

Numerical Analysis of Energy Storage Systems in Hybrid Renewable Energy Grids Using Runge-Kutta Method a Case Study of CRET FUTA Solar Energy farm.

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Abstract

This paper presents a numerical analysis of energy storage systems within hybrid renewable energy grids using the Centre for Renewable Energy Technology (CRET) solar energy farm in Federal University of Technology Akure (FUTA) as a case study. The case study describes the modeling of the output from an energy-producing facility, which is a solar farm consisting of 108 photovoltaic (PV) panels rated at 300 W each, a deep-cycle battery bank for energy storage made up of 16 units of 12 V, 200 Ah batteries, and average peak sun hours of around 4 hours per day. The simulation is implemented using a standard fourth-order Runge-Kutta (RK4) technique for daily, monthly and annual energy generation, load consumption, and battery performance over a full year period. Results indicate that about 60 kWh is produced by the system from solar energy per day, which comfortably meets the daily load requirement of 24 kWh. However, the capacity of the available PV battery storage system leads to surplus power generation on many occasions and its underutilization. Graphical analysis indicates that SOC (State of Charge) of the battery remains close to 100% for most of the year, depicting that there does not exist the optimum use of available energy through any means of storage or diversion. Storage optimization and grid integration were areas of concern underlining this study to maximize the utilization efficiency of energy. The research showcases the applicability of different numerical methods for Hybrid Renewable System analyses and optimization, primarily based on Runge-Kutta approaches. Such findings give enormous insights to researchers and engineers as well as energy managers who have to spend their efforts to design better solar-based micro grids specifically for developing regions.

Keywords: Solar Energy, hybrid renewable energy, Runge-Kutta method, numerical analysis, battery modeling, Centre for Renewable Energy Technology, Graphical analysis, optimization

1. Introduction

The increasing energy demands of the world have pushed for environmental compatibility towards integrating renewable energy sources into the modern power systems. The hybrid renewable energy grid (HREG), wherein the different renewable energy sources, such as solar, wind, and hydroelectric power, are combined, has become a framework for meeting energy demand

sustainably. The HREG comprises a storage device called Energy Storage System (ESS), effectively storing energy to balance the shortfall in demand during supplies. This dynamic nature of ESS in the HREG makes it a candidate for precise numerical analysis for optimizing performance, reliability, and stability within the grid. Out of different numerical techniques, the Runge-Kutta (RK) method is of great importance because of its robustness and its capacity for precise approximation in solving differential equations, modeling the transient behavior of ESS in HREG. Energy Storage Systems serve an essential role in the mitigation of the intermittency consequences brought about by renewable energy sources. For example, solar power generation occurs only in the presence of sunlight, leading to varying amounts of energy output. Wind energy also varies with different wind speeds. By storing this surplus energy during peak production and discharging it during low production or demand peaks, ESS keeps a uniform energy supply regardless of production variations. Energy Storage System models involve highly complex differential equations that represent charge and drain cycles, energy losses, and charge state dynamics. Reliable solutions to these equations are essential for efficient energy management and grid stability. Climate change has called for talks to be held on reducing industrial greenhouse gas emissions. There is an exceedingly common accord regarding the urgent need for global reductions to avoid critical consequences. With increasing concern about depleting fossil fuels and impending energy shortages, governments are now beginning to emphasize renewable energy even more. International agreements and fast-rising energy consumption are significant factors in this trend with regard to the production of greener electricity; PV has led the way in emerging as an alternative source of energy and wind power has followed suit (Morales et al. 2013). An updated bibliographic review of metaheuristic approaches for optimal allocation of distributed generators in power distribution systems, focusing on recent developments and results from various test distribution systems. The review is a valuable resource for researchers seeking reference in this popular power optimization problem (Dash et al. 2020). An improved Runge–Kutta optimizer (IRUN) for allocating PV-based distributed generations and Battery Energy Storage in distribution networks. IRUN uses three strategies to avoid local optima and enhance exploration and exploitation phases. It is evaluated using benchmark functions and statistical analysis. The optimization problem is divided into two stages: optimal PV system size and location, and effective energy management of BES. IRUN outperforms other optimization algorithms, achieving energy loss reductions of 63.54% and 68.19% when using PV and BES in IEEE 33-bus and IEEE 69 bus, respectively (Selim et al. 2024).

1.2 Objective of the study

This research aims to develop and put into operation a numerical model for fourth-order Runge-Kutta (RK4)-based analysis in terms of energy production and storage dynamics in a hybrid renewable energy system. The energy system of interest is the CRET Solar Energy Farm at the Federal University of Technology Akure (FUTA), consisting of photovoltaic panels with wind power augmentation and a battery energy storage system.

