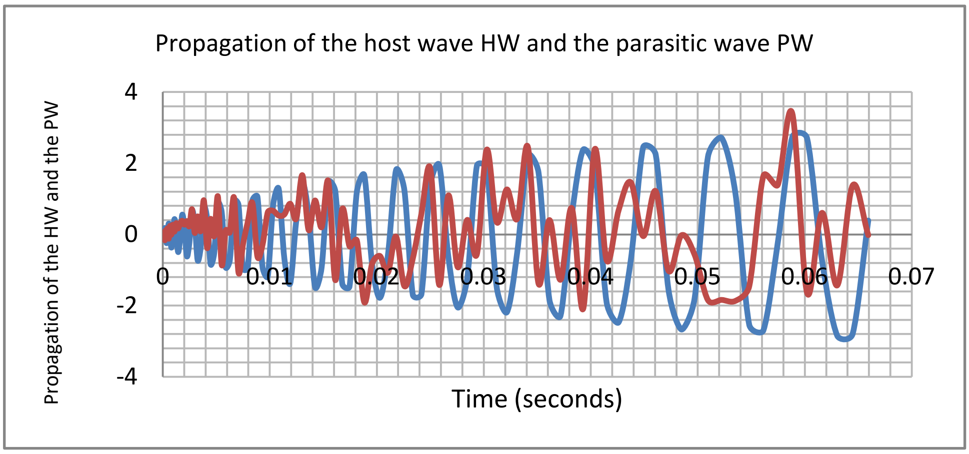

Fig. 3.6: The spectrum of the blue colour shows the oscillation of the propagating parasitic wave PPW as a function of only the multiplier $\lambda\,(0 \le \lambda \le 100)$ when the host wave multiplier $\beta = 0$. The spectrum of the brown colour shows only the oscillation of the propagating host wave (HW) as a function of only the multiplier $\beta\,(0 \le \beta \le 100)$ when the host multiplier $\lambda = 0$. The figure represents (A.13) and (A.15) respectively.

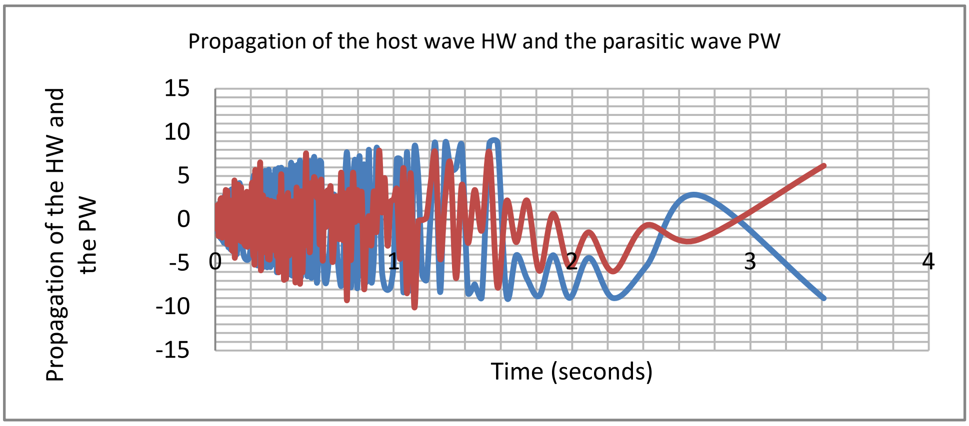

Fig. 3.7: The spectrum of the blue colour shows the oscillation of the propagating parasitic wave PW as a function of only the multiplier λ ($0 \le \lambda \le 303$) when the host multiplier $\beta = 0$. The spectrum of the brown colour shows the oscillation of the propagating host wave HW as a function of only the multiplier β ($0 \le \beta \le 303$) when the parasitic multiplier $\lambda = 0$. The figure represents (A.13) and (A.15) respectively.

