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Quasi Amarendra distribution and Its Properties

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Key Worlds

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Abstract

The life time distributions have been playing important role in field of applied sciences. This study on a new Quasi Amarendra distribution and discussed some its properties including moment generating function, rth moment about origin, mean, variance, coefficient of variation, dispersion index, reliability analysis measurements, Bonferroni and Lorenz curves, Renyi entropy, measure order statistics, graphical representation of some functions and discussed on maximum likelihood method.

1. Introduction

There are no of life time distributions like Weibull, Gamma, Exponential, lognormal, Lindley are more popular distributions are applying in different fields of science and Shanker, Sujatha, Amarendra, Akash, Ishtia, Pernave and Devya distributions are offering better performance than some other distributions like as the gamma and the lognormal distributions because the survival functions of the gamma and the lognormal distributions cannot be expressed in closed forms and both require numerical integration.. The probability density function (p.d.f.) and the cumulative distribution function (c.d.f.) of [1] Lindley (1958) distribution are respectively given as:

$$f_L(\mathrm{y},\theta) = \frac{\theta^2}{\theta+1}(1+y)e^{-\theta y} \quad y>0;\ \theta>0 \tag{1.1}$$

$$F_L(\mathrm{y},\theta) = 1-\left\{\frac{\theta y+\theta+1}{\theta+1}\right\}e^{-\theta y} \quad y>0;\ \theta>0 \tag{1.2}$$

The Lindley probability density function is a mixture of $f_1(\mathrm{y})$as an Exponential (θ) and $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$ with mixture proportions $P_1 = \frac{\theta}{\theta+1}$ and $P_2 = \frac{1}{\theta+1}$ such as:

$$f_L(\mathrm{y},\theta) = P_1\, f_1(\mathrm{y}) + (1-P_1)f_2(\mathrm{y}) \tag{1.3}$$

[2] Shanker and Mishra (2013a) proposed the probability density function (p.d.f.) and cumulative distribution function (c.d.f) of Quasi Lindley distribution (QLD) are

$$f_{QL}(\mathrm{y},\theta) = \frac{\theta}{\alpha+1}(\alpha+\theta y)e^{-\theta y} \qquad y>0;\ \theta>0;\ \alpha>-1 \tag{1.4}$$

$$F_{QL}(\mathrm{y},\theta) = 1-\left\{\frac{\theta y+\alpha+1}{\alpha+1}\right\}e^{-\theta y} \qquad y>0;\ \theta>0;\ \alpha>-1 \tag{1.5}$$

The Quasi Lindley probability density function is a mixture of $f_1(\mathrm{y})$ as an Exponential (θ) and $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$ with mixture proportions $P_1 = \frac{\alpha}{\alpha+1}$ and $P_2 = \frac{1}{\alpha+1}$ by putting in (1.3),

If $\alpha = \theta$ then (1.3) is Lindley probability density function.

[3] Shanker and Shukla (2018), introduced Quasi Aradhana distribution discussed some their properties and applications, pdf and cdf defined following,.

$$f_{QA}(\mathrm{y},\theta) = \frac{\theta}{\alpha^2+2\alpha+2}(\alpha+\theta y)^2e^{-\theta y} \quad y>0;\ \theta>0;\ \alpha^2+2\alpha+2>0 \tag{1.6}$$

$$F_{QA}(\mathrm{y},\theta) = 1-\left\{1+\frac{\theta y(\theta y+2\alpha+2)}{\alpha^2+2\alpha+2}\right\}e^{-\theta y} \quad y>0;\ \theta>0,\alpha^2+2\alpha+2>0 \tag{1.7}$$

The Quasi Aradhana probability density function is a mixture of $f_1(\mathrm{y})$as an Exponential (θ), $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$ and $f_3(\mathrm{y})$ as a Gamma $(3,\theta)$ with mixture proportions,

$P_1 = \frac{\alpha^2}{\alpha^2+2\alpha+2}\, P_2 = \frac{2\alpha}{\alpha^2+2\alpha+2}$ and $P_3 = \frac{2}{\alpha^2+2\alpha+2}$ such as:

$$f_{QA}(\mathrm{y},\theta) = P_1\, f_1(\mathrm{y}) + P_2f_2(\mathrm{y}) + (1-P_1-P_2)f_3(\mathrm{y}) \tag{1.8}$$

[4] Sujatha distribution introduced by Shanker (2016 a) pdf and cdf given respectively as:

$f_S(\mathrm{y},\theta)=\frac{\theta^3}{\theta^2+\theta+2}(1+y+y^2)e^{-\theta y} \quad y>0;\ \theta>0$ (1.9)

$F_S(\mathrm{y},\theta)=1-\left\{1+\frac{\theta y+\theta+2}{\theta^2+\theta+2}\right\}e^{-\theta y} \quad y>0;\ \theta>0$ (1.10)

The Sujatha probability density function is a mixture of $f_1(\mathrm{y})$as an Exponential (θ), $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$ and $f_3(\mathrm{y})$ as a Gamma $(3,\theta)$ with mixture proportions $P_1=\frac{\theta^2}{\theta^2+\theta+2}$ $P_2=\frac{\theta}{\theta^2+\theta+2}$ and $P_3=\frac{2}{\theta^2+\theta+2}$ such as:

$f_S(\mathrm{y},\theta)=P_1\,f_1(\mathrm{y})+\ P_2f_2(\mathrm{y})+(1-P_1-P_2)f_3(\mathrm{y})$ (1.11)

[5] Shanker (2016) Quasi Sujatha distribution pdf and cdf are defined as following:

$f_{QS}(\mathrm{y},\theta)=\frac{\theta^2}{\alpha\theta+\theta+2}(\alpha+\theta y+\theta y^2)e^{-\theta y} \quad y>0;\ \theta>0;\ \alpha>0$ (1.12)

$F_{QS}(\mathrm{y},\theta)=1-\left\{1+\left(\frac{\theta y+\theta+2}{\alpha\theta+\theta+2}\right)\right\}e^{-\theta y} \quad y>0;\ \theta>0;\alpha>0$ (1.13)

The Quasi Sujatha probability density function is a mixture of $f_1(\mathrm{y})$as an Exponential (θ), $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$ and $f_3(\mathrm{y})$ as a Gamma $(3,\theta)$ with mixture proportions,

