



SIMULATION OF WAITING LINE SYSTEM (A CASE STUDY OF FEDERAL POLYTECHNIC IDAH MEDICAL CENTER)

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KeyWords

Queueing, Simulation, Institutional Hospitals, Excess Capacity, Waiting Patients, Poison Arrivals, Double Channel Queues.

ABSTRACT

Organizations that provide services generally come under the intense pressure of balancing between reducing the time customers spend in their facilities and the cost of providing service. A great tendency would have been to save as much cost for the organization by employing fewer servers and thereby incurring higher customer cost resulting from longer waiting times. But with competitors providing similar services, the customers' times become important to service providing organizations. This research carried out a simulation analysis of the queuing situation at the Federal Polytechnic Idah Medical Center. Data on time between successive arrivals of patient at the consulting offices and service times during consultation were collected for a period of six days and used to develop a simulation model which was used to analyze the operation of the system. The data time between arrivals and service times were subjected first to a test of randomness to ensure that they were generated from a random process. The simulation model was replicated 30 times each time generating a Doctor Utilisation Factor (DUF). The study then constructed a 95% confidence interval for the mean DUF as $0.459 \leq \rho = 0.49 \leq 0.519$. The other system characteristics were found to be as follows: That the facility is without patients 34.2% of the peak period of 8 am to 12 noon, that a patient spends on the average 3.79 minutes before seeing a consultant. For most part of the time there are less than 2 patients in the facility. The study therefore recommends that the facility extends its coverage to include the neighboring communities and look into the possibility of serving as a referral centre to the neighboring clinics providing specialized service to patients referred from these other clinics.

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INTRODUCTION

In recent years, healthcare systems have been facing a massive increase in demand, which has led to a continuous need to improve and optimize operational processes and quality control methods. Along with this trend, managers of these systems have begun to encounter logistic and operational problems that previously did not present a most noticeable concern. Many of these problems relate to the deployment of resources and patient flow: How should we schedule nurse and Doctor shifts to optimize their schedules and maximize the quality of care that patients receive? How can we optimize the workflow of staff members faced with large numbers of incoming patients? How should we direct the patient flow in emergency and routine scenarios?

The above questions can be adequately answered by Queueing theory and models. Queueing theory and models have the capacity to address questions such as these in a systematic way, thereby providing managers with efficient solutions and tools for effective healthcare delivery.

Queueing theory basically addresses the question of finding just how much capacity to deploy such that there is a balance between the cost of providing service and the cost of waiting lines. However, there has been little attention given to the application of queueing theory or models to situations of excess capacity relative to customers waiting time, so as soon as there are no long queues, the system is efficient. This is worst when it comes to healthcare systems. But ironically, the cost of deploying excess capacity in healthcare provision can be enormous.

In this study, we present a queueing analysis where the likelihood of excess capacity was very high.

The federal polytechnic Idah medical center was established in 1977, like any other institutional health center, to provide health care services to the polytechnic community which is dominantly a student population; it is headed by the director medical services and has 47 staff (professional and non-professionals). There are two consultants and five units made up of: the pharmacy unit, laboratory unit, nursing unit, medical record unit and consulting unit. Patients' arrivals are typically between the hours of 8:00 am – 12:00 noon. This study was undertaken on the operations of the consulting unit. How patients arrive the unit from the medical records unit (having picked their folders), and queue up to see the next available consultant. The unit maintains a first come first served queuing discipline.

A typical walk through for the patients include: picking their folders from the medical record unit, working in to the waiting room where patients wait to see the consultants. Thereafter, the patient then either goes to the laboratory unit for test or to the pharmacy for drugs. Others cases may be admission or observation cases.

There exists an extensive world of literature on queueing theory and its application in the area of health care. Elalouf and Wachtel (2021) made an extensive review of literature on queueing studies. In fact, they reviewed about 229 peer-reviewed works, papers, and books from the last seven decades dealing with queueing theory or related healthcare and operations research topics as can be seen from table 1.