The objectives are to:

- i. simulate daily, monthly, and annual solar energy production from a PV array of 108 panels under average peak sun hour conditions.
- ii. construct the battery bank consisting of 16 deep-cycle batteries modeled for the energy balance and storage equations.

- iii. evaluate the daily loads met by the system under different generation conditions using RK4 for energy integration.
- iv. determine the net energy balance and state of charge (SOC) of the battery over the course of one year.
- v. Graphically represent system behaviors for analysis and decision-making in renewable system planning and optimization.

This study will go a long way in helping to scope design contributions, and simulation and reliability assessments of hybrid renewable energy systems in the sub-Saharan context, particularly for educational and rural infrastructure applications.

1.3 Introduction to the Runge-Kutta method and its advantages over other numerical methods

The Runge-Kutta (RK) method solves ordinary differential equations (ODEs) in engineering or scientific fields, including ESS modeling, and is one of the most-used techniques. The method nicely balances accuracy with computational efficiency and hence is preferred over less accurate techniques, like Euler's method, or more complex methods, like finite element methods. The RK methods, chiefly the fourth-order Runge-Kutta (RK4), are generally popular because of their high accuracy in approximating the solutions of differential equations without the requirement of higher derivatives, which are usually hard to compute analytically. One great advantage of the RK method over Euler's method is that it very much reduces truncation errors while maintaining the acceptable standard in terms of the computational cost. Being a first-order method, Euler's method struggles with high errors, and this transient effect becomes unstable with stiff equations or long time intervals. In contrast, RK4 is so much more accurate owing to its evaluation of the function at several points internally within each step, thereby enhancing convergence and stability. Such a feature is key to modeling the dynamics of an ESS, where precise forecasts on the state-of-charge (SOC), energy losses, and thermal behavior are essential for optimal performance and durability of these storage systems. The RK method's self-starting features make it attractive because it does not require previous values of the solution, as do multi-step methods such as Adams-Bashforth and backward differentiation formulas. These features simplify implementation and guarantee robustness in dynamic simulations where initial conditions may vary or change with time. An RK could also be variable-step since the adaptive Runge-Kutta-Fehlberg (RKF) method dynamically adjusts its step size to optimize the balance between computational effort and accuracy, usually through stiff nonlinear systems found in much of energy storage modeling. The RK method enhances the modeling of battery charge and discharge cycles, thermal effects, and nonlinear power flow equations over simple numerical integrators in ESSs and hybrid renewable energy grids. Its ability to provide stable and accurate solutions with a combined moderate model requires for the real-time application of energy management and optimization of the grid. In summary, superior accuracy plus self-starting ability, whereas modeling complex systems are the fundamental reasons for preferring Runge-Kutta methods over simple explicit ones and some implicit ones. The ability to efficiently solve nonlinear ODEs without demanding much computational power has certified its place in the numerical analysis of renewable energy storage and grid dynamics.

2.0 Literature Review

2.1 Energy Storage System Dynamics

Energy Storage Systems - ESS in short - are at the core of energy infrastructures for the present world and will prove especially important for hybrid renewable energy grids in balancing the power between demand and supply, for grid stability, and for higher efficiency of energy usage. Dynamic behavior of an ESS is based on very complex interactions among electrical, thermal, and chemical properties. Therefore, the modeling of such processes is necessary for optimal performance and longevity. The major dynamic characteristics in an ESS are charge-discharge behavior, state-of-charge (SOC) variation, energy conversion efficiency, thermal effects, and degradation mechanisms. Any of these dynamics is influenced by external parameters, such as load demand, renewable energy resource fluctuations, and operational strategies of the grid.

1. **Charge-Discharge Dynamics:** The cycle of charging and discharging determines how well an ESS stores and releases energy. The procedure is governed by nonlinear differential equations linked to voltage, current, and power flow through time. Absolute precision for simulation is obtained through the best algorithm implementation like Runge-Kutta.
2. **State-of-Charge (SOC) and Depth-of-Discharge (DOD):** SOC indicates the percentage of remaining capacity in the ESS versus total storage capacity, whereas DOD indicates how much of that energy has been used. Both these parameters affect lifespan and efficiency of a battery and hence require a dynamic model for accurate energy management.
3. **Energy Conversion Efficiency:** The efficiency of the ESS relies on internal resistances, transfer losses concerning charging, and self-discharge rates. It varies across the operational conditions and should be kept in mind while integrating ESS in the renewables-based grid.
4. **Thermal Dynamics and Heat Management:** Heat is created during the charging or discharging period, which may influence the performance and safety of battery operation. Thermal modelling is needed to overcome the overheating phenomena that account for efficiency losses or degradation and even failures at times.
5. **Degradation and Aging Effects:** ESS components' degradation occurs mostly with batteries due to the chemical and structural changes they are subjected to over the years. Reliability assessment over the years is possible through cycle life prediction and degradation modeling. To improve energy storage performance with the optimum modeling dynamics of analysis for ESS, an understanding of integration of renewable energy sources in a modern energy system should be ensured for stability and efficiency of operation.