$P_1=\frac{\alpha\theta}{\alpha\theta+\theta+2}$ $P_2=\frac{\theta}{\alpha\theta+\theta+2}$ and $P_3=\frac{2}{\alpha\theta+\theta+2}$ such as:

$f_S(\mathrm{y},\theta)=P_1\,f_1(\mathrm{y})+\ P_2f_2(\mathrm{y})+(1-P_1-P_2)f_3(\mathrm{y})$ (1.14)

[6] two-parameter Sujatha distribution (TPSD) proposed by Mussie and Shanker (2018), with mixture proportions $P_1=\frac{\alpha\theta^2}{\alpha\theta^2+\theta+2}$ $P_2=\frac{\theta}{\alpha\theta^2+\theta+2}$ and $P_3=\frac{2}{\alpha\theta^2+\theta+2}$ in (1.14) have $\alpha>0$. [7] New two-parameter Sujatha distribution (NTPSD), Mussie and Shanker (2018) developed as with mixture proportions $P_1=\frac{\theta^2}{\theta^2+\alpha\theta+2}$ $P_2=\frac{\alpha\theta}{\theta^2+\alpha\theta+2}$ and $P_3=\frac{2}{\theta^2+\alpha\theta+2}$ in (1.14) with $\alpha\geq 0$. [8] most recently Another two-parameter Sujatha distribution (ATPSD) by putting mixture proportions $P_1=\frac{\theta^2}{\theta^2+\alpha\theta+2\alpha}$ $P_2=\frac{\alpha\theta}{\theta^2+\alpha\theta+2\alpha}$ and $P_3=\frac{2\alpha}{\theta^2+\alpha\theta+2\alpha}$ in (1.14) have $\alpha\geq 0$. [9] Quasi Shanker distribution and its applications by developed by Shanker and Shukla (2017). [10] Shanker (2016 i) introduced Amarendra distribution and observed its various mathematical and statistical properties, estimation of its parameter and applications also find that it provides a better model than exponential, Lindley and Sujatha distributions for modeling lifetime data. The pdf and cdf of Amarendra distribution are defined respectively as:

$f_{Arm}(y;\ \theta)=\frac{\theta^4}{(\theta^3+\theta^2+2\theta+6)}\{1+y+y^2+y^3\}e^{-\theta y} \quad y>0;\ \theta>0$ (1.15)

$F_{Arm}(y;\alpha,\theta)=1-\left[1+\left\{\frac{y\theta(\theta^2+2\theta+6)+(y\theta)^2(\theta+3)+(y\theta)^3}{(\theta^3+\theta^2+2\theta+6)}\right\}\right]e^{-\theta y} \quad y>0;\ \theta>0$ (1.16)

The Amarendra distribution is a four component mixture like $f_1(\mathrm{y})$as an Exponential (θ), $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$, $f_3(\mathrm{y})$ as a Gamma $(3,\theta)$ and $f_4(\mathrm{y})$ as a Gamma $(4,\theta)$ with mixture proportions,

$P_1=\frac{\theta^3}{(\theta^3+\theta^2+2\theta+6)}$, $P_2=\frac{\theta^2}{(\theta^3+\theta^2+2\theta+6)}$, $P_3=\frac{2\theta}{(\theta^3+\theta^2+2\theta+6)}$ and $P_4=\frac{6}{(\theta^3+\theta^2+2\theta+6)}$such as:

$f_{Arm}(\mathrm{y},\theta)=P_1\,f_1(\mathrm{y})+\ P_2f_2(\mathrm{y})+P_3f_3(\mathrm{y})+(1-P_1-P_2-P_2)f_4(\mathrm{y})$ (1.17)

2. The New Quasi Armendra Distribution (QAMD)

Let be a random variable Y follow new Quasi Armendra distribution then pdf and cdf defined as following:

$f_{QArm}(y;\alpha,\theta)=\frac{\theta}{(\alpha^3+\alpha^2+2\alpha+6)}\{\alpha^3+\alpha^2\theta y+\alpha(\theta y)^2+(\theta y)^3\}e^{-\theta y}$ (2.1)

$\theta,y>0;\quad \alpha>-1$

The Quasi Amarendra distribution is a four component mixture of $f_1(\mathrm{y})$as an Exponential (θ), $f_2(\mathrm{y})$ as a Gamma $(2,\theta)$, $f_3(\mathrm{y})$ as a Gamma $(3,\theta)$ and $f_4(\mathrm{y})$ as a Gamma $(4,\theta)$ with mixture proportions,

$P_1 = \frac{\alpha^3}{(\alpha^3+\alpha^2+2\alpha+6)}$, $P_2 = \frac{\alpha^2}{(\alpha^3+\alpha^2+2\alpha+6)}$, $P_3 = \frac{2\alpha}{(\alpha^3+\alpha^2+2\alpha+6)}$ and $P_4 = \frac{6}{(\alpha^3+\alpha^2+2\alpha+6)}$such as:

$$f_{QArm}(\mathrm{y},\theta) = P_1\, f_1(\mathrm{y}) + P_2 f_2(\mathrm{y}) + P_3 f_3(\mathrm{y}) + (1 - P_1 - P_2 - P_2) f_4(\mathrm{y}) \tag{2.2}$$

Cumulative density function of Quasi Amarendra distribution is defined as:

$$F_{QArm}(y;\alpha,\theta) = 1 - \left[1 + \left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}\right] e^{-\theta y} \tag{2.3}$$
$$\theta, y > 0; \quad \alpha > -1$$

3. Some Properties

Moment generating function:

$$M_y(t) = \sum_{m=0}^{\infty} \frac{\alpha^3\Gamma(m+1)+\alpha^2\Gamma(m+2)+\alpha\Gamma(m+3)+\Gamma(m+4)}{(\alpha^3+\alpha^2+2\alpha+6)} \left(\frac{t}{\theta}\right)^m \tag{3.1}$$

Rth moments:

$$\mu_r' = \frac{\alpha^3\Gamma(r+1)+\alpha^2\Gamma(r+2)+\alpha\Gamma(r+3)+\Gamma(r+4)}{\theta^r(\alpha^3+\alpha^2+2\alpha+6)} \qquad r = 1,2,3,\ldots \tag{3.2}$$

1st four moment about origin are:

$$\mu_1' = \frac{\alpha^3+2\alpha^2+6\alpha+24}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.3}$$

$$\mu_2' = \frac{2\alpha^3+6\alpha^2+24\alpha+120}{\theta^2(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.4}$$

$$\mu_3' = \frac{6\alpha^3+24\alpha^2+120\alpha+720}{\theta^3(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.5}$$

$$\mu_4' = \frac{24\alpha^3+120\alpha^2+720\alpha+5040}{\theta^4(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.6}$$

