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SOLUTION OF LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF THE SECOND KIND USING THE MODIFIED ADOMIAN DECOMPOSITION METHOD

Okai J.O, Ilejimi D.O and Musa Ibrahim
Abubakar Tafawa Balewa University, Bauchi, Nigeria.
Federal University of Agriculture Abeokuta.
Federal College of Horticulture Dadin Kowa Gombe.
Jerry4complex@gmail.com

ABSTRACT

In this paper, linear volterra integro-differential equations are reduced to the standard linear volterra integral equation of the second kind in other to avoid unrealistic assumptions experienced with other methods and the exact solution are obtained using the modified Adomian decomposition method (MADM).

1.0 INTRODUCTION

In 1980 Adomian introduced a new method to solve linear and nonlinear functional equations. This method has since been termed the Adomian decomposition method (ADM) and has been subject of many investigations. This method generates a solution in the form of a series whose terms are determined by a recursive relation. Some fundamental works on various aspect of modification of Adomian decomposition method are given by Adrianov (1998), Venkatarangan (1995), Adamu (2016), Manjak (2017), Okai J.O (2018) and Wazwaz (2000).

Elgasery (2008) applied the Laplace decomposition method for the solution of Falkner-skan equation. Ilejimi D.O (2019) On The Numerical Solution of Picard Iteration Method for Fractional Integro - Differential Equation. Hussain and Khan (2010) applied the modified Laplace decomposition for solving some PDEs. Recently, the author's have used several methods for the numerical or the analytical solution of linear and nonlinear Fredholm and

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Volterra integral and integro-differential equations of the second kind. The linear volterra integro-differential equation is given by

$$u^{n}(x) = f(x) + \lambda \int_{a}^{x} k(x,t)\, u(t)dt \,, \qquad \dots (1)$$

Where $u^{n}(x)$ is the nth derivative of the unknown function $u(x)$ that will be determined, $k(x,t)$ is the kernel of the integration, $f(x)$ is an analytic function and $\boldsymbol{\lambda}$ is a parameter, the function $u(t)$ appears linearly under the integral sign.

The Taylor polynomial solution of integro-differential equations has been studied by Maleknejad and Mahmoudi (2003). The use of lagrange interpolation in solving integro-differential equation was investigated by Rashed (2004), Wazwaz (2006) used the modified decomposition method and the traditional methods for solving nonlinear integral equations. A variety of powerful methods has been presented, such as the Homotopy analysis method, homotopy perturbation method, Exp-function method, and variation iteration method. By using the MADM we obtain the exact solutions for the transformed integro-differential equations. It is well-known that the main disadvantage of the Laplace Transform method is that it involves large computational work, in this paper instead of using the Laplace transform method we introduce a transformation method in order to reduce the integro-differential equation to a standard integral equation of the second kind, the solution method using MADM becomes easier and faster. Our aim in this paper is to obtain the exact solutions by using the MADM. The remainder of the paper is organized as follows: In section 2, a brief discussion of the MADM is presented. In section 3, Implementation of this method is considered by solving two examples. Section 4 ends this paper with a brief conclusion.

2. METHODOLOGY

The Modified ADM

The Adomian decomposition method provides the solution in an infinite series of components. The components $u_n(x), n \geq 0$ are easily computed if the inhomogeneous term $f(x)$ in the integral equation consists of a polynomial. However, if the function $f(x)$ consists of a combination of two or more of polynomials, trigonometric functions, hyperbolic functions, and others, the evaluation of the components $u_n(x), n \geq 0$ requires cumbersome work. Going by the work of Wazwaz (2011) and Ghorbani (2007), a reliable modification of the Adomian decomposition method is presented. The modified decomposition method will facilitate the computational process and further accelerate the convergence of the series solution. The modified decomposition method will be applied, wherever it is appropriate. The modified ADM depends mainly on splitting the function $f(x)$ into two parts.

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To give a clear description of the technique, we recall that the standard ADM admits the use of the recurrence relation. Where the solution $u(x)$ is expressed by an infinite sum of components defined before by

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

The modified ADM introduces a slight variation to the recurrence relation that will lead to the determination of the components of $u(x)$ in an easier and faster manner. The function $f(x)$ can be set as the sum of two partial functions, namely $f_1(x)$ and $f_2(x)$.

In other words,

$$f(x) = f_1(x) + f_2(x) \qquad \ldots (2)$$

In view of (2), an introduction is made to change the formation of the original ADM, to minimize the size of calculations, we identified the zeroth component $u_0(x)$ by one part of $f(x)$, the other part of $f(x)$ can be added to the component $u_1(x)$ among other terms.

In other words, the modified Adomian's Decomposition method (MADM) introduces the modified recurrence relation:

$$u_0(x) = f_1(x),$$

$$u_1(x) = f_2(x) + \lambda \int_{\mathrm{a}}^{b} k(x,t) u_0 \, dt$$

$$\ldots$$

$$u_{n+1}(x) = \lambda \int_a^b k(x,t) u_n(t) \, dt, \qquad n \geq 1 \qquad \ldots (3)$$

Therefore

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \qquad \ldots (4)$$

3.0 IMPLEMENTATION.

In this section two examples are solved to illustrate the method

Example 1: Consider the second order linear volterra Integro-differential equation of the second kind of the form:

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$$u''(x) = 1 + x + \int_0^x (x-t)u(t)\,dt, u(0) = 1, u'(0) = 1 \qquad \dots(5)$$

Applying the n-fold integral formula on (5) and using the initial conditions we obtain

$$u(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{1}{3!}\int_0^x (x-t)^3 u(t)\,dt$$

Applying the MADM gives

$$u_0(x) = 1 + x$$

$$u_1(x) = \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{1}{3!}\int_0^x (x-t)^3(1+t)dt = \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

.

.

Therefore

$$u(x) = u_0(x) + u_1(x) + \cdots \ = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \dots \qquad \dots(6)$$

Which converges to the exact solution

$$u(x) = e^x$$

Example 2: Consider the second order linear volterra Integro-differential equation of the second kind

$$u''(x) = -x - \frac{x^3}{6} + \int_0^x (x-t)u(t)\,dt, \qquad u(0) = 0, u'(0) = 2 \qquad \dots(7)$$

With exact solution $u(x) = x + sinx$

Applying the n-fold integral formula and using the initial conditions we obtain

$$u(x) = 2x - \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{1}{3!}\int_0^x (x-t)^3 u(t)\,dt \qquad \dots(8)$$

Applying the MADM gives

$$u_0(x) = \ 2x - \frac{x^3}{3!}$$

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$$u_1(x) = \frac{1}{120}x^5 - \frac{1}{5040}x^7$$

$$u_2(x) = \frac{1}{39916800}x^{11} - \frac{1}{362880}x^9$$

.
.
.

Therefore

$$u(x) = 2x - \frac{x^3}{3!} + \frac{1}{120}x^5 - \frac{1}{5040}x^7 - \frac{1}{362880}x^9 + \frac{1}{39916800}x^{11} \qquad \ldots (9)$$

$$u(x) = x + \sin x$$

Which converges to the exact solution

$$u(x) = x + \sin x$$

4. CONCLUSION

The main idea of this work was to give a simple method for solving linear volterra integro-differential equations. We carefully transform the linear volterra integro-differential equation to a standard linear volterra integral equation of the second kind and we applied the MADM. The main advantage of this method is the fact that it gives the analytical solution in just few iterations which saves time.

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