Solving Nonlinear System of Equations Using the Adomian Decomposition Method

Amos Silungwe[1] and *Prof.Patrick* A. *Phiri*[2*]

Department of Mathematics, School of Natural Sciences, Copperbelt University, Zambia

Email: jedidahamos@gmail.com

* Corresponding author's email: patricka.phiri@cbu.ac.zm

Complete detail of the student

Full Name: Amos Silungwe

Affiliation (Department and College/University): Department of Mathematics, School

of Natural Sciences, Copperbelt University

Academic year/Semester: 2019-2022

Course level: Master

Course name: Master of Science in Mathematics

Course year/semester:2021

Email id:jedidahamos@gmail.com

ORCID:https://orcid.org/0009-0005-2707-5074

Complete detail of the supervisor

Full Name (Supervisor): Patrick A. Phiri

Affiliation (Department and College/University) with postal address: Department of

Mathematics, School of Natural Sciences, Copperbelt University

Post/Rank: Professor

Email id: patricka.phiri@cbu.ac.zm

ORCID:

Abstract

In this paper, we present the Adomian decomposition method (ADM) for solving systems
of nonlinear equations. MATHEMATICA program is introduced with an input program
for finding Adomian polynomials for nonlinear equations. ADM was compared with
Broyden's and Newton's methods and its results showed better and faster convergence to
exact solutions .

Keyword: Adomian Decomposition Method,Adomian's Polynomial, System of Nonlinear Equations, MATHEMATICA program.

1   Introduction

The Adomian Decomposition Method (ADM) separates the equation under investigation
into linear and nonlinear portions. The nonlinear portion is decomposed into a series of
Adomian polynomials. ADM produces a solution in form of a series whose terms are
determined by Adomian polynomials. The method decomposes the linear term x into an
infinite sum of components x_n defined by

$$x = \sum_{n=0}^{\infty} x_n \tag{1}$$

Among a number of already proposed methods of solving nonlinear equations, the Adomian Decomposition Method as gained astonishing popularity. Recently the method is applied to handling singular and non-singular initial, boundary value problems in ordinary,
partial differential equations and fractional differential equations. ADM is also applied to
obtain the solution for considered model under atmospheric internal wave phenomenon

2

referred to as gravity wave [1, 2, 3, 4, 5].

Researchers from earlier studies have investigated the ADM for solving nonlinear and system of nonlinear equations. For instance, Babolian, Biazar and Vahidi[6] applied the Adomian decomposition method to solve a system of nonlinear equations. Jafari and Varsha [7] suggested and used a modification of Adomian decomposition method to solve a system of nonlinear equations which showed a series of solution with accelerated convergence.

This papaer suggests the application of MATHEMATICA program to generate the Adomian polynomials which is an important procedure if we are to apply the ADM to solve system of nonlinear equations

2 Adomian Decomposition Method for a system of Nonlinear Equations

Consider the system of nonlinear equations;

$$f_1(x_1, x_2, \cdots x_n) = 0$$

$$f_2(x_1, x_2, \cdots x_n) = 0$$

$$\vdots$$

$$f_n(x_1, x_2, \cdots x_n) = 0$$

For this reason,we have the equations

$$f_i(x_1, x_2, \cdots x_n) = 0, i = 1, 2, \cdots, n \tag{2}$$

3

where f_i maps the vector $X = (x_1, x_2, \cdots x_n)^t$ of the n-dimentional space $\mathbb{R}^n$ into the real line $\mathbb{R}$

Application of the decomposition method

First we start by writting (2) in canonical form where the linear portion is written in terms of the constant and the nonlinear terms in each equation for $i = 1, 2, \cdots, n$

$x_i = c_i + N_i(x_1 + x_2 + \cdots x_n)$ where $N_i(x_1 + x_2 + \cdots x_n)$ is nonlinear portion and c_i is constant.