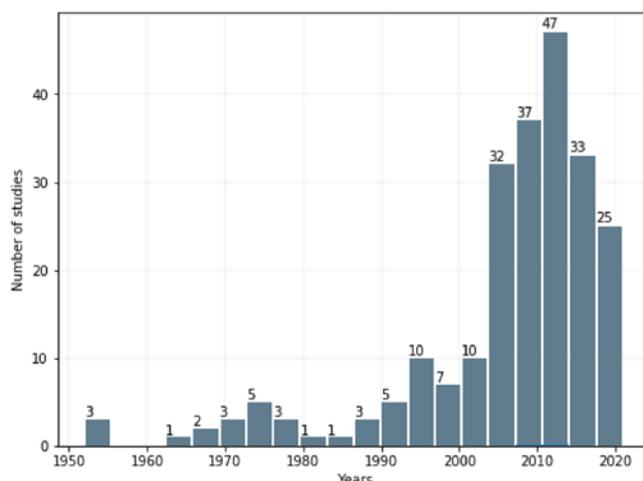


Figure 1: Frequency of published fast-track studies in the ED environment (Elalouf & Wachtel, 2021)

Lakshmi and Sivakumar (2013) is another comprehensive review of the contributions and applications of queueing theory in the field of health care management problems. This review provides sufficient information to analysts who are interested in using queueing theory to model a health care process and who want to locate the details of relevant models.

In a study to determine the optimal number of doctors at the zonal hospital Idah (Abu & Okolo, 2020), it was discovered that three Doctors were adequate compared to the ten Doctors currently being used. This is a clear case of excess capacity.

Bahadori et al (2014) applied simulation of queueing systems to a hospital in Tehran to determine the most efficient capacity for the military hospital pharmacy.

Umar et al (2011) carried out a cross-sectional queueing study in the outpatient clinic of the department of community health, Usman Danfodiyo University Teaching Hospital, Sokoto. It is a cross-sectional study that was carried out at the outpatient clinic of the university teaching hospital. A sample of one in four amounting to 384 patients attending the outpatient units of the hospital were randomly recruited into the study over a five-day period. This study aimed at reducing waiting time and thus ensuring an effective healthcare delivery system.

Oladejo and Aligwo (2014) also studied queueing in Shehu Muhammad Kangiwa medical center, Kaduna polytechnic. This study also determined appropriate capacity for the hospital.

METHODOLOGY

METHOD OF DATA COLLECTION

The data for this study was collected from The Federal Polytechnic Idah medical center for a period of six days. Records of time-between arrivals and service times were taken over this period. Direct personal investigation method was used where the arrival

time to the consulting section of the $(n+1)^{th}$ patient from the n^{th} patient was registered and the time from when the patient goes in to see a consultant until the time that patient leaves the consulting room (having been examined by the consultant). This was taken for the two consultants simultaneously. The data was collected over a period of six days, from Monday to Saturday, taking four (4) hours daily (8:00am to 12:00 noon). The data was recorded on the following table.

Table 1: data collection table proformer

Patient no.			1st consultant		2nd consultant		Waiting time
	Clock time at arrival	Clock time at previous arrival	Clock time at beginning of service	Clock time at end of service	Clock time at beginning of service	Clock time at end of service	
1							
2							
3							
-							
-							
-							

METHOD OF ANALYSIS

The hospital consultancy unit is made up of a single queue with two consultants and was analysed using the double channel (M/M/2) queuein model. We assumed that the inter arrival and service times are independent and identically distributed. All arriving customers enter the queuing system and remain there until service has been completed, the queuing discipline is first – come, first – served. A test of randomness was carried out on the time between arrivals and the service time to confirm if the sequences are a result of a random process.

To determine if the time between arrivals and service times generated from the hospital are actually from a random process, we employ the following procedure:

Hypothesis

1. H_0 : The occurrence of pattern for the Time Between Arrivals is generated by a random process
2. H_0 : The occurrence of pattern for the Service Times is generated by a random process

Table 2: Time Between Arrivals and Service Times and their codes

S/N	Service Times	Code	Time Btw Arrivals	Code
1	3	0	5	0
2	7	1	4	0
3	3	0	4	0
4	6	1	0	0
5	6	1	7	1
6	7	1	5	0
7	5	0	6	0
8	7	1	8	1
9	8	1	0	0
10	7	1	3	0
11	4	0	7	1
12	3	0	5	0
13	3	0	6	0
14	7	1	15	1
15	5	0	6	0
16	11	1	8	1
17	5	0	15	1
18	6	1	0	0
19	4	0	14	1
20	9	1	7	1
21	6	1	11	1
22	11	1	15	1
23	9	1	3	0
24	3	0	11	1
25	7	1	4	0
26	4	0	9	1
Mean	6		6.846	
N1	11		14	
N2	15		12	
R	17		16	

Given that:

r is the number of runs

μ_r is the expected number of runs; and

σ_r is the standard deviation of the number of runs.

