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Title: Reasoning and Argumentation in Teaching Mathematics and Mathematics Achievement in Oyo State, Nigeria

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Abstract

The purpose of this was to explore the role of reasoning and argumentation in teaching pedagogy and learning in primary mathematics in Nigeria by investigating the existing pedagogy of mathematics in Nigeria primary (basic level) schools, examine the extent of implementing reasoning and argumentative approach in teaching mathematics at the basic level and examining the degree at which students use reasoning and argumentation in solving mathematics. 220 teachers were surveyed about their beliefs and practices regarding the teaching of argumentation and these views were correlated with their students (n=440) performance on reasoning and argumentation tasks. A significant correlation was found between more favorable attitudes and more practices and student performance. This confirms previous findings that showed that the teaching critical thinking skills was also an effective means of enhancing students' understanding of Mathematics concepts even at the primary school level.

Introduction

Mathematics is a compulsory subject at all levels of education yet, more than half of children failed to meet minimum mathematic proficiency standards at the end of primary school in 1 in 4 countries, and at the lower secondary level in 1 in 3 countries (UNICEF, 2018). Several indicators such as the way the subject is being taught, lack of relevance of mathematics content to student's real life, inability and laziness of students to reason out suitable solution to a particular problem and lots more has been and attributed to this failure (George & Charles, 2019). A major objective of learning mathematics in school is to train students in reasoning, develop the ability to solve problems, provide information or communicate ideas through speech, writing, pictures, graphs, maps, diagrams, etc. Unfortunately, primary students have difficulty in mathematical reasoning and teachers rarely ask students to give written reasons, due to time pressure and students' difficulties to express their thoughts in writing (Pehkonen, 2000; Hasanah & Surya, 2017). It is, however, important in Mathematics teaching that thinking logically, analytical, systematic, critical, creative should be provided to all learners knowing so well that Mathematical thinking is different from doing mathematics. This will build a good foundation for formal proofs in high school (Ayinla, 2015; Risqi & Surya, 2017).

Science, Technology, Engineering and Mathematics (STEM) skills is increasing, the level of student participation in mathematics is steadily declining in many countries (OECD, 2006 cited in Wells, 2014) Mathematics as an important subject in modern society is useful in schools, workplaces, businesses and for personal decision-making. Mathematics is seen to be a language for everyday use whether in the market place, schools or even at home. Mathematics is fundamental to national prosperity in providing tools for understanding Science, Engineering, Technology and Economics. The importance of this subject may have led the Nigerian government to make it a compulsory subject in basic education and senior secondary schools as well as a prerequisite for admission to tertiary institutions. Despite the importance placed on Mathematics by the society, Maduabum and Odili (2006) observed that some students lack interest in the subject and perform poorly in it. This could be attributed to the students' inability to think critically and analyze mathematical concepts systematically. According to Osarenren and Asiedu (2007), and Owolabi (2003), it is particularly disappointing to find that mathematics has remained one of the subjects with high failure rate in Nigerian schools despite its importance, and the time it receives in an average school system.

Mathematics is taught at least four times in a week based on the Universal Basic Education (UBE) policy. UBE was launched in 1999 of which the programme consists of a 9-3-4 system involving 6 years of primary school and 3 years of junior secondary school culminating in 9 years of uninterrupted schooling (Chinyere & Uche, 2013). The 9-year Education Mathematics curriculum emphasizes entrepreneurship, affective domain and quantitative reasoning in order to boost learner's cognitive and psychomotor capabilities (NERDC 2012). Considering that learners spend more years of their schooling in basic schools in Nigeria and mathematics being a compulsory prerequisite to higher learning it is expected that their output justify the input but unfortunately many Nigerian school children lack proper understanding for mathematics (George & Charles, 2019). Mathematical reasoning encompasses both quantitative reasoning as well as other forms of reasoning as such measurement of mathematical practices which include application of mathematical proficiency in earning a living, reasoning and argumentation must be salient features of teachers' pedagogy and assessment design.

The role of reasoning and argumentation in teaching pedagogy and learning of mathematics among younger pupils cannot be over-emphasized. Findings show that in any classroom setting, reasoning and argumentation are germane to learning. Both factors organize students' ideas, build strong conceptual connections and foster mathematical thinking. The same holds in the field of education, either as a means to learn (argue to learn) or as a goal of instruction (learn to argue).This is very relevant to the society, as life situations are not static, hence the need for consideration of multiple factors, ability to express accurately their thoughts by forming arguments before arriving at a decision that shapes their reasoning in the future. According to Rapanta (2018) argumentation is the constructs of social and socio mathematical norms which are dialectical in nature because of the critical consideration of conflicting ideas to change the epistemic status of solution. The general procedure that gives rise to argument involves seeking justification over an idea, claim or conclusion. It inspires constructed self-discovery in order to contradict a position thus deepens understanding of the problem at hand. An argument presented with sufficient rigor will enlighten and convince more students, who in turn may convince their peers.

Argumentation is a holistic activity of making claims, challenging them, backing them up by producing evidence and reasoning, criticizing those reasons, rebutting those criticisms. While reasoning is used to describe the justification of evidence in support of a claim, so as to show how evidence justifiably leads to the claim. Argumentation as desired reasoning practice in classroom is well captured in literature except for what precise type of reasoning supports classroom argumentation (Jill 2014; Rapanta, 2018) The term argumentation is rarely used in mathematics until lately. Over the years, approaches to teaching mathematics has been 'drill and practice', or routine processes of problem-solving questions meanwhile the main objective of mathematics educators is to foster mathematical reasoning and understanding. Mathematics is very often about abstract entities; students have challenges grasping theories meanwhile argumentation in mathematics is absolutely crucial and multimodal. However, this factor has not being widely explored in the field of mathematics where it is assumed that students' quality of argumentative reasoning in basic mathematics is generally low (Bieda, 2010 cited in Elvis, 2017).