The linear term x is then expressed into an infinite sum of components x_n defined by

$$x_i = \sum_{m=0}^{\infty} x_{im} \tag{3}$$

and the nonlinear portion is given by

$$N_i(x_1 + x_2 + \cdots x_n) = \sum_{m=0}^{\infty} A_{im} \tag{4}$$

where A_{im}'s are Adomian polynomials depending on $x_{10}, \cdots, x_{1m}, \cdots, x_{n0}, \cdots x_{nm}$, hence $\sum_{m=0}^{\infty} x_{im} = c_i + \sum_{m=0}^{\infty} A_{im}$ and from this series equation we have

$$x_{i0} = c_i \tag{5}$$

$$x_{i,m+1} = A_{im} \tag{6}$$

for $m = 0, 1, \cdots$ and $i = 1, 2, \cdots, n$

To determine the Adomian polynomials we use [6, 8] whose formula is given by

$$A_{im}(x_{10}, \cdots, x_{1m}, \cdots, x_{n0}, \cdots x_{nm}) = \frac{1}{n!}[\frac{d^m}{d\lambda^m} N_i(x_i, \cdots, x_n)]_{\lambda=0} \tag{7}$$

4

We approximate the solution of x_i by using the series [6]

$$\psi_{ik} = \sum_{m=0}^{k-1} x_{im} \tag{8}$$

2.1 Application of MATHEMATICA program to generate Adomian polynomials

Following the work of Wazwaz ([9]), Choi and Shin ([10]). The computer algebra software MATHEMATICA has been applied to find Adomian Polynomials [11, 12, 13, 14].

2.1.1 MATHEMATICA code for the one-variable Adomian polynomials based on Duan's Corollary 3 algorithm ([11])

$AP[f\_,M\_] := Module[c,n,k,j,der,Table[c[n,k],n,1,M,k,1,n];$

$der = Table[D[f[Subscript[u,0]],Subscript[u,0],k],k,1,M];$

$A[0] = f[Subscript[u,0]];$

$For[n = 1,n <= M,n++,c[n,1] = Subscript[u,n];$

$For[k = 2,k <= n,k++,$

$c[n,k] = Expand[1/n * Sum[(j+1) * Subscript[u,j+1] * c[n-1-j,k-1],j,0,n-k]]];$

$A[n] = Take[der,n].Table[c[n,k],k,1,n]];$

$Table[A[n],n,0,M]]$

5

Application of MATHEMATICA code for a nonlinear term u^4

MATHEMATICA input for u^4

$\text{AP}[f\_, M\_] := Module[c, n, k, j, der, Table[c[n, k], n, 1, M, k, 1, n];$

$der = Table[D[f[Subscript[u, 0]], Subscript[u, 0], k], k, 1, M];$

$A[0] = f[Subscript[u, 0]];$

$For[n = 1, n <= M, n++, c[n, 1] = Subscript[u, n];$

$For[k = 2, k <= n, k++,$

$c[n, k] = Expand[1/n * Sum[(j + 1) * Subscript[u, j + 1] * c[n - 1 - j, k - 1], j, 0, n - k]]];$

$A[n] = Take[der, n].Table[c[n, k], k, 1, n]];$

$Table[A[n], n, 0, M]]\text{AP}\left(u^4, 9\right)$

MATHEMATICA output for u^4

$$A_0 = u^4(u_0) \tag{9}$$

$$A_1 = u_1\left(u^4\right)'(u_0) \tag{10}$$

$$A_2 = u_2\left(u^4\right)'(u_0) + \frac{1}{2}u_1^2\left(u^4\right)''(u_0) \tag{11}$$

$$A_3 = u_3\left(u^4\right)'(u_0) + u_2 u_1\left(u^4\right)''(u_0) + \frac{1}{6}u_1^3\left(u^4\right)^{(3)}(u_0) \tag{12}$$

$$A_4 = u_4\left(u^4\right)'(u_0) + \left(\frac{u_2^2}{2} + u_1 u_3\right)\left(u^4\right)''(u_0) + \frac{1}{24}u_1^4\left(u^4\right)^{(4)}(u_0) + \frac{1}{2}u_2 u_1^2\left(u^4\right)^{(3)}(u_0) \tag{13}$$