Then:

$$Z_{cal} = \frac{r - \mu_r}{\sigma_r}$$

The values of μ_r and σ_r are computed as follows:

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1$$

$$\sigma_r = \sqrt{\frac{(2n_1n_2)(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}$$

For the Service Times,

$$n_1 = 14$$

$$n_2 = 12$$

$$r = 16$$

$$\mu_r = \frac{2(14)(12)}{14 + 12} + 1 = 13.923$$

$$\sigma_r = \sqrt{\frac{(336)(336 - 14 - 12)}{(14 + 12)^2(14 + 12 - 1)}} = 2.4826$$

$$Z_{cal} = \frac{r - \mu_r}{\sigma_r}$$

$$Z_{cal} = \frac{15 - 13.923}{2.4826} = 0.434$$

$$Z_{tab} = Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

Since $Z_{cal} (= 0.434) < Z_{tab} (= 1.96)$, we accept H_0 and conclude that the Service Times come from a random process.

For the Time Between Arrivals,

$$n_1 = 11$$

$$n_2 = 15$$

$$r = 17$$

$$\mu_r = \frac{2(11)(15)}{11 + 15} + 1 = 13.692$$

$$\sigma_r = \sqrt{\frac{(330)(330 - 11 - 15)}{(11 + 15)^2(11 + 15 - 1)}} = 2.436$$

$$Z_{cal} = \frac{r - \mu_r}{\sigma_r}$$

$$Z_{cal} = \frac{17 - 13.692}{2.436} = 1.358$$

$$Z_{tab} = Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

Since $Z_{cal} (= 1.358) < Z_{tab} (= 1.96)$, we accept H_0 and conclude that the Time Between Arrivals come from a random process.

The simulation model is then developed as follows:

Table 3: Probability Distributions of Time Between Arrivals and Service Times and Their Mapping to Random Numbers

Time Between Arrivals	Service Times
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ITA	Prob.	C/Prob.	Mapping	ST	Prob.	C/Prob.	Mapping
0	0.12	0.12	0.00 - 0.12	1	0.04	0.04	0.00-0.04
3	0.08	0.20	0.13 - 0.20	2	0.04	0.08	0.05-0.08
4	0.12	0.32	0.21 - 0.32	3	0.12	0.19	0.09-0.19
5	0.12	0.44	0.33 - 0.44	4	0.15	0.35	0.20-0.35
6	0.12	0.56	0.45 - 0.56	5	0.08	0.42	0.36-0.42
7	0.12	0.68	0.57 - 0.68	6	0.12	0.54	0.43-0.54
8	0.08	0.76	0.69 - 0.76	7	0.15	0.69	0.55-0.69
9	0.04	0.80	0.77 - 0.80	8	0.08	0.77	0.70-0.77
11	0.08	0.88	0.81 - 0.88	9	0.12	0.88	0.78-0.88
14	0.04	0.92	0.89 - 0.92	10	0.04	0.92	0.89-0.92
15	0.12	1.00	0.93 - 0.99	11	0.08	1.00	0.93-0.99

We then use the above mapping to generate random arrivals of patients and random service times for the simulation process. The main simulation table is shown in table 4.