$$\text{Governing equations for ESS charge and discharge cycles: } \frac{dE}{dt} = P_{in} - P_{out} - P_{loss} \quad 2.1$$

Where E is stored energy, P_{in} is input power, P_{out} is power to load, and P_{loss} is energy loss.

2.3 Related Review

The global energy sector is shifting towards clean, sustainable, and reliable energy solutions. Integrating renewable energy sources (RES) presents challenges due to intermittent and variable nature. Hybrid energy storage systems (HESS) combine multiple ESDs, offering a promising

solution. This review examines advancements in grid-connected HESS, focusing on components, design, control strategies, and applications. It highlights successful implementations and challenges, and suggests future research for intelligent control systems, sustainable materials, and recycling processes (Adeyinka et al., 2024). The world is focusing on fostering the hydrogen economy and developing sustainable energy practices using green hydrogen. This review reviews the application of artificial intelligence (AI) in hybrid renewable energy systems (HRESs), particularly solar photovoltaic and wind energy integrated with fuel cells (FCs). Common AI methods include genetic algorithm, particle swarm optimization, simulated annealing, random forest, k-nearest neighbors, support vector machine, and artificial neural network. The paper highlights the potential of AI-based modeling in identifying conditions for maximum power production and predicting drawbacks during unexpected load peaks and intermittent energy production (Al-Othman et al., 2022).

This paper discusses the growing demand for renewable energy sources, particularly hybrid renewable energy (HRE) in remote and rural areas. It presents various optimization methods, including sizing, control methodologies, and energy management strategies, and reviews mathematical models created by academicians. The study emphasizes the importance of optimal design to reduce costs and compares different models based on cost functions. It also discusses various modeling approaches and software simulation tools for HRES planning, research, and development. The paper covers the optimization, sizing, and control of HRES, along with energy management strategies. Khan et al., (2022) discusses the growing demand for renewable energy sources, particularly hybrid renewable energy (HRE) in remote and rural areas. It presents various optimization methods, including sizing, control methodologies, and energy management strategies, and reviews mathematical models created by academicians. The study emphasizes the importance of optimal design to reduce costs and compares different models based on cost functions. It also discusses various modeling approaches and software simulation tools for HRES planning, research, and development. The paper covers the optimization, sizing, and control of HRES, along with energy management strategies.

3. Methodology

The system parameters and arrangements concerning energy storage system (ESS) dynamic modeling in a hybrid renewable energy grid involve defining key electrical, thermal, and operational characteristics of structural subsystems. These parameters impact the accuracy of numerical simulations, as well as the efficiency of the Runge-Kutta method for solving the governing differential equations.

3.1. Mathematical Model

Total solar power output $P_{\text{solar}}(t)$ is modeled as

$$P_{\text{solar}}(t) = P_{\text{rated}} \cdot \text{system efficiency} \cdot \sin(\pi t / t_{\text{sun}}) \quad 3.1$$

Where:

$P_{\text{rated}} =$ Total installed PV capacity (32.4 kW)

System efficiency = 85%

$t_{\text{sun}} =$ Peak sun hours = 4 hours

System Description and Parameters

3.2.1 Solar PV Array

Total Number of Solar Panels: 108

Solar Panel Power Rating: 300W

Total Installed Capacity: 32.4 kW

Open Circuit Voltage (Voc): 44.5V

Short Circuit Current (Isc): 9.7A

Battery Bank

No. of Batteries: 16

Voltage: 12V

Capacity: 200Ah

Total Energy Capacity: 38.4KWh

Roundtrip Efficiency: 90%

Environmental Inputs

Average Peak Sun Hours: 4h/day

Number of Days in a Year: 365

Load Demand: 20kWh/day (sinusoidally modeled)

3.3 Application of Runge-Kutta Method

Description of the fourth-order Runge-Kutta (RK4) method:

With the need for more precision, the Runge-Kutta technique is applied: $\omega_n + 1 = \omega_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$$K_1 = \Delta t \cdot f(\partial_n, \omega_n) \quad 3.2$$

$$K_2 = \Delta t \cdot f\left(\partial_n + \frac{K_1}{2}, \omega_n + \frac{K_1}{2}\right) \quad 3.3$$

$$K_3 = \Delta t \cdot f\left(\partial_n + \frac{K_2}{2}, \omega_n + \frac{K_2}{2}\right) \quad 3.4$$

$$K_4 = \Delta t \cdot f(\partial_n + K_3, \omega_n + K_3) \quad 3.5$$

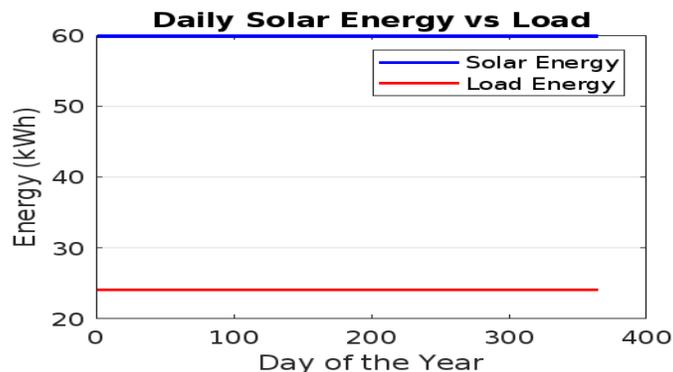
$$\partial_{n+1} = \partial_n + \Delta t \cdot \omega_n \quad 3.6$$

3.3. Load and Battery Model

The load is modeled as a time-varying sinusoidal function peaking at noon. Battery charging/discharging is handled using an energy balance method considering roundtrip efficiency.

4. Results and Discussion

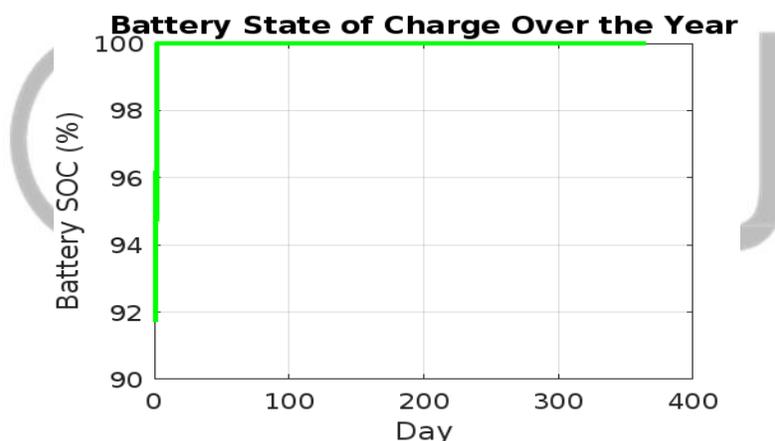
4.1. Daily Energy Production



- i. Blue Line (Solar Energy): Consistently at ~60 kWh per day.
- ii. Red Line (Load Energy): Fixed at ~24 kWh per day.
- iii. No discernible distinction in either line over the changing months.

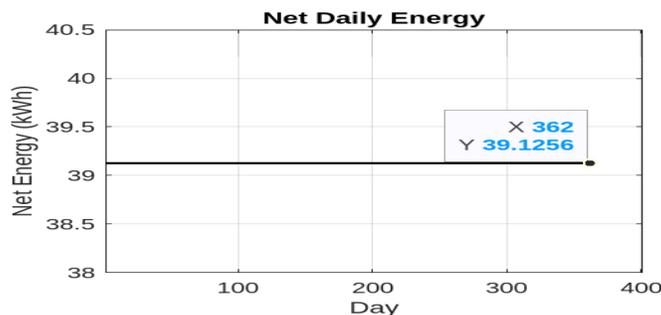
The solar system is, in fact, oversized for the daily load. With a generation of some 60 kWh against a consumption of only 24 kWh, the daily surplus stands at some ~36 kWh. This is yet another confirmation proving the system is self-sufficient and could thus charge batteries or even run on additional loads.