By using (3.3), (3.4), (3.5) and (3.6) in following (3.7), (3.8) and (3.9) the moments about mean could be obtain:

$$\mu_2 = \mu_2' - (\mu_1')^2 \tag{3.7}$$

$$\mu_3 = \mu_3' + 3\mu_2'\mu_1' - 2(\mu_1')^3 \tag{3.8}$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4 \tag{3.9}$$

$$\mu_2 = \frac{(\alpha^6+4\alpha^5+18\alpha^4+96\alpha^3+72\alpha^2+96\alpha+144)}{\theta^2(\alpha^3+\alpha^2+2\alpha+6)^2} \tag{3.10}$$

$$\mu_3 = \frac{(2\alpha^9+12\alpha^8+72\alpha^7+484\alpha^6+652\alpha^5+936\alpha^4+1344\alpha^3+1440\alpha^2+1728\alpha+1728)}{\{\theta(\alpha^3+\alpha^2+2\alpha+6)\}^3} \tag{3.11}$$

$$\mu_4 = \frac{(9\alpha^{12}+54\alpha^{11}+534\alpha^{10}+7308\alpha^9+6036\alpha^8+27582\alpha^7+229800\alpha^6+74496\alpha^5+390096\alpha^4+550008\alpha^3+738720\alpha^2+1088640\alpha+1047168)}{\{\theta(\alpha^3+\alpha^2+2\alpha+6)\}^4} \tag{3.12}$$

The mean, variance, coefficient of variation, index of dispersion, mean deviation about mean and median of QAMD are obtained as:

Mean: $\frac{\alpha^3+2\alpha^2+6\alpha+24}{\theta(\alpha^3+\alpha^2+2\alpha+6)}$ (3.13)

Variance: $\frac{(\alpha^6+4\alpha^5+18\alpha^4+96\alpha^3+72\alpha^2+96\alpha+144)}{\theta^2(\alpha^3+\alpha^2+2\alpha+6)^2}$ (3.14)

Coefficient of Variation:

$C.V = \frac{\sigma}{\mu_1{}'} \times 100$ by using (3.13) and (3.14) (3.15)

CV= $\frac{\sqrt{(\alpha^6+4\alpha^5+18\alpha^4+96\alpha^3+72\alpha^2+96\alpha+144)}}{\alpha^3+2\alpha^2+6\alpha+24} \times 100$ (3.16)

Index of Dispersion:

$\gamma = \frac{\sigma^2}{\mu_1{}'}$ by using (3.13) and (3.14) (3.17)

$$\gamma = \frac{(\alpha^6+4\alpha^5+18\alpha^4+96\alpha^3+72\alpha^2+96\alpha+144)}{\theta(\alpha^3+\alpha^2+2\alpha+6)(\alpha^3+2\alpha^2+6\alpha+24)} \quad (3.18)$$

Coefficient of Skewness:

$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2{}^{3/2}}$ by using (3.10) and (3.11) (3.19)

$$\sqrt{\beta_1} = \frac{(2\alpha^9+12\alpha^8+72\alpha^7+484\alpha^6+652\alpha^5+936\alpha^4+1344\alpha^3+1440\alpha^2+1728\alpha+1728)}{(\alpha^6+4\alpha^5+18\alpha^4+96\alpha^3+72\alpha^2+96\alpha+144)^{3/2}} \quad (3.20)$$

Coefficient of Kurtosis:

$\beta_2 = \frac{\mu_4}{\mu_2{}^2}$ by using (3.10) and (3.12) (3.21)

$$\beta_2 = \frac{(9\alpha^{12}+54\alpha^{11}+534\alpha^{10}+7308\alpha^9+6036\alpha^8+27582\alpha^7+229800\alpha^6+74496\alpha^5+390096\alpha^4+550008\alpha^3+738720\alpha^2+1088640\alpha+1047168)}{(\alpha^6+4\alpha^5+18\alpha^4+96\alpha^3+72\alpha^2+96\alpha+144)^2} \quad (3.22)$$

Mean deviations:

The amount of variation in a population is generally measured to some extent by the totality of deviations usually either from the mean or the median. These are known as the mean deviation about the mean and the mean deviation about the median and are defined

$$\varphi_1(y) = \int_0^\infty |Y-\mu| f_{QArm}(\mathrm{y})dy \quad (3.23)$$

$$\mu = E(y)$$

$$\varphi_2(y) = \int_0^\infty |y-M| f_{QArm}(\mathrm{y})dy \quad (3.24)$$

$M = Median(y)$ The measure of $\varphi_1(y)$ and $\varphi_2(y)$ can be calculated as:

$$\varphi_1(y) = \int_0^\mu (\mu-y) f_{QArm}(\mathrm{y})dy + \int_\mu^\infty (y-\mu) f_{QArm}(\mathrm{y})dy$$

$$= \mu F(\mu) - \int_0^\mu y f_{QArm}(\mathrm{y})dy - \mu[1-F(\mu)] + \int_\mu^\infty y f_{QArm}(\mathrm{y})dy$$

$$= 2\mu F(\mu) - 2\mu + 2\int_\mu^\infty y f_{QArm}(\mathrm{y})dy$$

$$= 2\mu F(\mu) + 2\int_0^\mu y f_{QArm}(\mathrm{y})dy \quad (3.25)$$

Also

$$\varphi_2(y) = \int_0^\infty |y-M| f_{QArm}(\mathrm{y})dy$$

$$\varphi_2(y)=\int_0^M (M-y)f_{QArm}(\mathrm{y})dy+\int_M^\infty (y-M)f_{QArm}(\mathrm{y})dy$$

$$=MF(M)-\int_0^M yf_{QArm}(\mathrm{y})dy-M[1-F(M)]+\int_M^\infty yf_{QArm}(\mathrm{y})dy$$

$$=2MF(M)-\int_0^M yf_{QArm}(\mathrm{y})dy-M+\int_M^0 yf_{QArm}(\mathrm{y})dy+\int_0^\infty yf_{QArm}(\mathrm{y})dy$$

$$=-\int_0^M yf_{QArm}(\mathrm{y})dy+\int_M^0 yf_{QArm}(\mathrm{y})dy+\mu$$

$$=\mu-2\int_0^M yf_{QArm}(\mathrm{y})dy \tag{3.26}$$

So by using pdf (2.1) found that;

$$\int_0^\mu yf_{QArm}(\mathrm{y})dy=\frac{[\{1-(1+\mu\theta)e^{-\theta\mu}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(\mu\theta)^2(\alpha^2+3\alpha+12)+(\mu\theta)^3(\alpha+4)+(\mu\theta)^4\}e^{-\theta\mu}]}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.27}$$

$$\int_0^M yf_{QArm}(\mathrm{y})dy=\frac{[\{1-(1+M\theta)e^{-\theta M}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(M\theta)^2(\alpha^2+3\alpha+12)+(M\theta)^3(\alpha+4)+(M\theta)^4\}e^{-\theta M}]}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.28}$$