Traditionally, mathematics students are taught theories and models first, and then progress to exercises before its application to real-life situation. This approach has been argued not to produce real learning particularly in mathematics (Rapanta, 2018). Asking students to memorize mathematical formula, procedures, principles and structure will not allow creative reasoning. Emphasis must be on full understanding of the subject matter which includes the capacity to engage in the process of mathematical thinking to solve problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on (Elvis, 2017). Mathematics can be practiced in our day-to-day living as it can be a situated problem such that the context itself, and the tools appropriate to the context, can serve as a support. For instance, to find three-quarters of two-thirds of a cup, instead of calculating $^3/_4$ x $^2/_3$ as an algorithm, you measure $^2/_3$ of a cup, divide it into four parts and remove one quarter. The cup measure provided the tool to approach the problem concretely, quickly and efficiently. This contextualized approach helps learners to develop informal understanding of mathematics before being exposed to the subject formally and further discourages teaching mathematics as an abstract subject (Jill, 2014). An example of mathematical reasoning from pupils in basic class:

What is $2\frac{1}{2}$ divided by $\frac{1}{4}$? (Battista 1999). Many students solve this problem using the "invert and multiply" procedure they memorize and almost never understand:

$2\frac{1}{2} \div \frac{1}{4} = \frac{5}{2} \times \frac{4}{1} = \frac{20}{2} = 10$

They do not make conceptual sense of this procedure, and the only way they can justify it is by saying something like "That's the way my teacher taught me." In contrast, students who have made sense of and understand division of fractions do not need a symbolic procedure to compute an answer to this problem. They can think about the symbolic problem physically as one that requires finding the number of pieces of size one-fourth that fit in a quantity of size two and one-half (as shown in figure 1).

They reason that, since there are 4 fourths in each 1, and 2 fourths in $\frac{1}{2}$, there are 10 fourths in $2\frac{1}{2}$.

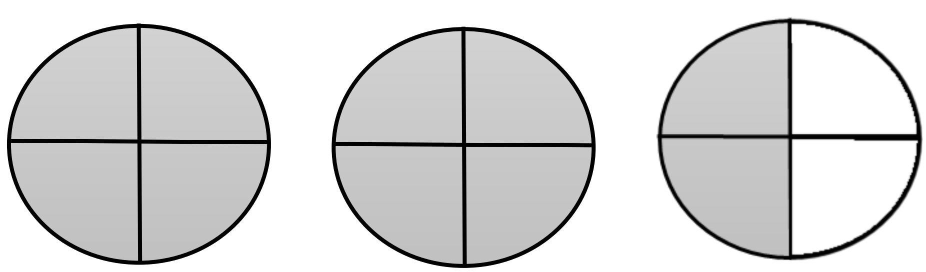


Figure 1: Finding the number of fourths in $2\frac{1}{2}$ using pictorial means

Also, having this mental-model–based intuitive understanding of division of fractions can help students start to make personal reasoning of the symbolic algorithm. In the problem $2\frac{1}{2} \div \frac{1}{4}$ why do we change division by $\frac{1}{4}$ to multiplication by 4? Because there are 4 fourths in each whole, to determine how many fourths are in the divided $2\frac{1}{2}$, we must multiply the number of wholes in the dividend (including fractional parts) by 4.

$2\frac{1}{2} \div \frac{1}{4} = 2\frac{1}{2} \times 4 = 8 + 2 = 10$

As another example, what is 10 divided by $\frac{1}{4}$? Because there are 4 fourths in each whole, and there are 10 ones in 10, there are 10 times 4 fourths in 10. So, the answer is found by multiplying the dividend 10 by 4; that is, $10 \div \frac{1}{4} = 10 \times 4 = 40$. To have students continue this reasoning, teachers in basic school can ask pupils to describe similar problems (Battista, 2012).

The above example shows mathematical reasoning in early grades where reasoning means explaining your mathematical argument in a way to clarify your personal ideas to others. A mathematics teacher should reason together with the students with the aim of promoting some level of argument on the taught contents. Abductive, plausible and defeasible hypothetical reasoning should be used as main tools in guiding students' argumentation in the mathematics classroom. Mathematical argument is a line of reasoning that shows why a mathematical outcome is true. It might be a formal or informal proof, an explanation of how a student or teacher arrived at a conjecture, how a student or teacher reasoned (Schwarz, Hershkowitz, Prusak, 2010). To understand how to change the ways in which students learn to reason in school mathematics communities, there is need to research how teachers, students, and the curricula vis-à-vis instructional materials existing at school mathematics communities interact in classroom settings. There is need attempted to facilitate mathematical reasoning skills by means of solving problems in students. This will create chances for discussing the validity of arguments they construct and relating knowledge and experiences to gain deep understanding. The above aforementioned problems and many more are the main reasons why the researcher examines reasoning and argumentation in elementary mathematics education

Problem statement

Problem-solving is a major skill in the 21st century which can be gotten through critical thinking thus the supply of capable mathematically-trained pupils, in an increasingly technological society cannot be overemphasized; as it prepares citizens with productive skills to meet societal needs. A problem or task may have multiple entry hence the need for student to be allowed to reason differently so as to bring solutions out across various mathematics concept. Mathematics education is a major discipline that should respond positively to these needed skills unfortunately it appears that many students see the subject as an unattainable task and difficult subject to understand yet necessary to pass because of the predominant traditional way of teaching the subject. A great majority of students believe that mathematics competence is reserved for a selected few. Considering this major gap between learning mathematics and doing mathematics, there is need for reform in Nigeria mathematics classrooms through changes in pedagogical approaches to focus on reasoning and argumentation.