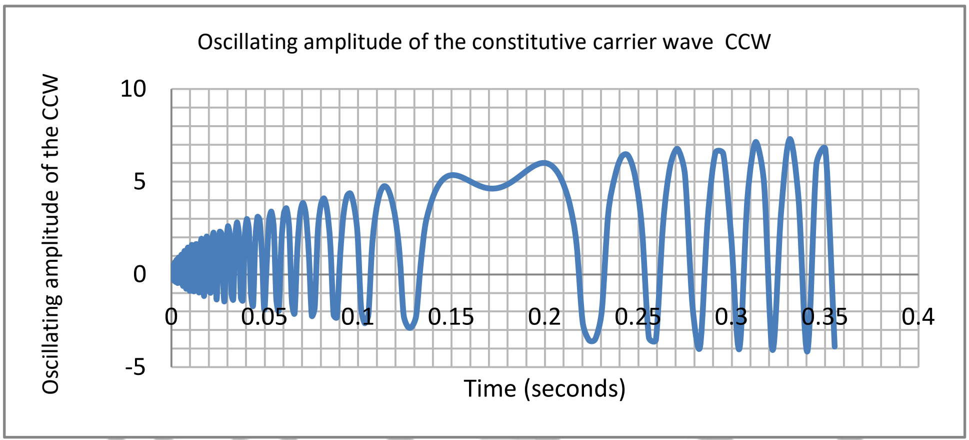

Fig. 3.8: Shows the spectrum of only the oscillating amplitude of the constitutive carrier wave in the interval of the raising multipliers $\lfloor 0-200 \rfloor$ for both the parasitic wave and the host wave. The figure represents (A.7) in the absence of oscillating phase ϕ .

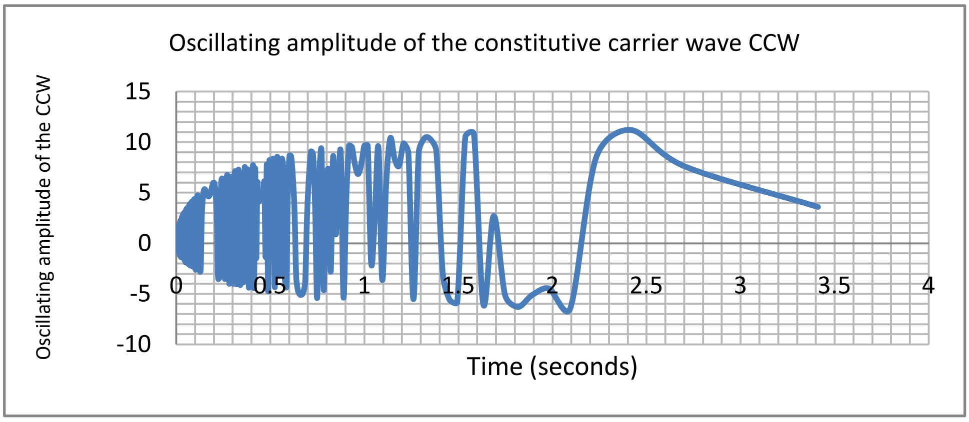

Fig. 3.9: Shows the spectrum of only the oscillating amplitude of the constitutive carrier wave in the interval of the raising multiplier $\lfloor 0-303 \rfloor$ for both the parasitic wave and the host wave. The figure represents (A.7) in the absence of oscillating phase.

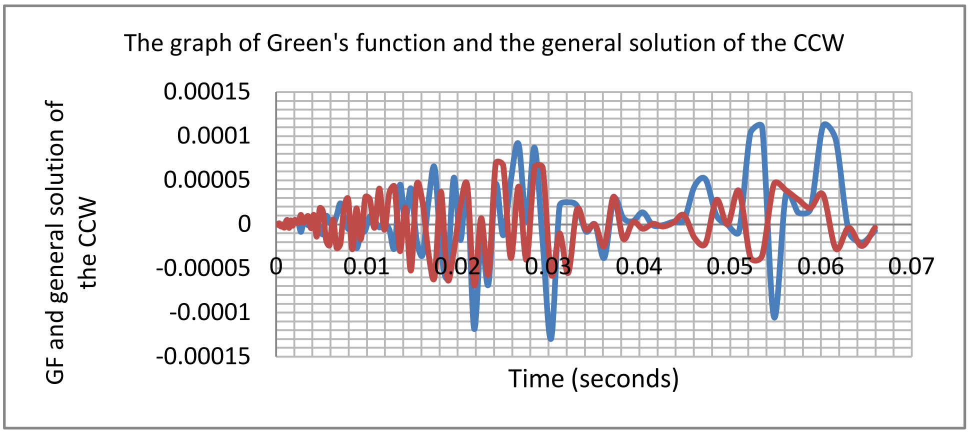

Fig. 3.10: Shows the spectrum of the Green's function (brown colour) and the general solution (blue colour) of the CCW in the interval of the multipliers $\lfloor 0-100 \rfloor$ for both the parasitic wave and the host wave. The figure represents (2.44) and (2.54) respectively.