$$\begin{aligned}
A_5 = u_5\left(u^4\right)'(u_0) + (u_2 u_3 + u_1 u_4)\left(u^4\right)''(u_0) + \left(\frac{1}{2}u_3 u_1^2 + \frac{1}{2}u_2^2 u_1\right)\left(u^4\right)^{(3)}(u_0) \\
+ \frac{1}{6}u_1^3 u_2\left(u^4\right)^{(4)}(u_0) + \frac{1}{120}u_1^5\left(u^4\right)^{(5)}[u_0].
\end{aligned} \tag{14}$$

6

$$A_6 = u_6 \left(u^4\right)'(u_0) + \left(\frac{u_3^2}{2} + u_2 u_4 + u_1 u_5\right)\left(u^4\right)''(u_0) + \frac{1}{720} u_1^6 \left(u^4\right)^{(6)}(u_0) + \frac{1}{24} u_2 u_1^4 \left(u^4\right)^{(5)}(u_0)$$

$$+ \left(\frac{u_2^3}{6} + u_1 u_3 u_2 + \frac{1}{2} u_1^2 u_4\right)\left(u^4\right)^{(3)}(u_0)$$

$$+ \left(\frac{1}{6} u_3 u_1^3 + \frac{1}{4} u_2^2 u_1^2\right)\left(u^4\right)^{(4)}(u_0)$$

$$(15)$$

$$A_7 = u_7 \left(u^4\right)'(u_0) + \left(u_3 u_4 + u_2 u_5 + u_1 u_6\right)\left(u^4\right)''(u_0) + \frac{u_1^7 \left(u^4\right)^{(7)}(u_0)}{5040} + \frac{1}{120} u_2 u_1^5 \left(u^4\right)^{(6)}(u_0)$$

$$+ \left(\frac{1}{2} u_5 u_1^2 + \frac{1}{2} u_3^2 u_1 + u_2 u_4 u_1 + \frac{1}{2} u_2^2 u_3\right)\left(u^4\right)^{(3)}(u_0)$$

$$+ \left(\frac{1}{6} u_4 u_1^3 + \frac{1}{2} u_2 u_3 u_1^2 + \frac{1}{6} u_2^3 u_1\right)\left(u^4\right)^{(4)}(u_0) + \left(\frac{1}{24} u_3 u_1^4 + \frac{1}{12} u_2^2 u_1^3\right)\left(u^4\right)^{(5)}(u_0)$$

$$(16)$$

$$A_8 = u_8 \left(u^4\right)'(u_0) + \left(\frac{u_4^2}{2} + u_3 u_5 + u_2 u_6 + u_1 u_7\right)\left(u^4\right)''(u_0) + \frac{u_1^8 \left(u^4\right)^{(8)}(u_0)}{40320} + \frac{1}{720} u_2 u_1^6 \left(u^4\right)^{(7)}(u_0)$$

$$+ \left(\frac{1}{2} u_6 u_1^2 + u_3 u_4 u_1 + u_2 u_5 u_1 + \frac{1}{2} u_2 u_3^2 + \frac{1}{2} u_2^2 u_4\right)\left(u^4\right)^{(3)}(u_0)$$

$$+ \left(\frac{u_2^4}{24} + \frac{1}{2} u_1 u_3 u_2^2 + \frac{1}{2} u_1^2 u_4 u_2 + \frac{1}{4} u_1^2 u_3^2 + \frac{1}{6} u_1^3 u_5\right)\left(u^4\right)^{(4)}(u_0)$$

$$+ \left(\frac{1}{24} u_4 u_1^4 + \frac{1}{6} u_2 u_3 u_1^3 + \frac{1}{12} u_2^3 u_1^2\right)\left(u^4\right)^{(5)}(u_0) + \left(\frac{1}{120} u_3 u_1^5 + \frac{1}{48} u_2^2 u_1^4\right)\left(u^4\right)^{(6)}(u_0)$$