Purpose of the Study

The main objective of this research is to identify the role of reasoning and argumentation in pedagogical contexts as well as learning basic mathematics and to provide mathematics teachers with tools to help them identify, assess, and use this approach to inculcate knowledge.
Specifically, this research seeks to:

- Investigate the existing pedagogy of mathematics in Nigeria primary schools
- Examine the extent of implementing reasoning and argumentative approach in teaching mathematics at basic levels.
- Examine the degree at which basic pupils use reasoning and argumentation in solving mathematics.

Significance and Contribution to Knowledge

The study will add to existing theories through its findings. Policy makers on education through this publication update mathematics curriculum in Nigeria to accommodate reasoning and learning in mathematics. Teacher's pedagogy style and students will shift from 'learn mathematics' to 'do mathematics' thereby enhancing reflective thinking and creativity, giving priority to personal discovery, expression, team work and vitality of young people to enable them face uncertain and demanding future.

Theoretical Framework

Many educational theorists support the view that argumentative reasoning is a way of learning and can be relevant in mathematics education. These include Jean Piaget's theory on distinguishing between three types of interaction among pupils who contribute to cognitive development, Toulmin (1958), focus on the component parts of an argument, the form and role of these elements. She opined that the use and identification of the elements of an argument offer opportunities for analysis of the argument in terms of components and their linkages. Moreover, this study shall consider the Deanna Kuhn approach to learning and argumentation. According to Kuhn (2010), all learning can be conceptualized as argument. Specifically, the theory related science learning to science argumentation, developing the view of teaching and learning science as argument. Also the theory explained the relationship between argumentation and learning, stressed that the first and most crucial development is an increase in students' ability and willingness to attend critically to the others dialogue, of which only abductive type of reasoning is dialogical thus potentially argumentative (Kuhn, 2010). Researchers in Science agree that scientific reasoning is hypothetical-deductive, meaning that it is based on hypothesis formation, which is an abductive process.

Literature Review
Concept of Reasoning

In order to evaluate the thinking ability of students who study mathematics, it is necessary to know the components in students' thinking skills, especially students who study mathematics. According to Elvis (2017) one level of mathematics learning outcomes is the students' reasoning ability. The reasoning is one mathematical thinking skills in addition to conjecturing, proving, making connections, abstraction, generalization, and specialization. Being able to reason is essential to understanding mathematics (Lithner, 2000). Mathematics reasoning is the process of making sense, understanding mathematical ideas and concepts inherent to procedures (Rapanta, 2018). Reasoning is a process in which the reasoner is conscious that a judgment, the conclusion, is determined by other judgment or judgments, the premises, according to a general habit of thought. The rule of reasoning, must be truth-conducive, and truth must be based on knowledge and justification (Psillos, 2011).Reasoning is an attempt to relate facts, concepts, or principles, look for pattern emerged, make effort to generalize or logical conclusion, make conjecture and simultaneously its proof (Lithner, 2000). Lithner (2000) pointed two ways in which a process of reasoning can confer justification on a belief. The first is by making the case that if the premises are true, the conclusion must be true. The second is by rendering a belief plausible and thus making it available for further testing. Only the second way called abductive reasoning, may produce new knowledge, and thus may be related to scientific reasoning.

Lithner (2000) opined that reasoning is the bedrock of mathematics of which if the skill is not properly developed in students then the ultimate purpose of learning mathematics will be defeated. He further stressed that reasoning is a way of thinking adopted to produce assertions and reach conclusion by transferring properties from one familiar situation to another task solving situation that has some level of superficial resemblance to the familiar situation. Whereas argumentation is the substantiation, the part of the reasoning that aims at convincing oneself, or someone else, that the reasoning is appropriate. Lithner (2003) further classified reasoning into plausible reasoning (PR), established experience (EE), and identification of similarity (IS). This classification based on three parts, which he called reasoning structure, components and properties, and reasoning characteristics. According to Lithner (2003, pp. 31-32), one way to structure the reasoning is:

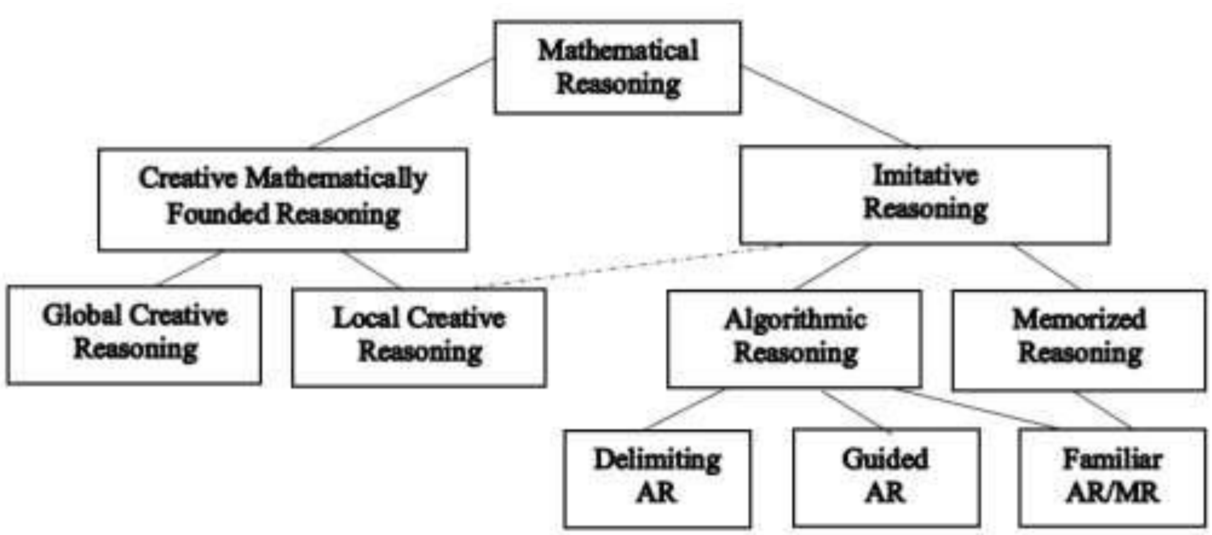
1. A problematic situation is met where it is not obvious how to proceed.
2. Strategy choice: Try to choose (in a wide sense: choose, recall, construct, discover, etc.) a strategy that can solve the difficulty. This choice can be supported by predictive argumentation: Will the strategy solve the difficulty?
3. Strategy implementation: This can be supported by verificative argumentation: Did the strategy solve the difficulty?
4. Conclusion: A result is obtained.