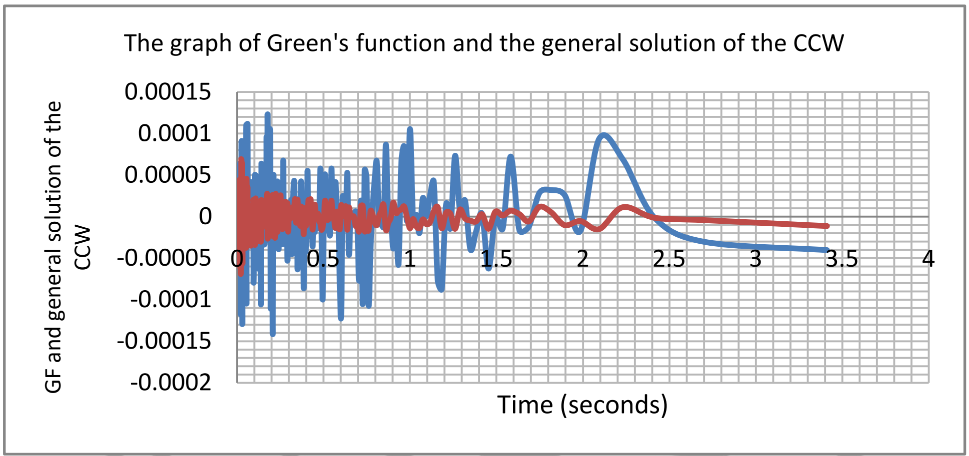

Fig. 3.11: Shows the spectrum of the Green's function (brown colour) and the general solution (blue colour) of the CCW in the interval of the raising multiplier $\lfloor 0-303 \rfloor$ for both the parasitic wave and the host wave. The figure represents (2.44) and (2.54) respectively.

4.0 DISCUSSION OF RESULTS.

From the spectrum given by fig. 3.1, it is very clear that the distance covered is not directly proportional to the time taken. Although, the distance covered by the CCW first increases rapidly as the time is increased. This relationship is maintained until when the time is about 1.8945 seconds with a corresponding coordinate distance travelled of 2970 m. Hence the total distance covered and the corresponding total time taken up to this point is 442530 m and 99.4381 seconds. Beyond this coordinate, the distance covered by the CCW becomes stable with increased in time.

The sum of the remaining distance covered under this stable condition is 18030 m. The simple explanation here is that the propagating CCW is now moving with constant angular velocity and with a-zero angular acceleration. Under this situation, the parasitic component of the CCW is directly taking predominant control over the host component.

It must first be mentioned here that fig. 3.2 is an excerpt of fig. 3.3. This is to enable us study the initial behaviour of the total phase angle $E(t)$ and the characteristic angular velocity $Z(t)$ of the CCW. Hence fig. 3.2 reveals that within the given interval of the raising multiplier, the total phase angle maintains stable amplitude of about ± 1.5 radians and also with regular frequency. Meanwhile, the amplitude of $Z(t)$ initially increases with regular frequency as it leaves the source.

However, the amplitude of $Z(t)$ CCW is much lower than the $E(t)$. In fig. 3.3 the $E(t)$ almost maintain stable amplitude except at the time coordinate of 2 seconds when it fluctuates to much lower amplitude between $0 \le E(t) \le -1.5$.The total phase angle of the CCW then increases to a maximum value of 1.5 radians with a time coordinate of 2.4161 seconds before it attenuates to zero.

It is also shown in the figure that around the coordinate of 1.1604 seconds the characteristic angular velocity increases to a maximum of about $-2.3389 \le Z(t) \le 2.3787$ rad/s. It also shows similar fluctuation with much lower amplitude about the same time with the total phase angle and thereafter it increases to a maximum value of 2.4151 rad/s before it finally goes to zero. However, $Z(t)$ of the CCW does not attenuate to equilibrium position like the $E(t)$. This is because there is some residual signal still present in the characteristic angular velocity even in the absence of other driving forces.

Again fig. 3.4 is an excerpt of fig. 3.5. They both show the behaviour of the oscillating amplitude of the propagating host wave (PHW) and the propagating parasitic wave (PPW). It is shown in fig. 3.4 that within the given interval of the raising multiplier for both vibratory components of the CCW, the oscillation of both the propagating host wave and parasitic wave initially show similar behaviour. The amplitude of both vibrations increases with equal and regular frequency as they leave the source.

Now from fig. 3.5 within the time interval $1.5 \le t \le 2.5$ both waves show similar fluctuation with reduced oscillating amplitude. Although, within this interval, while the host wave show positive and negative values that ranges from $+5$ to -6.8161 m, the oscillating parasitic wave show only negative value range of -4.0872 and -8.7379 m. Thereafter, both waves propagate with increasing oscillating amplitude to a maximum positive value of about 2.8235 m for the parasitic wave and 7.1355 m for the host wave, before they attenuate to zero in like manner.