Table 4: The Main Simulation Table

Opening:		7:00AM				Consultant 1		Consultant 2		
Patient S/N	RND	TBA	CTA	RND	ST	CTBS	CTES	CTBS	CTES	WT
1	0.62	7	7	0.49	6	7	13			0
2	0.20	3	10	0.06	2	0	0	10	12	0
3	0.44	5	15	0.64	7	15	22	FALSE	0	0
4	0.58	7	22	0.12	3	22	25	FALSE	0	0
5	0.16	3	25	0.47	6	25	31	FALSE	0	0
6	0.06	0	25	0.01	1	0	0	25	26	0
7	0.38	5	30	0.41	5	30	35	FALSE	0	0
8	0.11	0	30	0.89	9	0	0	30	39	0
9	0.49	6	36	0.69	7	36	43	FALSE	0	0
10	0.99	15	51	0.24	4	51	55	FALSE	0	0
11	0.47	6	57	0.53	6	57	63	FALSE	0	0
12	0.50	6	63	0.74	8	63	71	FALSE	0	0
13	0.76	8	71	0.65	7	71	78	FALSE	0	0
14	0.99	15	86	0.36	4	86	90	FALSE	0	0
15	0.28	4	90	0.80	9	90	99	FALSE	0	0
16	0.43	5	95	0.90	10	0	0	95	105	0
17	0.75	8	103	0.84	9	103	112	FALSE	0	0
18	0.36	5	108	0.61	7	0	0	108	115	0
19	0.28	4	112	0.70	8	112	120	FALSE	0	0
20	0.86	11	123	0.09	3	123	126	FALSE	0	0
21	0.14	3	126	0.43	5	126	131	FALSE	0	0
22	0.18	3	129	0.19	3	0	0	129	132	0
23	0.07	0	129	0.35	4	129	133	FALSE	0	0
24	0.56	6	135	0.01	1	135	136	FALSE	0	0
25	0.34	5	140	0.47	6	140	146	FALSE	0	0
26	0.25	4	144	0.95	11	0	0	144	155	0

From the main simulation table above,

The mean Time Between Arrivals of patients $\left(\frac{1}{\lambda}\right) = 5.539 \text{ minutes}$

Therefore, mean Patients Arrival Rate, $\lambda = 0.181 \text{ patients per minute}$ or 10.86 patients per hour

The mean Doctors' Service time $\left(\frac{1}{\mu}\right) = 5.808 \text{ minutes}$

Therefore, mean Doctors' Service Rate, $\mu = 0.172 \text{ patients per minute}$ or 10.32 patients per hour

The Doctors' utilization Factor (since this is an M/M/2 system) is:

$$\rho = \frac{\lambda}{2\mu} = \frac{0.181}{2(0.172)} = 0.526$$

We computed a 95% confidence interval for this value of the Doctor Utilization Factor (DUF) as follows:

The simulation model for the operations of the system for the two doctors for the 26 patients (Table 4) was replicated 30 times.

This generated the following 30 values for the doctor utilization factor:

0.66	0.48	0.46	0.62	0.48
0.36	0.40	0.49	0.49	0.62
0.53	0.59	0.39	0.43	0.57
0.46	0.55	0.39	0.51	0.46
0.40	0.43	0.59	0.43	0.47
0.65	0.49	0.44	0.40	0.43

From the above values,

Sample Size	30
Mean (DUF)	0.49
Std Dev.	0.08339

For 95% confidence interval for the value of the doctor utilization factor (DUF); we have

$$\bar{x} \pm Z \left(\frac{\sigma}{\sqrt{n}} \right) = 0.49 \pm 1.960 \left(\frac{0.08339}{\sqrt{30}} \right)$$

i.e

$$(0.49 - 0.029841) \leq \rho \leq (0.49 + 0.029841)$$

or

$$0.459 \leq \rho \leq 0.519$$

With 95% confidence interval established for our Doctor Utilization Factor, we can say that the facility has a utilization factor for her Doctors of 0.49.

With this value of the Doctor Utilization Factor, we compute the other system characteristics as follows:

Probability that there are no patients in the facility: $P_0 = \frac{1}{M}$

$$\text{Where } M = \sum_{j=0}^2 \frac{(2\rho)^j}{j!} + \frac{(2\rho)^2}{2!} \left(\frac{\rho}{1-\rho} \right)$$

$$M = 2.9216$$

Therefore:

$$P_0 = 0.342$$

$$\text{Mean number of patients in the Queue: } L_q = \frac{\rho(2\rho)^2 P_0}{2!(1-\rho)^2} = \frac{0.49(2 \times 0.49)^2 0.342}{2!(1-0.49)^2} = 0.309 \text{ Patients per hour}$$

$$\text{Mean number of patients in the System: } L_s = \frac{\rho(2\rho)^2 P_o}{2!(1-\rho)^2} + 2\rho = \frac{0.49(2 \times 0.49)^2 \times 0.342}{2!(1-0.49)^2} + 2 \times 0.49 = 1.289 \text{ Patients per hour}$$

$$\text{Mean time a patient spends in the Queue: } W_q = \frac{(2\rho)^2 P_o}{2!(1-\rho)^2 2\mu} = \frac{(2 \times 0.49)^2 \times 0.342}{2!(1-0.49)^2 \times 2 \times 10.32} = 0.0612 \text{ hours} \approx 3.79 \text{ minutes}$$

$$\text{Mean time a patient spends in the System: } W_s = \frac{(2\rho)^2 P_o}{2!(1-\rho)^2 2\mu} + \frac{1}{\mu} = \frac{(2 \times 0.49)^2 \times 0.342}{2!(1-0.49)^2 \times 2 \times 10.32} + \frac{1}{10.32} = 0.158 \text{ hours} \approx 9.49 \text{ minutes}$$