A sequence of mathematical reasoning is classified as PR if the strategy choice and strategy implementation: (i) is founded on intrinsic mathematical properties of the components involved in the reasoning, and (ii) is meant to guide towards what probably is the truth, without necessarily having to be completed or corrected. Meanwhile, the reasoning is classified as EE if the argumentation (i) is founded on notions and procedures established on the basis of the individual's previous experiences from the learning environment, and (ii) is meant to guide towards what probably is the truth, without necessarily having to be completed or corrected. Last, the reasoning is classified as IS if it fulfills (i) the strategy choice is founded on identifying similar surface properties in an example, theorem, rule, or some other situation described earlier in the text. (ii) The strategy implementation is carried through by mimicking the procedure from the identified situation.

Various researches revealed that reasoning can be categorized into quantitative reasoning, creative reasoning, deductive reasoning, inductive reasoning, abductive reasoning, imitative reasoning

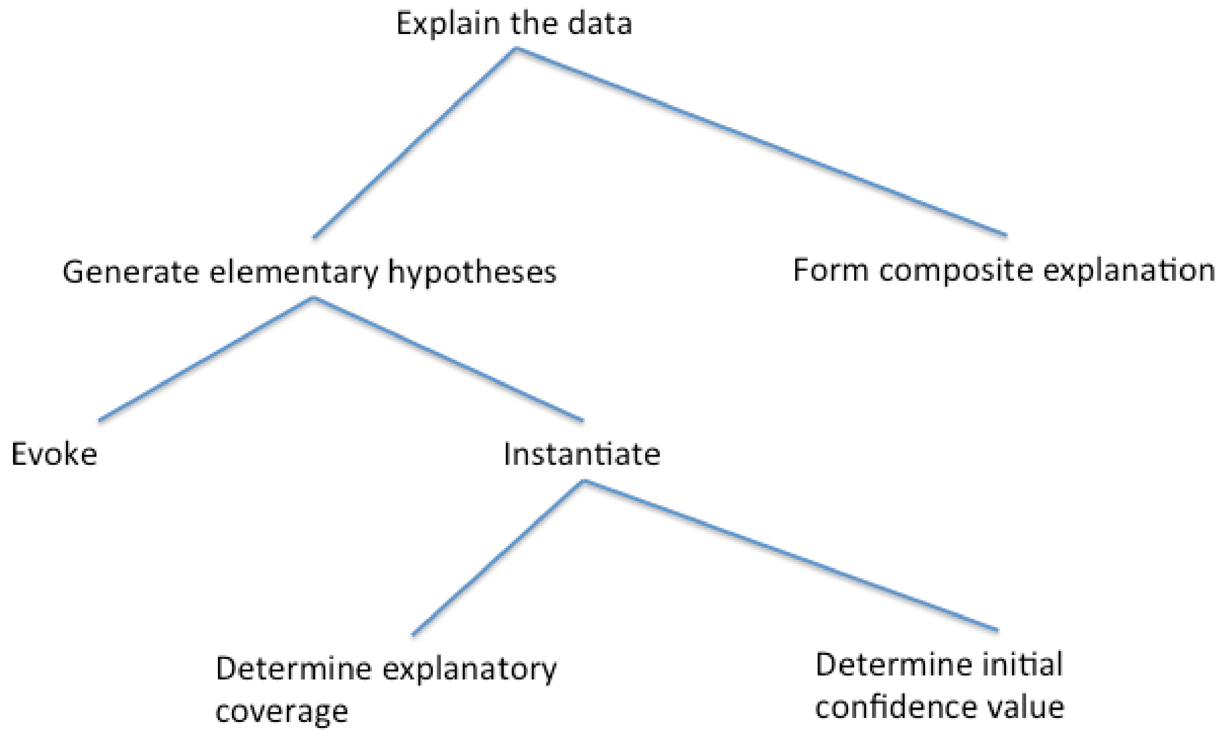
(Memorized Reasoning) (MR) and Algorithmic Reasoning (AR). Quantitative reasoning domain tests students' ability to use numbers and mathematical concepts to solve mathematical problems as well as ability to analyze data presented in variety of ways, such as in table or graph. Mathematical creative thinking refers to a combination of logical and divergent thinking on the basis of situation, but it has a conscious purpose (Lila et al., 2019). Deductive reasoning is a method of reasoning by which premises understood to be true produce logical certain conclusion. It moves from general statement to a specific conclusion. It is a possibility that does not require further justification. Inductive reasoning is a method of reasoning in which the sufficient premises are viewed as supplying a strong evidence for the truth of a given conclusion. It is about arriving at a probability rather than 'certain conclusion' so there is need for other facts and data to test the conclusion. Imitative reasoning is students reasoning that often suitable in routine tasks. The MR strategy choice is founded on recalling; by memory, an answer, and the strategy implementation consist only of writing it down. The AR strategy choice is to recall an algorithm, which is a sequence of rules for solving a particular task type whereas the following three conditions define CMR, i.e., novelty, plausibility and mathematical foundation (Elvis, 2017; Rapanta, 2018). Thus, there are five levels of mathematical reasoning skills.