Also fig. 3.6 is an excerpt of fig. 3.7. They both represent the behaviour of the propagating host wave and the parasitic wave if they are allowed to propagate independently of one another. Note that the spectra represented by fig. 3.6 and 3.7 are simply determined by multiplying the oscillating amplitude given by the spectra of fig. 3.4 and 3.5 by the cosine oscillating phase. Initially, as shown in fig. 3.6, the amplitude of the propagating parasitic wave increases with regular frequency of oscillation, while the amplitude of the propagating host wave is irregular with almost sinusoidal in character.

The amplitude of both waves increase with equal and regular frequency as they propagate away from the source. In fig. 3.7 within the time interval $1.5 \le t \le 2.5$ both waves show similar fluctuating behaviour with reduced oscillating amplitude. Within this interval, while the host wave show positive and negative values that range from $+2.1706$ m to -5.9476 m, the propagating parasitic wave show only negative value range of -4.0861 and -8.9910 m. After this time, the parasitic wave propagates with increasing amplitude to a maximum positive value of about 2.8235 m before it starts to attenuate to zero. However, the spectra shows that the host wave increases without attenuating to zero.

The reader should note that fig. 3.8 is an excerpt of fig. 3.9. They respectively represent the initial and total behaviour of the oscillating amplitude of the CCW, if allowed to propagate dependently when both multipliers are effective. Initially, in fig. 3.8 the amplitude of the CCW increases with regular frequency and bandwidth up to 0.1242 seconds. The sum of the total time taken to display this anomalous gap is 11.3130 seconds. Under this situation the host wave is now responding to the presence of the manifestation of a strange velocity-like wave whose interference may be constructive or destructive. However, this effect is annulled and the CCW propagates further with the usual frequency.

In fig. 3.9 the oscillating amplitude increases with irregular frequency. Also within the time interval $1.6 \le t \le 2.2$ the oscillating amplitude of the CCW display anomalous behaviour with a wide gap in the band spectrum. It is clear that within the interval, the host wave is now responding to the negative influence of the parasitic wave. After this time, the CCW propagates with increasing amplitude to a maximum positive value of about 11.2110 m before it starts to attenuate to zero. However, the band spectrum shows that the CCW does not attenuate completely to zero due to the presence of residual signal still present in it even when the parasitic wave would have completely eroded the constituent wave characteristics of the host wave.

Fig. 3.10 and 3.11 shows the initial and complete behaviour of the propagating CCW as determined by the general solution and the Green's function method. Note that the propagation is dependent on the varying wave characteristics of both the host wave and the parasitic wave. Initially, as shown in fig. 3.10 the amplitude of the propagating CCW increases with irregular frequency and bandwidth. The spectrum shows minimum amplitude in the time interval $0.0302 \le t \le 0.0522$ before it starts to increase again. The wave pattern of the oscillating CCW is sinusoidal as it progresses from the source.

In fig. 3.11 the amplitude of the CCW increases with irregular frequency. Also, within the time interval $1.6 \le t \le 2.2$ the oscillating amplitude of the CCW display anomalous behaviour with a wide gap in the band spectrum. It is clear that within this interval, the host wave is now responding fully to the negative manifestation of the parasitic wave. After this time, the CCW propagates with increasing amplitude to a maximum positive value of about 11.2110 m before it starts to attenuate to zero. The band spectrum shows that the CCW does not attenuate completely to zero

5.0 CONCLUSION

The results given by the Green's function approach has smaller amplitude of oscillation and very high frequency compared to that of the general solution of the wave equation with the constitutive carrier wave as the source distribution function. Although, both solutions show similar asymptotic behaviour by exhibiting zero-oscillating amplitude and zero-frequency after a prolonged time. Consequently, the anomalous behaviour exhibited by the constitutive carrier wave during the decay process, is due to the resistance posed by the intrinsic parameters of the host in the constitutive carrier wave in an attempt to annul the destructive tendency of the parasitic wave. The results show that the retarded behaviour of the constitutive carrier wave described by the Green's function at some point away from the origin is much greater than the general wave solution of the constitutive carrier wave at the origin. The Green's function is spherically symmetric about the source, and falls off smoothly with increasing distance from the source. The attenuation time of the Green's function representation of the constitutive carrier wave leads the attenuation time for the general wave solution.

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