$$(17)$$

$$A_9 = u_9 \left(u^4\right)'(u_0) + \left(u_4 u_5 + u_3 u_6 + u_2 u_7 + u_1 u_8\right)\left(u^4\right)''(u_0) + \frac{u_1^9 \left(u^4\right)^{(9)}(u_0)}{362880} + \frac{u_2 u_1^7 \left(u^4\right)^{(8)}(u_0)}{5040}$$

$$+ \left(\frac{u_3^3}{6} + u_2 u_4 u_3 + u_1 u_5 u_3 + \frac{1}{2} u_1 u_4^2 + \frac{1}{2} u_2^2 u_5 + u_1 u_2 u_6 + \frac{1}{2} u_1^2 u_7\right)\left(u^4\right)^{(3)}(u_0)$$

$$+ \left(\frac{1}{6} u_6 u_1^3 + \frac{1}{2} u_3 u_4 u_1^2 + \frac{1}{2} u_2 u_5 u_1^2 + \frac{1}{2} u_2 u_3^2 u_1 + \frac{1}{2} u_2^2 u_4 u_1 + \frac{1}{6} u_2^3 u_3\right)\left(u^4\right)^{(4)}(u_0)$$

$$+ \left(\frac{1}{24} u_5 u_1^4 + \frac{1}{12} u_3^2 u_1^3 + \frac{1}{6} u_2 u_4 u_1^3 + \frac{1}{4} u_2^2 u_3 u_1^2 + \frac{1}{24} u_2^4 u_1\right)\left(u^4\right)^{(5)}(u_0)$$

$$+ \left(\frac{1}{120} u_4 u_1^5 + \frac{1}{24} u_2 u_3 u_1^4 + \frac{1}{36} u_2^3 u_1^3\right)\left(u^4\right)^{(6)}(u_0) + \left(\frac{1}{720} u_3 u_1^6 + \frac{1}{240} u_2^2 u_1^5\right)\left(u^4\right)^{(7)}(u_0)$$

$$(18)$$

7

The above demostration of the application of MATHEMATICA code gives us an advantage of find the adomian polynomials for any given nonlinear term much quicker than resolving the derivatives of Adomian polynomials manually, hence helping us to generate solutions to any given nonlinear or system of nonlinear equations.

application of ADM

Consider the following system of nonlinear equations

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0$$

$$x_1 x_2^2 + x_1 - 10x_2 + 8 = 0$$

Rewriting each nonlinear equation in canonical form we have

$$x_1 = 0.8 + 0.1x_1^2 + 0.1x_2^2$$

$$x_2 = 0.8 + 0.1x_1 x_2^2 + 0.1x_1$$

Applying the Adomian Decomposition Method we have the linear term of each equation decomposed into infinity series (3) and the nonlinear terms decomposed into adomian polynomial series (4).

$$x_1 = \sum_{m=0}^{\infty} x_{1m} = 0.8 + 0.1 \sum_{m=0}^{\infty} A_{1m}(x_1^2) + 0.1 \sum_{m=0}^{\infty} A_{2m}(x_2^2)$$

$$x_2 = \sum_{m=0}^{\infty} x_{2m} = 0.8 + 0.1 \sum_{m=0}^{\infty} A_{2m}(x_1 x_1^2) + 0.1 \sum_{m=0}^{\infty} A_{1m}(x_1)$$

Using Mathematica we have the following Adomian polynomials:

MATHEMATICA code input for Adomian polynomials for x^2

$\text{AP}[f\_, M\_] := Module[c, n, k, j, der, Table[c[n,k], n, 1, M, k, 1, n];$