Figure 1
Level of mathematical reasoning skills



Abductive reasoning is when the conclusion is the hypothesis or a best guess based on the given knowledge and evidence at that moment. Abductive reasoning is the only one that leads to new knowledge because it is about reasoning from effect to cause. The causes are based on tested hypothesis rather than just facts. Josephson and Josephson (1996) provide a task analysis framework of abductive hypothesis formation. According to these authors, learning is the acquisition of knowledge. The generation of elementary hypotheses, which is the main part of the abductive reasoning process, can mainly be achieved through: a) evocation and b) instantiation. It is during instantiation that explanatory coverage and confidence values of the initial hypotheses are determined, re-determined, or refined.

Figure 2
A framework for abductive reasoning (from Josephson & Josephson 1996; p. 140).



What usually happens in a classroom, especially in a science classroom, is that the teacher explains the theory that is already decided by the scientific community as the most valid or acceptable. Instead, what Josephson and Josephson (1996) suggest is that it is the data, not the theories (usually composed of a claim plus data plus a warrant) that should be explained. Education practitioners have generally referred to teaching styles as being either inductive, i.e., presenting phenomena leading to principles, or deductive, i.e., presenting principles leading to phenomena (Felder & Silverman, 1988). However, no explicit use of abductive reasoning has been reported thus far in science learning and teaching literature, with the exception of mathematics.

Concept of Argumentation

In order to undertake any study of argumentation, it is important to first explicate what is meant by the term. Many people describe an argument by this common purpose, the achievement of a win, or the persuading or convincing of others to carry out an action. The most common view of argument, and the widely accepted lay position, is a model of confrontation, whereby each side makes claims, defends them and argues against any opposing claims until a winning position has been established. By default then, there must also be a losing position. However, this confrontational view of argumentation is largely inadequate and one which is limiting. There is no universally endorsed definition for argumentation. According to Nettel and Roque (2012); O'Keefe (2012) argumentation systematically vary in two ways, namely, (a) the communicative ends specified and (b) the communicative means specified. The first explained the place of persuasion in argumentation theory and the extent to which persuasion and argumentation overlap. Rather than aiming to achieve a winning position, argumentation in the scientific sense involves collaborative discussion to explore and resolve an issue in order to construct an explanation which best fits available evidence and logic. In our everyday living, arguments are often considered to occur in a somewhat confrontational manner whereby each side makes claims, defends them and argues against the opposing claims until a winning position has been established. So, by default there must also be a losing position.

However, scientifically, argumentation is quite different, and the difference is identifiable by the aim or goal of the argument geared towards position rather than aiming to achieve a winning position. Argumentation involves collaborative discussion to explore and resolve an issue in order to construct best fit evidence, using that evidence to make a claim, and then articulation of how the evidence leads to the claim through reasoning. In essence, argumentation provides possibilities to intentionally direct students to focus on the mathematics content, and the ways in which such can be used, to respond to a problem or dilemma (Jill 2014).

The mathematical outcome might be a public proposition about some class of mathematical objects or it might simply be the solution to a mathematical problem that has been posed. Argumentation in mathematics education can mean two things:

1. The mathematical arguments that students and teachers produce in mathematics classrooms
2. The arguments that mathematics education researchers produce regarding the nature of mathematics learning and the efficacy of mathematics teaching in various contexts.

Lumer (2010) and Nettel and Roque (2012) proposed epistemic argumentation as those which collectively seek the truth through critical reasoning and justification. The goal is to reach justified consensus where both parties not only share the final opinion but ideally also their subjective justification for it. It is based on cognizing procedures that guarantee the truth or at least the acceptability, i.e. truth, high probability or verisimilitude, of the results (Lumer, 2010). Also stressed that argumentation in the classroom emphasizes cognitive and metacognitive processes, epistemic criteria and reasoning, as well as the enculturation of learners into the practices and discourses of a subject. Epistemological argumentation distinguishes itself by evaluating the strength and validity of an argument through epistemic criteria only. Mathematics needs to be embedded in a context that propels argumentation which will help student to focus on the need for quality evidence of the mathematical content. Few researches focused on argumentation practices of students undertaking inquiry of this nature by contrast. Mathematical inquiry offers the opportunity for students to engage in ill-structured, ambiguous problems that have neither a defined solution path nor a single correct answer since the subject has tendencies to be taught at a predominantly abstract level (Jill, 2014). He however stressed that to implement argumentation practices in mathematics, teaching would require the establishment of an environment which is conducive to the exploration of such alternate pathways and alternate answers. To achieve this, Inquiry-Based Learning (IBL) which is about situating argumentation within a classroom will facilitate the basis of social construct and does not treat knowledge as absolute.

Studies on current pedagogical practices

Rapanta (2018) discusses abductive reasoning as the most adequate for students' arguments to emerge in a classroom discussion. He explains the unique characteristics of abductive reasoning in relation to deductive and inductive reasoning. He conducted empirical research on students' 12th and 13th grade showing that the nature of the explicit argumentation process in the classroom is mainly abductive. His finding shows that a type of reasoning generally called "argumentative reasoning" yields significantly positive results in terms of learning and reasoning quality. Elvis (2017) reported a study on developing teaching materials in inculcating upper secondary students' mathematical reasoning skills (MRS). The researcher implemented a Four-D Model. The study took subjects from five public schools in Province of North Sumatera, Indonesia, designed and developed students' work sheet (SWS) and instrument to measure MRS. He found that students lack pattern relationship to analyze situation, to make analogy, or to generalize.