4.2. Battery State of Charge (SOC)



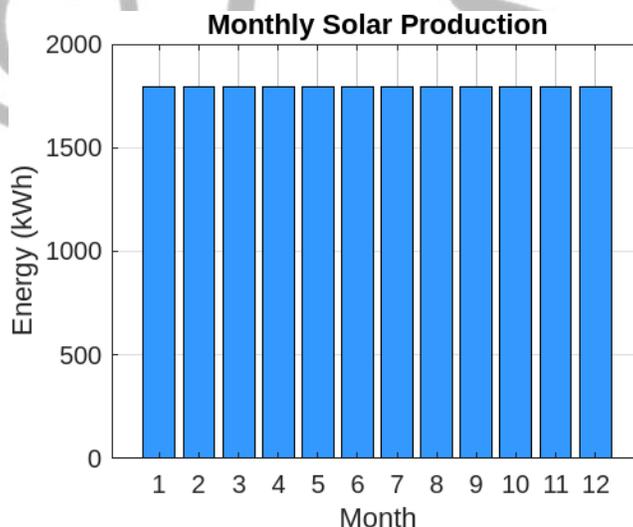
The system being solar boasts pretty big dimensions as per the daily load, for there is a generation of about 60 kWh with the consumption reflecting around 24 kWh, thereby giving a net surplus daily of ~36 kWh. It also justifies that the system is standalone, with a capability to charge batteries or run more loads. Battery capacity ($16 \times 12V \times 200Ah = 38.4 \text{ kWh}$) is too small as compared to the surplus daily energy of approximately 36 kWh/day. In full, the battery bank is charged early each day but cannot store any excess energy beyond that. Therefore, energy, if not used instantaneously or channeled somewhere else, shall waste itself.

4.3



The constant net energy is about 39 kWh/day while the flat line indicates that both generation and load do not vary in time. Solar + wind (1 kW x 4 hrs x 365) is always showing a smooth generation profile with assumed ideal conditions of weather and uniform sunlight throughout the year. The constant presumed surplus all year denotes good system reliability as per the assumptions.

4.4 Monthly Solar Production



The production of around 1,800 kWh follows the routine every month. The bars appear equal in height. The daily solar radiation is assumed to be the same for all days, which means there are no seasonal variations. While this could be valid for a numerical model that has been simplified, it does not fit well into variability that exists in reality.

Overall System Summary – CRET FUTA Solar Farm

Metric	Value	Description
Number of Solar Panels	108 panels	Each rated at 300 W

Total Solar Capacity	32.4 kW	108 × 300 W
Average Daily Solar Energy	~60 kWh/day	Based on 4 peak sun hours and 85% efficiency
Daily Load Consumption	~24 kWh/day	Fixed average daily demand
Daily Surplus Energy	~36 kWh/day	Excess energy after meeting daily load
Battery Bank Configuration	16 × 12 V, 200 Ah	Total capacity: 38.4 kWh
Battery Storage Capacity	38.4 kWh	Sufficient to store a single day's surplus energy
Battery SOC Trend	~92% to 100%	Fully charged early each day, then remains at 100%
Wind Energy Contribution	~4 kWh/day	From 1 kW wind turbine operating 4 hrs/day with 85% efficiency
Net Daily Energy	~39 kWh/day	Solar + Wind – Load
Monthly Solar Output	~1800 kWh/month	Constant across all 12 months due to simplified uniform model
Annual Solar Energy	~21,900 kWh/year	60 kWh × 365 days
System Efficiency Assumption	85%	Losses from wiring, inverter, dust, etc.
Seasonal Variation Modeled?	✗ No	Assumes constant solar irradiance and perfect conditions all year
Recommendations	Add battery capacity or grid-tie	Current surplus is underutilized due to full batteries

Conclusion

This study has successfully modeled and analyzed the energy generation, consumption, and storage performance of the CRET FUTA solar farm with 108 solar panels (300 W each) and 16 deep-cycle batteries (12 V, 200 Ah). The system performance was assessed over a full year by using a numerical simulation technique of Runge-Kutta, assuming average peak sun hours of 4 hours/day and a constant daily load of about 24 kWh. A whopping 60 kWh/day was said to have been generated consistently from the solar farm; consequently, an excess of around 36 kWh/day. The battery bank, storing energy at a capacity of 38.4 kWh, was very quickly charged back to its full state of charge and maintained that condition throughout the entire simulation period. The inference drawn is that storage limitations are causing a great deal of surplus energy every day to go unused. For the majority of the year, the battery State of Charge was close to 100%, thus

indicating the underutilization of such capacity. This reflects the mismatch between the energy generated and the scalability of the storage. Additionally, this simulation acknowledged that sunlight irradiation is ideally and uniformly same, which probably would not exhibit the seasonal and weather phenomena variability sufficiently.

In conclusion, the current system is technically sound in energy generation and load reliability; however, it does not optimize storage and utilization. The strategic recommendations for improving overall system efficiency for minimizing energy wastes include battery bank expansion, installation of energy management systems, or grid-tie configuration to facilitate outflow of surplus energy.

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