Put (3.27) in (3.25) and (3.28) into (3.26) have following:

$$\varphi_1(y)=\frac{2\mu\theta\{(\alpha^3+\alpha^2+2\alpha+6)-e^{-\theta\mu}(2\alpha^3+3\alpha^2+2\alpha(3+\theta^2)+30)\}+4(\mu\theta)^2\{(\alpha^2+2\alpha+6)+\mu\theta(\alpha+3)+(\mu\theta)^2\}e^{-\theta\mu}-2(\alpha^3+2\alpha^2+6\alpha\theta^2+24)(1-e^{-\theta\mu})}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.29}$$

$$\varphi_2(y)=\frac{\mu\theta(\alpha^3+\alpha^2+2\alpha+6)-2[\{1-(1+M\theta)e^{-\theta M}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(M\theta)^2(\alpha^2+3\alpha+12)+(M\theta)^3(\alpha+4)+(M\theta)^4\}e^{-\theta M}]}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{3.30}$$

4. Bonferroni and Lorenz Curves

The Bonferroni and Lorenz curves by [11] Bonferroni, 1930 and Bonferroni and Gini indices have utilized in economics to study the variation in income, poverty, also in other fields like reliability, vital statistics, insurance and medicine. The Bonferroni and Lorenz curves are defined as

$$B(p)=\frac{1}{p\mu}\int_0^q yf_{QArm}(\mathrm{y})dy \tag{4.1}$$

$$=\frac{1}{p\mu}\int_0^\infty yf_{QArm}(\mathrm{y})dy-\frac{1}{p\mu}\int_q^\infty yf_{QArm}(\mathrm{y})dy$$

$$=\frac{1}{p}-\frac{1}{p\mu}\int_q^\infty yf_{QArm}(\mathrm{y})dy$$

By using (2.1) pdf of QAMD find:

$$\int_0^q yf_{QArm}(\mathrm{y})dy=\frac{\{1-(1+q\theta)e^{-\theta q}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(q\theta)^2(\alpha^2+3\alpha+12)+(q\theta)^3(\alpha+4)+(q\theta)^4\}e^{-\theta q}}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{4.2}$$

Put (4.2) into (4.1).

$$B(p)=\frac{1}{p\mu}\frac{[\{1-(1+q\theta)e^{-\theta q}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(q\theta)^2(\alpha^2+3\alpha+12)+(q\theta)^3(\alpha+4)+(q\theta)^4\}e^{-\theta q}]}{\theta(\alpha^3+\alpha^2+2\alpha+6)} \tag{4.3}$$

And

$$L(p)=\frac{1}{\mu}\int_0^q yf_{QArm}(\mathrm{y})dy \tag{4.4}$$

$$=\frac{1}{\mu}\int_0^\infty yf_{QArm}(\mathrm{y})dy-\frac{1}{\mu}\int_q^\infty yf_{QArm}(\mathrm{y})dy$$

$$=1-\frac{1}{\mu}\int_q^\infty yf_{QArm}(\mathrm{y})dy$$

Put (4.2) into (4.4).

$$L(p) = \frac{\{1-(1+q\theta)e^{-\theta q}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(q\theta)^2(\alpha^2+3\alpha+12)+(q\theta)^3(\alpha+4)+(q\theta)^4\}e^{-\theta q}}{\mu\theta(\alpha^3+\alpha^2+2\alpha+6)} \quad (4.5)$$

Or both equivalent to $B(p) = \frac{1}{p\mu}\int_0^p F(y)^{-1}dy$ and $L(p) = \frac{1}{\mu}\int_0^p F(y)^{-1}dy$ where $q = F(p)^{-1}$

The Bonferroni indice is defined as:

$$B = 1 - \int_0^1 B(p)dp \quad (4.6)$$

Put (4.3) into (4.6) have,

$$B = 1 - \frac{(1-e^{-\theta q}-q\theta e^{-\theta q})(\alpha^2+2\alpha+6)-(q\theta)^2(\alpha+3+q\theta)e^{-\theta q}}{\theta(\alpha^2+\alpha+2)} \quad (4.7)$$

The Gini indice defined as following:

$$G = 1 - 2\int_0^1 L(p)dp \quad (4.8)$$

Put (4.5) into (4.8) determined as:

$$G = 1 - \frac{2[\{1-(1+q\theta)e^{-\theta q}\}(\alpha^3+2\alpha^2+6\alpha\theta^2+24)-\{(q\theta)^2(\alpha^2+3\alpha+12)+(q\theta)^3(\alpha+4)+(q\theta)^4\}e^{-\theta q}]}{\mu\theta(\alpha^3+\alpha^2+2\alpha+6)} \quad (4.9)$$

5. Reliability Measures

There are different reliability measures namely Survival Function, Hazard Rate Function, Mean Residual Life Function, Cumulative Hazard Function and Reversed Cumulative Hazard Function.
Let Y be a continuous random variable with pdf $f_{QArm}(y;\alpha,\theta)$ and cdf $F_{QArm}(y;\alpha,\theta)$ of Quasi Armarindra distribution. Then the Survival $S_{QArm}(y;\alpha,\theta)$, Hazard rate $h_{QArm}(y;\alpha,\theta)$, Mean residual life function $m_{QArm}(y;\alpha,\theta)$, Cumulative hazard function $CH_{QArm}(y;\alpha,\theta)$ and $H_{QArm}(y;\alpha,\theta)$ Reversed cumulative hazard function given below:

Survival function:

Let Y be a continuous random variable with pdf $f_{QArm}(y;\alpha,\theta)$ (2.1) and cdf $F_{QArm}(y;\alpha,\theta)$ (2.3) of QAMD the Survival function obtain as:

$$S_{QArm}(y;\alpha,\theta) = \left[1 + \left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}\right]e^{-\theta y} \quad (5.1)$$

Hazard function:

Let Y be a continuous random variable with p.d.f. $f_{QArm}(y;\alpha,\theta)$ (2.1) and c.d.f. $F_{QArm}(y;\alpha,\theta)$ (2.3) of new Quasi Amrandra distribution (QAMD) The hazard rate function known as the failure rate function defined as:

$$h_{QArm}(\mathrm{y}) = \lim_{\Delta y\to 0}\frac{P(Y<y+\Delta y|Y>y)}{\Delta y} = \frac{f(y)}{1-F(y)} \quad (5.2)$$

By using (2.1) and (2.3) find as:

$$h_{QArm}(y;\alpha,\theta) = \frac{\theta\{\alpha^3+\alpha^2\theta y+\alpha(\theta y)^2+(\theta y)^3\}}{[(\alpha^3+\alpha^2+2\alpha+6)+y\theta(\alpha^2+2\alpha+6)+(\theta y)^2(\alpha+3)+(y\theta)^3]} \quad (5.3)$$

Here note that:

$$h_{QArm}(0;\alpha,\theta) = f_{QArm}(0;\alpha,\theta)$$

Mean Residual Life Function:

Let Y be a continuous random variable with p.d.f. $f_{QArm}(y;\alpha,\theta)$ and cdf $F_{QArm}(y;\alpha,\theta)$ of new Quasi Amarendra distribution (QAMD) The Mean Residual Life function defined as:

$$m(y;\alpha,\theta)=E(Y-y|Y>y)=\frac{1}{1-F(y;\alpha,\theta)}\int_x^\infty\{1-F(t;\alpha,\theta)\}dt \quad (5.4)$$

By using (2.1) and (2.3) in (5.4) find as:

$$m_{QArm}(y;\alpha,\theta)=\frac{(\alpha^3+2\alpha^2+6\alpha+24)+x\theta(\alpha^2+4\alpha+18)+(x\theta)^2(\alpha+6)+(x\theta)^3}{\theta\{(\alpha^3+\alpha^2+2\alpha+6)+y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3\}} \quad (5.5)$$

Also note that put y=0 in (5.5) and get (3.3):

$$m_{QArm}(0;\alpha,\theta)=\mu_1{}'=\frac{\alpha^3+2\alpha^2+6\alpha+24}{\theta(\alpha^3+\alpha^2+2\alpha+6)}$$

Cumulative hazard function:

Let Y be a continuous random variable with p.d.f. $f_{QArm}(y;\alpha,\theta)$ and c.d.f. $F_{QArm}(y;\alpha,\theta)$ of (QAMD) then the Cumulative hazard function defined as:

$$CH_{QArm}(y;\alpha,\theta)=-\ln|F(y;\alpha,\theta)| \quad (5.6)$$

By putting (2.3) we have,

$$CH_{QArm}(y;\alpha,\theta)=-\ln\left|1-\left[1+\left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}\right]e^{-\theta y}\right| \quad (5.7)$$

Reversed hazard function:

Let Y be a continuous random variable with p.d.f. $f_{QArm}(y;\alpha,\theta)$ and c.d.f. $F_{QArm}(y;\alpha,\theta)$ of (QAMD) then the reversed hazard function defined as:

$$H_{QArm}(\mathrm{y})=\frac{f(y;\alpha,\theta)}{F(y;\alpha,\theta)} \quad (5.8)$$

By putting (2.1) and (2.3) into (5.8) have found

$$H_{QArm}(y;\alpha,\theta)=\frac{\theta^4\{1+y+y^2+y^3\}e^{-\theta y}}{(\alpha^3+\alpha^2+2\alpha+6)-\{(\alpha^3+\alpha^2+2\alpha+6)+y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3\}e^{-\theta y}} \quad (5.9)$$

6. Oder Statistics (OS)

The density function $f_{(i,j)}(\mathrm{y})$ of "ith" order statistics (i=1, 2, …, n) from independent and identically distributed (i.i.d), random variable $y_1, y_2, \dots y_n$. The order statistics say as $y_{(1)}, y_{(2)}, \dots y_{(n)}$ the function of order statistics defined as:

$$f_{(i,j)}(y;\alpha,\theta)=\frac{f(y)}{B(i,n-i+1)}\sum_{j=0}^{n-i}(-1)^j\binom{n-j}{j}[F(y)]^{i+j-1} \quad (6.1)$$

$$f_{(i,j)}(y;\alpha,\theta)=$$
$$\frac{\theta^4\{1+y+y^2+y^3\}e^{-\theta y}}{B(i,n-i+1)(\alpha^3+\alpha^2+2\alpha+6)}\sum_{j=0}^{n-i}(-1)^j\binom{n-j}{j}\left[1-\left[1+\left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}\right]e^{-\theta y}\right]^{i+j-1}$$

Where $B(i,n-i+1)$ is the beta function. Here, present an expansion for the density function of new Quasi Amarendra distribution (QAMD) (2.1), $f_{QArm}(y;\alpha,\theta)$ *and* cdf (2.3) $F_{QArm}(y;\alpha,\theta)$ into (6.1) have pdf of ith order statistics is:

$$f_{(i,j)}(y;\alpha,\theta)=$$
$$\frac{\theta^4\{1+y+y^2+y^3\}e^{-\theta y}}{B(i,n-i+1)(\alpha^3+\alpha^2+2\alpha+6)}\sum_{j=0}^{n-i}(-1)^j\binom{n-j}{j}\sum_{m=0}^{\infty}\binom{i+j-1}{m}e^{-\theta my}\left[1+\left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}\right]^m$$

$$f_{(i,j)}(y;\alpha,\theta) = \frac{\theta^4\{1+y+y^2+y^3\}e^{-\theta y}}{B(i,n-i+1)(\alpha^3+\alpha^2+2\alpha+6)}\sum_{j=0}^{n-i}(-1)^j\binom{n-j}{j}\sum_{m=0}^{\infty}\binom{i+j-1}{m}e^{-\theta my}\sum_{l=0}^{m}\binom{m}{l}\left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}^{m-l} \quad (6.2)$$

The ith order statistics c.d.f is:

$$F_{(i,j)}(y;\alpha,\theta) = \sum_{j=0}^{n}\sum_{i=0}^{n-j}(-1)^i\binom{n}{j}\binom{n-j}{i}\left\{1-\left[1+\left\{\frac{y\theta(\alpha^2+2\alpha+6)+(y\theta)^2(\alpha+3)+(y\theta)^3}{(\alpha^3+\alpha^2+2\alpha+6)}\right\}\right]e^{-\theta y}\right\}^{j+i} \quad (6.3)$$

For maximum order statistics put i=n, for minimum order statistics put i=1 in equation.