Wells (2014) researched on how students can have deeper understanding of Inquiry-Based Argument practices and possibilities, how students' developing use of evidence in argumentation could be understood and supported. Qualitative research design was used and data was generated through interview and observations of 27 students single inquiry classroom of Year 4-5 students (n=27, aged 8-10). The analysis identified several significant results of introducing Inquiry-Based Argument into the classroom and suggests that there is potential for argumentation to have a significant role in mathematics education. Akanmu (2019) worked on the performance of Nigerian students in the Senior School Certificate Mathematics Examinations. His study examined the effects of think-pair share on senior school students' performance in mathematics in Ilorin, Nigeria. The researcher employed a quasi-experimental design for the study. The sample consisted of 118 SS II students. The instruments used for the study was Mathematics Performance Test (MPT) with reliability values of 0.78 using Pearson Product Moment Correlation procedure. Independent Sample t-tests and Analysis of Covariance (ANCOVA) was used to analyse data. The study concluded that, the use of think-pair-share improved students' performance in Mathematics, gender of a student does not affect his or her performance in Mathematics, and the use of think-pair-share improved the retention ability of the students.

Ukobizaba et al. (2019) explored insights of teachers and students regarding mathematics teaching and learning in Rwanda and found that peer learning, group work and expository were found to be the most applied teaching methods in the selected schools. The study was a survey designed involving 217 ordinary level secondary school students and 25 secondary school teachers who teach Mathematics, from 5 schools in Karongi District, Western Province, in Rwanda. The results analysis was confined to three components namely; preferred mathematics teaching methods, motivation to teach and learn mathematics, and the usability of mathematics in daily life.

Asuai (2013) worked on impact of critical thinking on performance in mathematics among senior secondary school students in Lagos state. A quasi-experimental design was adopted for the study while multi-stage sampling was used to generate a sample of 195 students. Mathematics performance test and Watson-glaser Critical Thinking Appraisal were used for the study. The study found that critical thinking skills was also an effective means of enhancing students' understanding of Mathematics concepts. Chinyere and Uche (2013) found out how mathematics teachers can help

Nigerian school children overcome mathematics phobia. They pointed out the consequences of poor performance of students in mathematics during West African Senior School Certificate Examination (WASSCE). It then looked at the attributes of a mathematics teacher that can bring about a change in the teaching and learning of mathematics.

The major gaps identified by the researcher are absence of a study that investigates both reasoning and argumentation in teaching mathematics together with reasoning and argumentation among basic pupils in solving mathematical problems. Most of the studies reviewed examined: abductive reasoning (Rapanta, 2018); developing teaching materials in inculcating upper secondary students' mathematical reasoning skills (Elvis, 2017); how students can have deeper understanding of Inquiry-Based Argument practices and possibilities (Wells, 2014); the effects of think-pair share on senior school students' performance in mathematics (Akanmu, 2019); impact of critical thinking on performance in mathematics among senior secondary school students (Asuai, 2013). While this present study seeks to contribute to bridging these literature gap by identifying the role of reasoning and argumentation in pedagogical contexts as well as learning basic mathematics.

Methodology

This study made use of mixed methods of both qualitative (pupils' questions and answers) and quantitative (teachers' questionnaires) approach that reported the results of both qualitative and quantitative analyses that was performed to achieve the objectives of the study. Two hundred and twenty (220) mathematic teachers and four hundred and forty (440) (represent 6.4% of the pupils in the locality) pupils (who were preparing for terminal examination into junior secondary schools, usually within the age range of 8 to 11 years) in Nigerian primary schools successfully participated in the teachers' questionnaire and pupils' questions and answers respectively. These make total respondents of six hundred and sixty (660): two hundred and twenty (220) mathematic teachers and four hundred and forty (440) pupils for this study. The study made use of snowball and purposive sampling technique due to the nature of the respondents (mathematic teachers and pupils in terminal class). These mathematic teachers were able to recommend their colleagues who were involved in teaching mathematics at primary schools. This was done in a ratio one teacher to two pupils (a male and a female). The pupils were asked five different mathematic questions from national common entrance examination series and their responses were under closed observations. The questionnaire comprised two main sections namely; personal information of respondents and the role of reasoning and argumentation in pedagogical teaching and learning of basic mathematics. The reliability of the scale was calculated with the Cronbach's alpha value of 0.79. Respondents used the modified Likert scale of SA- Strongly Agree (4), A- Agree (3), D- Disagree (2), SD- Strongly Disagree (1).

Data Analysis

The gathered data were analysed using descriptive statistics and analysis of observation. Descriptive statistics such as frequency counts, simple percentages were used for teachers' personal information of the respondents. Frequency counts and simple percentages were also used to achieve the first and second research objectives, while Pearson Product-Moment Correlation (PPMC) analysis of observation was used to achieve the last research objective.

Results

Descriptive analysis of personal information of respondents

This section presents the personal information of respondents. Responses were received from two hundred and sixty-five (265) mathematic teachers in Nigerian primary schools as at 31 July, 2020 (data cutoff collection date for this study). Two hundred and twenty (220) respondents had successfully completed the online questionnaires (completion rate: 83%).

Table 1 below shows the demographic details of the respondents. Majority 119 (54.1%) of the respondents were females. The table also shows the age of the respondents. It also revealed that most of the respondents 75 (34.1%) fell within the age range of 26 to 35 years. Lastly, the table revealed the teaching experience of the respondents, with 86 (39.1%) having spent 11 to 15 years in teaching mathematics at primary school in the country.

Table 1

Demographic information of teacher respondents

Demographic		**Frequency (N=220)**	**Percentage (%)**
Sex	Male	101	45.9
	Female	119	54.1
Age (Years)	less than 25	29	13.2
	26 to 35	75	34.1
	36 to 45	66	30.0
	Above 45	50	22.7
Teaching Experience (Years)	fewer than 5	20	9.1
	6 to 10	65	29.5
	11 to 15	86	39.1
	Above 15	49	22.3

Existing Pedagogy of Mathematics in Nigeria Primary Schools

Table 2 below reveals the existing pedagogy of mathematics in Nigeria primary schools. It shows that mere 80.4% of the respondents agreed that the curriculum enabled creativity in teaching mathematics among Nigeria primary school pupils, 59.5% of the respondents agreed that Nigeria primary school pupils found mathematics to be interesting subject, 50% of the respondents agreed that the existing designs of teaching mathematics made teaching difficult, 65% of the respondents agreed that the existing designs of teaching mathematics made learning interesting among Nigeria primary school pupils, while 61.8% of the respondents agreed that the existing designs of teaching mathematics made pupils reason and argue in classroom discussion.