$der = Table[D[f[Subscript[x,0]], Subscript[x,0], k], k, 1, M];$

8

$A[0] = f[Subscript[x,0]];$

$For[n = 1, n <= M, n + +, c[n,1] = Subscript[x,n];$

$For[k = 2, k <= n, k + +,$

$c[n,k] = Expand[1/n * Sum[(j+1) * Subscript[x, j+1] * c[n-1-j, k-1], j, 0, n-k]]];$

$A[n] = Take[der, n].Table[c[n,k], k, 1, n]];$

$Table[A[n], n, 0, M]] AP(x^2, 10)$

Adomian Polynomials for $A_{1m}(x_1^2)$

$A_{10} = x_{10}^2$

$A_{11} = 2x_{10}x_{11}$

$A_{12} = 2x_{10}x_{12} + x_{11}^2$

$A_{13} = 2x_{10}x_{13} + 2x_{11}x_{12}$

$A_{14} = 2x_{10}x_{14} + 2x_{11}x_{13} + x_{12}^2$

$A_{15} = 2x_{10}x_{15} + 2x_{11}x_{14} + 2x_{12}x_{13}$

$A_{16} = 2x_{10}x_{16} + 2x_{11}x_{15} + 2x_{12}x_{14} + x_{13}^2$

$A_{17} = 2x_0x_{17} + 2x_{11}x_{16} + 2x_{12}x_{15} + 2x_{13}x_{14}$

$A_{18} = 2x_{10}x_{18} + 2x_{11}x_{17} + 2x_{12}x_{16} + 2x_{13}x_{15} + x_{14}^2$

$\vdots$

Adomian Polynomials for $A_{2m}(x_2^2)$

$A_{20} = x_{20}^2$

$A_{21} = 2x_{20}x_{21}$

$A_{22} = 2x_{20}x_{22} + x_{21}^2$

$A_{23} = 2x_{20}x_{23} + 2x_{21}x_{22}$

9

$$A_{24} = 2x_{20}x_{24} + 2x_{21}x_{23} + x_{22}^2$$

$$A_{25} = 2x_{20}x_{25} + 2x_{21}x_{24} + 2x_{22}x_{23}$$

$$A_{26} = 2x_{20}x_{26} + 2x_{21}x_{25} + 2x_{22}x_{24} + x_{23}^2$$

$$A_{27} = 2x_0x_{27} + 2x_{21}x_{26} + 2x_{22}x_{25} + 2x_{23}x_{24}$$

$$A_{28} = 2x_{20}x_{28} + 2x_{21}x_{27} + 2x_{22}x_{26} + 2x_{23}x_{25} + x_{24}^2$$

$$\vdots$$

Adomian Polynomials for the nonlinear term $x_1x_2^2$

$$A_0 = x_{10}x_{20}^2$$

$$A_1 = 2x_{10}x_{20}x_{21} + x_{11}x_{20}^2$$

$$A_2 = x_{10}x_{21}^2 + 2x_{10}x_{20}x_{22} + 2x_{11}x_{20}x_{21}$$

$$A_3 = 2x_{10}x_{20}x_{23} + 2x_{10}x_{21}x_{22} + x_{11}x_{21} + 2x_{11}x_{20}x_{22} + 2x_{12}x_{20}x_{21} + x_{13}x_{20}^2$$

$$A_4 = \left(2x_{20}x_{24} + 2x_{21}x_{23} + x_{22}^2\right)x_{10} + \left(2x_{20}x_{23} + 2x_{11}x_{22}\right)x_{11} + \left(x_{21}^2 + 2x_{20}x_{22}\right)x_{12} + 2x_{20}x_{21}x_{13} + x_{20}^2 + x_{14}$$

Using the recurrence relations (5) and (6) we have the table 1 with decomposed components of x_{im}

Table 1: x_{im} components

x_{1m}	x_{2m}
$x_{10} = 0.80000$	$x_{20} = 0.80000$
$x_{11} = 0.12800$	$x_{21} = 0.13120$
$x_{12} = 0.014147$	$x_{22} = 0.03779$
$x_{13} = 0.01604$	$x_{23} = 0.01564$
$x_{14} = 0.00712$	$x_{24} = 0.00729$
$x_{15} = 0.00344$	$x_{25} = 0.00366$