7. Renyi Entropy Measure

A popular entropy measure is [12] Renyi entropy (1961), an entropy of a random variable Y is a measure of the variation of uncertainty. Let Y is a continuous random variable having probability density function (QAMD) $f_{QArm}(y;\alpha,\theta)$, then Renyi entropy is defined as

$$T_{RE}(y) = \frac{1}{1-\delta}\log\left\{\int_0^\infty f(y)^\delta\,dy\right\} \quad (7.1)$$

Let $f_{QArm}(y;\alpha,\theta)$ pdf of (QAMD) the Renyi entropy such that:

$$T_{RE}(y;\alpha,\theta) = \frac{1}{1-\delta}\log\left\{\int_0^\infty f_{QArm}(y;\alpha,\theta)^\delta\,dy\right\}$$

$$= \frac{1}{1-\delta}\log\left\{\int_0^\infty\left[\frac{\theta}{(\alpha^3+\alpha^2+2\alpha+6)}\{\alpha^3+\alpha^2\theta y+\alpha(\theta y)^2+(\theta y)^3\}e^{-\theta y}\right]^\delta dy\right\}$$

$$= \frac{1}{1-\delta}\log\left\{\frac{\theta^\delta}{(\alpha^3+\alpha^2+2\alpha+6)^\delta}\int_0^\infty\{\alpha^3+\alpha^2\theta y+\alpha(\theta y)^2+(\theta y)^3\}^\delta e^{-\theta\delta y}dy\right\}$$

$$= \frac{1}{1-\delta}\log\left\{\frac{\theta^\delta\alpha^{3\delta}}{(\alpha^3+\alpha^2+2\alpha+6)^\delta}\int_0^\infty\left\{1+\frac{\theta y}{\alpha}+\left(\frac{\theta y}{\alpha}\right)^2+\left(\frac{\theta y}{\alpha}\right)^3\right\}^\delta e^{-\theta\delta y}dy\right\}$$

$$= \frac{1}{1-\delta}\log\left\{\frac{\theta^\delta\alpha^{3\delta}}{(\alpha^3+\alpha^2+2\alpha+6)^\delta}\int_0^\infty\sum_{j=0}^{\infty}\binom{\delta}{j}\left\{\frac{\theta y}{\alpha}+\left(\frac{\theta y}{\alpha}\right)^2+\left(\frac{\theta y}{\alpha}\right)^3\right\}^j e^{-\theta\delta y}dy\right\}$$

$$= \frac{1}{1-\delta}\log\left\{\frac{\theta^\delta\alpha^{3\delta}}{(\alpha^3+\alpha^2+2\alpha+6)^\delta}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\binom{\delta}{j}\binom{j}{k}\binom{k}{l}\left(\frac{1}{\alpha\delta}\right)^{j+k+l}\int_0^\infty(y)^{j+k+l}e^{-\theta\delta y}\,dy\right\}$$

$$= \frac{1}{1-\delta}\log\left\{\frac{\theta^{\delta-1}\alpha^{3\delta}}{\delta(\alpha^3+\alpha^2+2\alpha+6)^\delta}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\binom{\delta}{j}\binom{j}{k}\binom{k}{l}\left(\frac{1}{\alpha\delta}\right)^{j+k+l}\Gamma(j+k+l+1)\right\} \quad (7.2)$$

8. Graphs

The graphs of c.d.f **Fig. 8.1 (a)**, p.d.f **Fig. 8.1 (b)**, hazard **Fig. 8.1 (c)** and survival function **Fig. 8.1 (d)** given below:

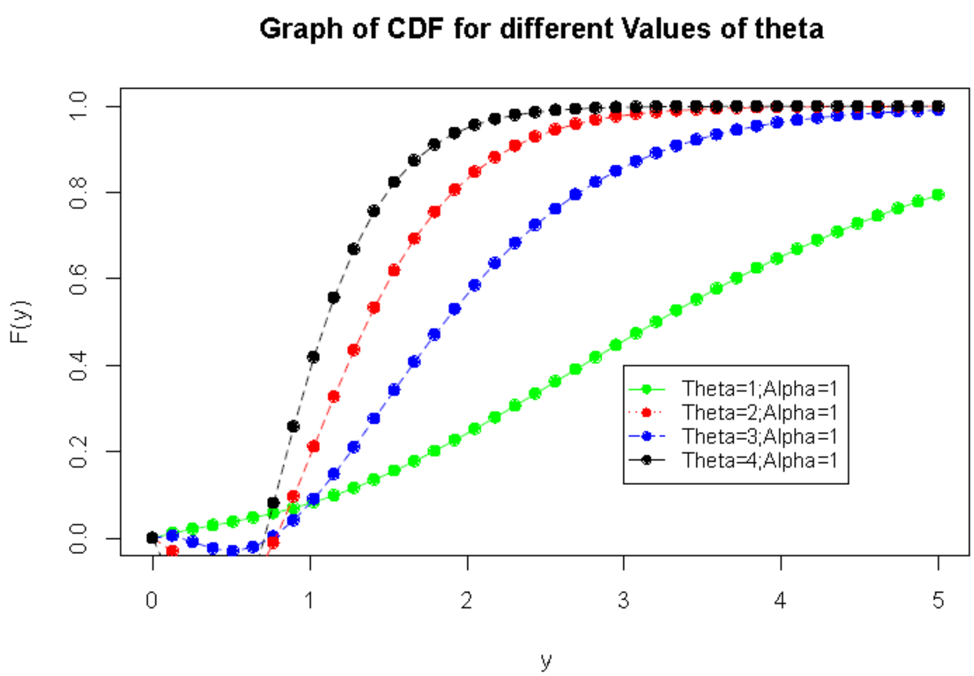

Fig. 8.1 (a)