Table 2

Teachers' perceptions of the existing curriculum and its design

S/N	STATEMENTS	SA	A	D	SD
1)	The curriculum enables creativity in teaching mathematics among Nigeria primary school pupils	45 (20.4)	132 (60.0)	36 (16.4)	7 (3.2)
2)	Nigeria primary school pupils find mathematics to be interesting subject	51 (23.2)	80 (36.3)	62 (28.2)	27 (12.3)
3)	The existing designs of teaching mathematics make teaching difficult	40 (18.2)	70 (31.8)	76 (34.5)	34 (15.5)
4)	The existing designs of teaching mathematics make learning interesting among Nigeria primary school pupils	54 (24.5)	89 (40.5)	50 (22.7)	27 (12.3)
5)	The existing designs of teaching mathematics make pupils reason and argue in classroom discussion	50 (22.7)	86 (39.1)	55 (25.0)	29 (13.2)

NOTE: SA- Strongly Agree, A- Agree, D- Disagree, SD- Strongly Disagree. () indicates percentage. Highlighted cells indicate the most prevalent response.

Reasoning and Argumentative Approach in Teaching and Learning Mathematics

Table 3 below reveals the extent of implementing reasoning and argumentative approach in teaching mathematics at basic levels. It shows that mere 91.8% of the respondents agreed that they applied their logic in teaching mathematics at basic levels, 82.2% of the respondents agreed that they used their views that are different from scheme of work in teaching mathematics at basic levels, 63.6% of the respondents agreed that pupils were allowed to use different techniques in solving mathematics at basic levels, 87.7% of the respondents agreed that pupils asked questions outside the subject matter in mathematic classroom at basic levels, 89.1% of the respondents agreed that reasoning and argumentative were allowed in teaching mathematics at basic levels, 97.2% of the respondents agreed that there was no time for individual reasoning of pupils in mathematics class, 98.6% of the respondents agreed that there was no sufficient time for pupils to exhibit their own way of thinking in mathematics class, 29.1% of the respondents agreed that they allowed pupils to form personal opinion about the topic taught after which positive superior opinion was concluded and generalized.

Table 3

Teachers' perceptions of current pedagogical practices

S/N	STATEMENTS	SA	A	D	SD
1)	I apply my logic in teaching mathematics at basic levels	70 (31.8)	132 (60.0)	16 (7.3)	2 (0.9)
2)	I use views that are different from scheme of work in teaching mathematics at basic levels	51 (23.2)	130 (59.0)	27 (12.3)	12 (5.5)
3)	Pupils are allowed to use different techniques in solving mathematics at basic levels	56 (25.4)	84 (38.2)	46 (20.9)	34 (15.5)

4)	Pupils ask questions outside the subject matter in mathematic classroom at basic levels	74 (33.6)	119 (54.1)	20 (9.1)	7 (3.2)
5)	Reasoning and argumentative are allowed in teaching mathematics at basic levels	80 (36.4)	116 (52.7)	20 (9.1)	4 (1.8)
6)	There is no time for individual reasoning of pupils in mathematics class	94 (42.7)	120 (54.5)	4 (1.8)	2 (0.9)
7)	There is no sufficient time for pupils to exhibit their own way of thinking. in mathematics class	77 (35.0)	140 (63.6)	3 (1.4)	- (-.-)
8)	I allow pupils to form personal opinion about the topic taught after which positive superior opinion is concluded and generalized	14 (6.4)	50 (22.7)	96 (43.6)	60 (27.3)

NOTE: SA- Strongly Agree, A- Agree, D- Disagree, SD- Strongly Disagree. () indicates percentage. Highlighted cells indicate the most prevalent response.

Table 4 reveals the degree at which basic pupils in terminal class used reasoning and argumentation in solving mathematics. Five questions were given, right answers were identified by examples and justification, while wrong answers were identified by wrong and restating the question without given answer. The table shows that 33.2% of the pupils got question one wrongly, 52.7% of the pupils got the answer correctly with some element of reasoning while 14.1% of the pupils got the answer correctly with some element of argumentation. Concerning the second question, 46.8% of the pupils got it wrongly, 40.9% of the pupils got the answer correctly with some element of reasoning while 12.3% of the pupils got the answer correctly with some element of argumentation. Regarding the third question, 20.5% of the pupils got it wrongly, 60.4% of the pupils got the answer correctly with some element of reasoning while 19.1% of the pupils got the answer correctly with some element of argumentation. Concerning the fourth question, 61.4% of the pupils got it wrongly, 29.5% of the pupils got the answer correctly with some element of reasoning while 9.1% of the pupils got the answer correctly with some element of argumentation. Lastly on the fifth question, 50% of the pupils got it wrongly, 38.6% of the pupils got the answer correctly with some element of reasoning while 11.4% of the pupils got the answer correctly with some element of argumentation.

Table 4

Pupils' Reasoning and Argumentation in Solving Mathematical problems

S/N	Questions	Wrong	Reasoning Examples	Argument Justify	Restate
1)	Question one	102 (23.2)	232 (52.7)	62 (14.1)	44 (10.0)
2)	Question two	154 (35.0)	180 (40.9)	54 (12.3)	52 (11.8)
3)	Question three	80 (18.2)	266 (60.4)	84 (19.1)	10 (2.3)
4)	Question four	204 (46.4)	130 (29.5)	40 (9.1)	66 (15.0)

5)	Question five	184 (41.8)	170 (38.6)	50 (11.4)	36 (8.2)

Note: Highlighted cells indicate the most prevalent response.