From equation (8) we have the following Approximate solution for the first five decomposed terms

$$\psi_{15} = x_{10} + \cdots + x_{14} = 0.9961$$

$$\psi_{25} = x_{20} + \cdots + x_{24} = 0.9956$$

2.2 Example 2

Consider the following system of nonlinear equations:

$$15x_1 + x_2^2 + 4x_3 = 13$$

$$x_1^2 10x_2 + e^{-x_3} = 11$$

$$x_2^3 - 25x_3 = -22$$

writing each equation in canonical form we have

$$x_1 = 0.86667 - 0.06667x_2^2 + 0.26667x_3$$

$$x_2 = 1.1 - 0.1x_1^2 + 0.1e^{-x_3}$$

$$x_3 = 0.88 + 0.04x_2^2$$

expressing each equation using Adomian decomposition series we have

11

$$x_1 = 0.86667 - 0.06667 \sum_{m=0}^{\infty} A_{2m}(x_2^2) + 0.26667 \sum_{m=0}^{\infty} A_{3m}(x_3)$$

$$x_2 = 1.1 - 0.1 \sum_{m=0}^{\infty} A_{1m}(x_1^2) + 0.1 \sum_{m=0}^{\infty} A_{3m}(e^{-x_3})$$

$$x_3 = 0.88 + 0.04 \sum_{m=0}^{\infty} A_{2m}(x_2^2)$$

The Adomian polynomials for $A_{1m}(x_1^2)$

$$A_{10} = x_{10}^2$$

$$A_{11} = 2x_{10}x_{11}$$

$$A_{12} = x_{11}^2 + 2x_{10}x_{12}$$

$$A_{13} = 2x_{10}x_{13} + 2x_{11}x_{12}$$

$$A_{14} = 2x_{10}x_{14} + 2x_{11}x_{13} + x_{12}^2$$

$$\vdots$$

The Adomian polynomials for $A_{2m}(x_2^2)$

$$A_{20} = x_{20}^2$$

$$A_{21} = 2x_{20}x_{21}$$

$$A_{22} = x_{21}^2 + 2x_{20}x_{22}$$

$$A_{23} = 2x_{20}x_{23} + 2x_{21}x_{22}$$

$$A_{24} = 2x_{20}x_{24} + 2x_{21}x_{23} + x_{22}^2$$

$$\vdots$$

The Adomian polynomials for $A_{2m}(x_3)$

$$A_{30} = x_{30}$$

$$A_{31} = x_{31}$$

$$A_{32} = 0$$

12

$A_{33} = 0$

$A_{34} = 0$

$\vdots$

The Adomian polynomials for $A_{2m}(e^{-x_3})$

$A_{31} = e^{-x_{30}}$

$A_{32} = -e^{-2x_{30}}$

$A_{33} = 1.5e^{-3x_{30}}$

$A_{34} = 267e^{-4x_{30}}$

$\vdots A_{3m}(e^{-x_3}) = (-1)^{m+1}\frac{m^{m-1}}{m!}e^{-mx_{30}}$ for $m = 1, 2, \cdots,$.

Therefore, the decomposed components of x are given as follows:

$x_{10} = 0.86667$

$x_{20} = 1.10000$

$x_{30} = 0.88000$

$x_{11} = -0.06667(x_{20}^2) + 0.2666679(x_{30}) = -0.06667(1.1)^2 + 0.26667(0.88) = 0.1539989$

$x_{21} = -0.1(x_{10})^2 + 0.1(e^{-x_{30}}) = -0.1(0.86667)^2 + 0.1(e^{0.88}) = -0.0336334$

$x_{31} = 0.04(x_20)^3 = 0.04(1.1)^3 = 0.05324.$

By following the same procedure and using the Adomian polynomials defined above we have the following values of the components of x given in table 2