From the results of the analysis, the system is fairly free most of the time. The doctors are free up to 34.2% of the time during peak hours. Recall that the system was observed at peak hours of 8:00 am to 12:00 noon.

The patients spend less than 4 minutes on average waiting to see a consultant and spend on average nine and a half minutes in the facility.

The average number of patients waiting to see a consultant is less than two signifying that one consultant is free most of the time.

CONCLUSION

From the system characteristics computed above, it can clearly be seen that the facility has excess capacity in terms of the consultant services. The number of consultants is in excess of the need of the polytechnic community.

This is however, common with health care systems. In health care, speciality is a necessary requirement for a facility to attain a required status. E.g. a hospital requires a medical doctor to be considered a secondary health care facility irrespective of the size of its patronage. The institutional facility is a special facility and may have this excess capacity because of its special nature.

RECOMMENDATION

For a hospital facility to have excess capacity in a place where medical care is still less than desired would be a great undoing.

This study believes that the following suggestions would have great implication for policy decisions regarding the health centre.

1. That the management of the facility considers extending its services to the neighbouring community especially as a referral for the neighbouring clinics and dispensaries.
2. That other special services be introduced to utilize the excess capacity.
3. In line with (1) above, publicity should be made of the available services to the neighbourhood, especially the neighbouring clinics
4. Service costs should be subsidized

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References

- [1] Abu, O. and Okolo, P.N. (2018). Application of queuing model in healthcare center. (A case study of Zonal hospital, Idah, Kogi State). International Journal PAS, 11(9): 165-172.
- [2] Bahadori, M., Mohammadnejhad, S. M., Ravangard, R., and Teymourzadeh, E. (2014). 'Using Queuing Theory and Simulation Model to Optimize Hospital Pharmacy Performance', Iranian Red Crescent Medical Journal, 16(3): 84-91.
- [3] Elalouf, A. and Wachtel, G. (2021). Queuing Problems in Emergency Departments: A Review of Practical Approaches and Research Methodologies, Operations Research Forum, 3(2). <https://doi.org/10.1007/s43069-021-00114-8>.
- [4] Green, L.V., Soares, J., Gigilio, J.F., and Green, R.A. (2006). 'Using Queuing Theory to Increase the Effectiveness of Emergency Department Provider Staffing', Academic Emergency Medicine, 13(1): 61-68
- [5] Kandemir-Cavas, C., Cavas, L., An Application of Queuing Theory to the Relationship between Insulin Level and Number of Insulin Receptors, Turkish Journal of Biochemistry, 32(1): 34
- [6] Lakshmi, C. and Sivakumar, A. (2013). Application of queuing theory in health care: a literature review. Oper Res Health Care, 2(1-2):25-39. <https://doi.org/10.1016/j.orhc.2013.03.002>
- [7] Lakshmi, C. and Sivakumar, A. I. (2013). Application of queuing theory in health care: A literature review, Operations Research for Health Care, 2(1-2): 25-39. <https://doi.org/10.1016/j.orhc.2013.03.002>.
- [8] Oladejo, M.O. and Aligwo, M.C (2014). Queuing model for medical center (A case study of Shehu Muhammed Kangiwa Medical centers, Kaduna Polytechnic). IOSR Journal of Mathematics, 10(1): 18-22.
- [9] Swed, F. S. and Eisenhart, C. (1943). Tables for testing randomness of grouping in a sequence of alternatives. Ann Math Statist., 14(1):66-87. <https://doi.org/10.1214/aoms/117731494>
- [10] Umar, I., Oche, M.O. and Umar, A.S. (2011). Patient Waiting Time in a Tertiary Health Institution in Northern Nigeria (Department of Community Health, Usman Danfodiyo University, Sokoto, Nigeria). Journal of Public Health and Epidemiology, 3 (2): 78-82