Table 5 indicated Pearson Product-Moment Correlation showing the relationship between greater use of reasoning and argumentative pedagogical approach by teachers and students' academic performance. From table 5, there was a positive strong correlation between greater use of reasoning and argumentative pedagogical approach by teachers and students' academic performance, r = (679), 0.002, p < .05. Based on this analysis, main objective of this study which is to identify the role of reasoning and argumentation in pedagogical contexts as well as learning basic mathematics was therefore achieved. This indicates that greater use of reasoning and argumentative pedagogical approach by teachers in basic mathematics yield high students' academic performance in the subject.

Table 5: Reasoning and Argumentative Pedagogical Approach and Students' Performance

Variable	N	m	SD	r	p
Teachers' Pedagogy in Reasoning and Argumentative	220	3.11	1.049	0.679	0.002
Students' Performance	440	3.62	0.886		

Discussion

This study broadly examined the role of reasoning and argumentation in teaching pedagogy and learning in basic mathematics and specifically focused on the existing pedagogy of mathematics in Nigeria primary schools. The results of the findings indicated that the existing designs of teaching mathematics made pupils reason and argue in classroom discussion. The findings of this study also indicated that the curriculum enabled creativity in teaching mathematics among Nigeria primary school pupils. This indicates that the existing designs of teaching mathematics allow basic teachers to use different approaches in the pedagogical contexts of mathematics.

These findings are in agreement with earlier studies of Asuai (2013) and Chinyere and Uche (2013) who found out that mathematics teachers can help Nigerian school children overcome mathematics phobia. They pointed out the consequences of poor performance of students in mathematics and then looked at the attributes of a mathematics teacher that can bring about a change in the teaching and learning of mathematics. Their studies also found out that critical thinking skills was also an effective means of enhancing students' understanding of Mathematics concepts.

This study also specifically examined the extent of implementing reasoning and argumentative approach in teaching mathematics at basic levels. The findings again indicated that they used their views that are different from the scheme of work in teaching mathematics at basic levels, Pupils were allowed to use different techniques only when solving mathematics but there was not enough time for pupils' reasoning and argumentation during class at basic levels, The findings equally indicated that pupils asked questions outside the subject matter in mathematic classroom at basic levels, while reasoning and argumentative were allowed in teaching mathematics at basic levels. The findings again indicated that there was not sufficient time for pupils to exhibit their own way of thinking in mathematics class.

These findings are in agreement with earlier study like Rapanta (2018) who submitted that abductive reasoning as the most adequate for students' arguments to emerge in a classroom discussion. This study's findings are also in consonant with Lumer (2010); Nettel and Roque (2012) who submitted that epistemic argumentation collectively seek the truth through critical reasoning and justification. Also stressed that argumentation in the classroom emphasizes cognitive and metacognitive processes, epistemic criteria and reasoning, as well as the enculturation of learners into the practices and discourses of a subject.

This study also examined the degree at which basic pupils in terminal class used reasoning and argumentation in solving mathematics. The results from the findings indicated that at average level, pupils in primary schools used reasoning in solving mathematics more than the way they used argumentation in solving mathematics. This agrees with the findings of Akanmu (2019) who worked on the performance of Nigerian students and concluded that the use of think-pair-share improved students' performance in Mathematics, and the use of think-pair-share improved the retention ability of the students.

Lastly, this study majorly identified the role of reasoning and argumentation in pedagogical contexts as well as learning basic mathematics. The results from the findings indicated that there was a positive strong correlation between greater use of reasoning and argumentative pedagogical approach by teachers and students' academic performance, $r = (0.679)$, 0.002, $P< .05$. This indicates that greater use of reasoning and argumentative pedagogical approach by teachers in basic mathematics yield high students' academic performance in the subject. This is in line with the findings of Elvis (2017) who emphasized full understanding of the subject matter which includes the capacity to engage in the process of mathematical thinking to solve problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on. This is also similar to the work of Ukobizaba, Ndihokubwayo, Mukuka and Uwamahoro (2019) who explored insights of teachers and students regarding mathematics teaching and learning and found that peer learning, group work and expository were found to be the most applied teaching methods in the selected schools.

Conclusion

Considering the objectives of this study, various relevant literatures that were reviewed, methodology and the findings, this study arrived at the following conclusions;

The existing designs of teaching mathematics made pupils reason and argue in classroom discussion. The curriculum and scheme of work enabled creativity in teaching mathematics among Nigeria primary school pupils. Teaching mathematics allow basic teachers to use different approaches in the pedagogical contexts of mathematics.

Mathematic teachers at the basic level of education applied their logics in teaching mathematics but there was no sufficient time for pupils to exhibit their own way of thinking in mathematics class. They used their views that are different from scheme of work in teaching mathematics at basic levels, Pupils at the basic level of education were allowed to use different techniques when solving mathematics but there was no enough time for pupils' reasoning and argumentation during class, Pupils asked questions outside the subject matter in mathematic classroom at basic levels, while reasoning and argumentative were allowed in teaching mathematics at basic levels. On average, pupils in primary schools used reasoning in solving mathematics, while very few of them used argumentation in solving mathematics.

Recommendations

Based on the objectives of this study, various relevant literatures that were reviewed, methodology and the findings, the following recommendations were made; Government, school administrators, teachers, parents and students should all intensify their effort on the existing designs of teaching mathematics to be accommodate rigorous abductive reasoning and scientific argumentation. Mathematic teachers at the basic level of education should make sure that as they are using reasoning and argumentation in the pedagogical contexts of mathematics, they should also create time to allow pupils' reasoning and argumentation during learning. All the stakeholders in basic education are to provide professional development to teachers who have not been using reasoning and argumentative pedagogical approach in basic mathematics to start doing so, this will enable all students to come up to the same level. Lastly, mathematic teachers at the basic level of education should also encourage their pupils to be more rigorous in reasoning and argumentation by allowing them to express their feelings without fear.

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