Approximating the first six terms we have

$\psi_{15} = x_{10} + \cdots + x_{15} = 1.04223716$

$\psi_{25} = x_{20} + \cdots + x_{25} = 1.01568782$

13

Table 2: x components

x_{1i}	x_{2i}	x_{3i}
$x_{10} = 0.86667$	$x_{20} = 1.10000$	$x_{30} = 0.880000$
$x_{11} = 0.15400$	$x_{21} = -0.003363$	$x_{31} = 0.05324000$
$x_{12} = 0.01913$	$x_{22} = -0.04389$	$x_{32} = 0.00488349$
$x_{13} = 0.00506$	$x_{23} = 0.00501670$	$x_{33} = 0.006328724$
$x_{14} = -0.00262$	$x_{24} = -0.01179828$	$x_{34} = 0.00111667$
$x_{15} = -0.00000284$	$x_{25} = -0.0302776$	$x_{35} = -0.022424424$

$$\psi_{35} = x_{30} + \cdots + x_{35} = 0.92314446$$

which is a good approximation of the exact solution $x = (1.04214966, 1.03109169, 0.92384809)^t$.

3   Numerical Comparison

3.1   Numerical Comparison of ADM with Broyden's Method

Consider a system of nonlinear equations solved using Broyden's Method by (Douglas and Burden) ([15]) on solutions of systems of nonlinear equations

$$3x_1 - \cos(x_2 x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin(x_3) + 1.06 = 0$$

$$e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

with the exact solution $X* = (x*_1, x*_2, x*_3)^t = (0.5, 0, -\frac{\pi}{6})^t$. We solve the same system of nonlinear equations using ADM

14

Application of ADM

Rewriting the equations in canonical form we have

$$x_1 = \tfrac{1}{6} + \tfrac{1}{3}\cos(x_2 x_3)$$

$$x_2 = \tfrac{0.25}{16.2} + \tfrac{1}{16.2}x_1^2 - \tfrac{81}{16.2}x_2^2 + \tfrac{1}{16.2}\sin(x_3)$$

$$x_3 = \tfrac{-10\pi - 3}{60} - \tfrac{1}{20}e^{-x_1 x_2}$$

The Adomian polynomials for nonlinear term (x_1^2)

$$A_{10} = x_{10}^2$$

$$A_{11} = 2x_{10}x_{11}$$

$$A_{12} = x_{11}^2 + 2x_{10}x_{12}$$

$$A_{13} = 2x_{10}x_{13} + 2x_{11}x_{12}$$

$$A_{14} = 2x_{10}x_{14} + 2x_{11}x_{13} + x_{12}^2$$

$$\vdots$$

The Adomian polynomials for nonlinear term (x_2^2)

$$A_{20} = x_{20}^2$$

$$A_{21} = 2x_{20}x_{21}$$

$$A_{22} = x_{21}^2 + 2x_{20}x_{22}$$

$$A_{23} = 2x_{20}x_{23} + 2x_{21}x_{22}$$

$$A_{24} = 2x_{20}x_{24} + 2x_{21}x_{23} + x_{22}^2$$

$$\vdots$$

15

Adomian polynomials for nonlinear term $sinx_3$

$$A_{30} = \sin x_{30}$$

$$A_{31} = x_{31} \cos x_{30}$$

$$A_{32} = x_{32} \cos x_{30} + \tfrac{1}{2!} x_{31}^2 \sin x_{30}$$

$$A_{33} = x_{33} \cos x_{30} - x_{31} x_{32} \sin_{30} - \tfrac{1}{3!} x_{31}^3 \cos x_{30}$$

$$A_{34} = -x_{34} \cos x_{30} - \left(\tfrac{1}{2!} x_{32}^2 + x_{31} x_{33}\right) \sin x_{30} - \tfrac{1}{2!} x_{31}^2 x_{32} \cos x_{30} + \tfrac{1}{4!} x_{31}^4 \sin x_{30}$$