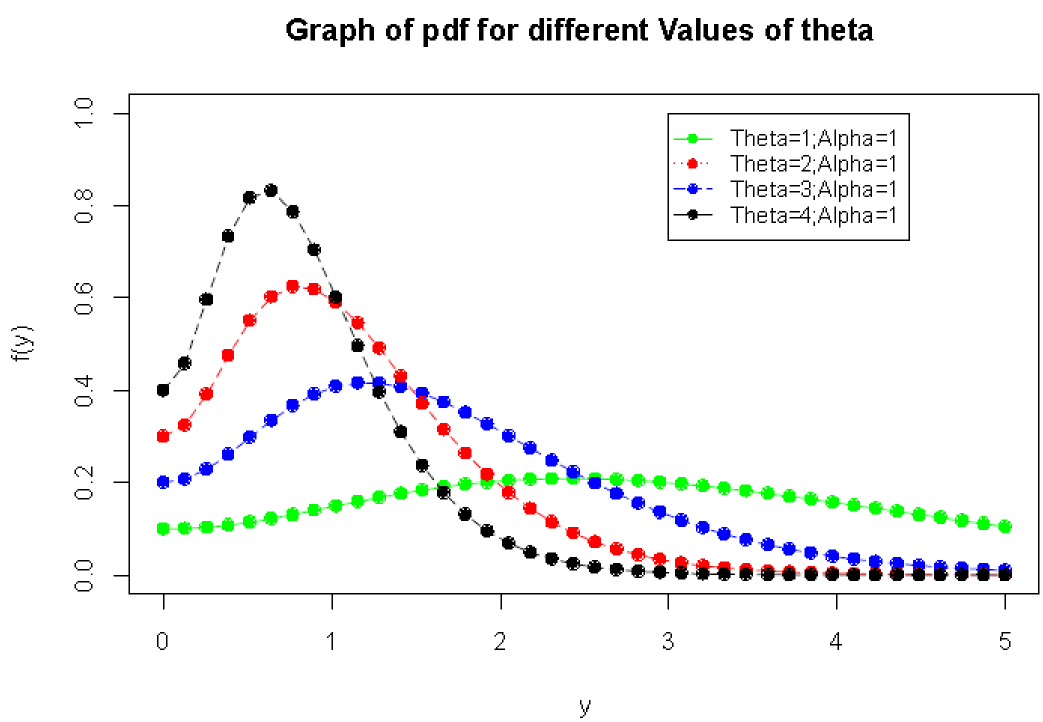

Fig. 8.1 (b)

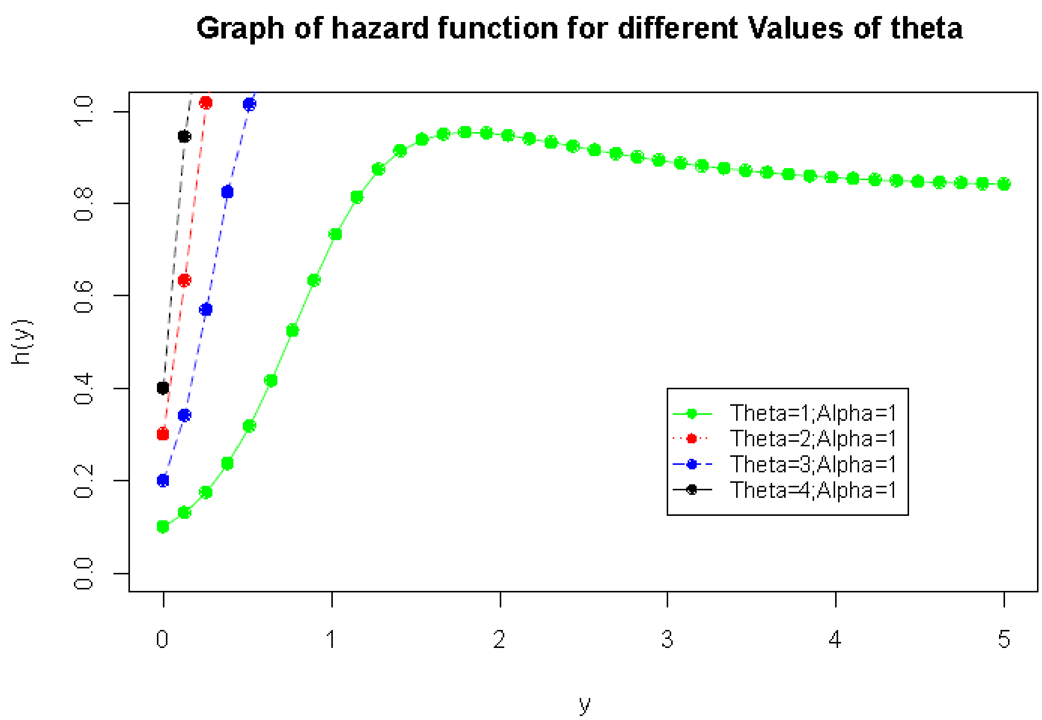

Fig. 8.1 (c)

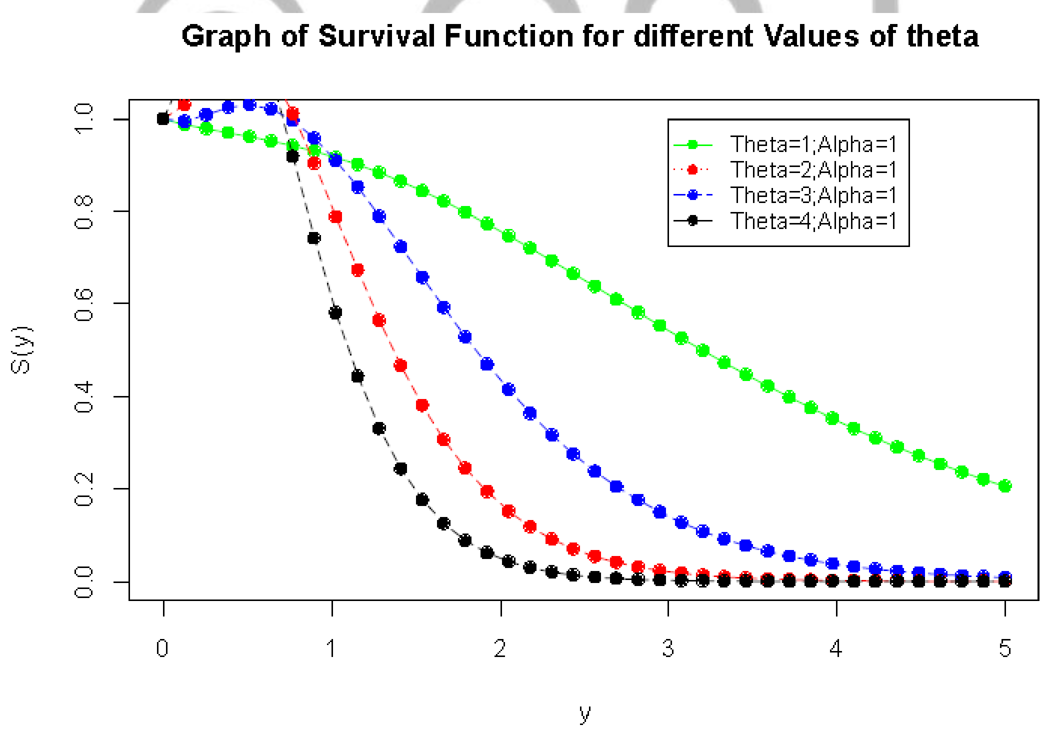

Fig. 8.1 (d)

9. Maximum Likelihood method

Let be a random sample $y_1, y_2, \dots y_n$ from $f_{QArm}(y_i; \alpha, \theta)$ (QAMD), then the Maximum Likelihood (ML) function defined as:

$$L(y;\, \theta, \alpha) = \prod_{i=0}^{n} \frac{\theta e^{-\theta y_i}}{(\alpha^3+\alpha^2+2\alpha+6)} \{\alpha^3 + \alpha^2\theta y_i + \alpha(\theta y_i)^2 + (\theta y_i)^3\} \tag{9.1}$$

$$lnL(y;\, \theta, \alpha) = nln\theta - nln(\alpha^3 + \alpha^2 + 2\alpha + 6) + \sum_{i=0}^{n} \ln\{\alpha^3 + \alpha^2\theta y_i + \alpha(\theta y_i)^2 + (\theta y_i)^3\} - \theta \sum_{i=0}^{n} y_i$$

$$\frac{\partial lnL(y;\, \theta,\alpha)}{\partial\theta} = \frac{n}{\theta} + \sum_{i=0}^{n} \frac{\alpha^2 y_i + 2\alpha\theta(y_i)^2 + 3\theta^2(y_i)^3}{\ln\{\alpha^3+\alpha^2\theta y_i+\alpha(\theta y_i)^2+(\theta y_i)^3\}} - \sum_{i=0}^{n} y_i \tag{9.2}$$

$$\frac{\partial lnL(y;\, \theta,\alpha)}{\partial\alpha} = \frac{n(3\alpha^2+2\alpha+2)}{(\alpha^3+\alpha^2+2\alpha+6)} + \sum_{i=0}^{n} \frac{3\alpha^2+2\alpha\theta y_i+(\theta y_i)^2}{\{\alpha^3+\alpha^2\theta y_i+\alpha(\theta y_i)^2+(\theta y_i)^3\}} \tag{9.3}$$

By putting (9.2) and (9.3) as $\frac{\partial lnL(y;\, \theta,\alpha)}{\partial\theta} = 0;$ $\qquad \frac{\partial lnL(y;\, \theta,\alpha)}{\partial\alpha} = 0$

The (9.2) and (9.3) natural log likelihood equations do not seem to be solved directly because they are not in closed form, so the Fisher's scoring method can be applied to solve these equations. We have

$$\frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\theta^2} = -\frac{n}{\theta^2} + \sum_{i=0}^{n} \frac{2(y_i)^2\{\alpha+3\theta^2\}\ln\{\alpha^3+\alpha^2\theta y_i+\alpha(\theta y_i)^2+(\theta y_i)^3\} - \{\alpha^2 y_i+2\alpha\theta(y_i)^2+3\theta^2(y_i)^3\}^2}{\{\alpha^3+\alpha^2\theta y_i+\alpha(\theta y_i)^2+(\theta y_i)^3\}[\ln\{\alpha^3+\alpha^2\theta y_i+\alpha(\theta y_i)^2+(\theta y_i)^3\}]^2} \tag{9.4}$$

$$\frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\alpha^2} = \frac{2n(\alpha^2+\alpha+2)-(2\alpha+1)^2}{(\alpha^2+\alpha+2)^2} + \sum_{i=0}^{n} \frac{2\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\}\ln\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\} - (2\alpha+\theta y_i)^2}{\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\}[\ln\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\}]^2} \tag{9.5}$$

$$\frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\theta\partial\alpha} = \sum_{i=0}^{n} \frac{y_i\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\}\ln\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\} - \{\alpha y_i+2\theta(y_i)^2\}^2}{\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\}[\ln\{\alpha^2+\alpha\theta y_i+(\theta y_i)^2\}]^2} \tag{9.7}$$

The iterative solution of the equations (9.4) to (9.7) using matrix given following will be the MLEs $\hat{\theta}$ $\hat{\alpha}$ of parameters θ α of QAMD.

$$\begin{bmatrix} \frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\theta^2} & \frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\alpha\partial\theta} \\ \frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\theta\partial\alpha} & \frac{\partial lnL^2(y;\, \theta,\alpha)}{\partial\alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta} - \theta_0 \\ \hat{\alpha} - \alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial lnL(y;\, \theta,\alpha)}{\partial\theta} \\ \frac{\partial lnL(y;\, \theta,\alpha)}{\partial\alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

Where θ_0 and α_0 initial values of parameters of θ α of QAMD.

Conclusion

In this study Quasi Amarendra (QAMD) distribution with two parameters special case of Amarendra distribution as $\theta = \alpha$, discussed some properties, variation measurement Bonferroni and Lorenz curves, reliability measures, Order statistics, specific measure of variation in uncertainty is Renyi entropy, Behaviour of (QAMD) c.d.f in Fig. 8.1 (a) is monotonically increasing function and p.d.f in Fig. 8.1 (b) is positively skewed, monotonically decreasing function with varying parameter values showing it may performed better model for real lifetime data analysis, behavior of Hazard function in Fig. 8.1 (c) and Survival function in Fig. 8.1 (d) of (QAMD) is monotonically increasing and decreasing function respectively. Under the ML method using variance covariance matrix could be estimate the MLEs $\hat{\theta}$ $\hat{\alpha}$ of parameters θ α of (QAMD) and use in real life applications.

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