$$\vdots$$

Adomian polynomial for nonlinear term $cos(x_2 x_3)$

$$A_0 = cos(x_{2,0} x_{3,0})$$

$$A_1 = -(x_{3,0} x_{2,1} + x_{2,0} x_{3,1}) sin(x_{2,0} x_{3,0})$$

$$A_2 = \left(\tfrac{-1}{2!} x_{2,1}^2 x_{3,0}^2 - \tfrac{1}{2!} x_{3,1}^2 x_{2,0}^2 + x_{2,0} x_{3,0} x_{2,1} x_{3,1}\right) cos(x_{2,0} x_{3,0}) - (x_{2,1} x_{3,1} + x_{2,2} x_{3,0} + x_{3,2} x_{2,0}) sin(x_{2,0} x_{3,0})$$

$$A_3 = \left(\tfrac{1}{3!} x_{3,0}^2 x_{2,1}^3 + \tfrac{1}{3!} x_{2,0}^3 x_{3,1}^3 + \tfrac{1}{2!} x_{1,1} x_{3,1}^2 x_{2,0}^2 x_{3,0} + \tfrac{1}{2!} x_{2,1}^2 x_{3,1} x_{2,0} x_{3,0}^2 + \tfrac{1}{2!} x_{1,1} x_{3,1}^2 x_{2,0}^2 x_{3,0} - \right.$$

$$x_{3,1} x_{2,2} - x_{2,1} x_{3,2} - x_{2,3} x_{3,0} - x_{3,3} x_{2,0}) sin(x_{2,0} x_{3,0}) - (x_{1,1} x_{2,1}^2 x_{3,0} + x_{2,1}^2 x_{3,1} x_{3,0} + x_{1,1} x_{3,1}^2 x_{2,0} +$$

$$x_{3,1} x_{3,2} x_{2,0}^2 - x_{3,1} x_{2,2} x_{3,0} x_{2,0} + x_{2,1} x_{3,2} x_{2,0} x_{3,0}) cos(x_{2,0} x_{3,0}),$$

Adomian polynomial for nonlinear term $e^{x_1 x_2})$

$$A_0 = e^{-x_{1,0} x_{2,0}}$$

$$A_1 = -(x_{2,0} x_{1,1} + x_{2,1} x_{1,3}) e^{-x_{1,0} x_{2,0}}$$

$$A_2 = \left(\tfrac{1}{2!} x_{1,1}^2 x_{2,0}^2 + \tfrac{1}{2!} x_{2,1}^2 x_{1,0}^2 - x_{1,1} x_{2,1} + x_{1,1} x_{2,1} x_{1,0} x_{2,0} - x_{1,2} x_{2,0} - x_{2,2} x_{1,0}\right) e^{-x_{1,0} x_{2,0}}$$

$$A_3 = \left(-\tfrac{1}{3!} x_{1,1}^3 x_{2,0}^3 - \tfrac{1}{3!} x_{2,1}^3 x_{1,0}^3 + x_{1,1}^2 x_{2,1} x_{2,0} \tfrac{1}{2!} x_{1,1}^2 x_{2,1} x_{1,0} x_{2,0} x_{2,0}^2 + x_{1,1} x_{1,2} x_{2,0}^2 + x_{2,2} x_{1,0}^2 x_{2,1} - \right.$$

$$x_{2,1} x_{1,2} + x_{2,1} x_{1,2} x_{1,0} x_{2,0} - x_{1,1} x_{2,2} + x_{1,1} x_{2,2} x_{1,0} x_{2,0} - x_{1,3} x_{2,0} - x_{2,3} x_{1,0})) e^{-x_{1,0} x_{2,0}}$$

By applying the Adomian Decomposition Method and calculating the components of the

16

series we have

$x_{10} = \frac{1}{6}$

$x_{20} = \frac{5}{324}$

$x_{30} = -0.4735987756$

$x_{11} = cos(\frac{5}{324}(-0.5735987756)) = 0.3333333274$

$x_{21} = \frac{1}{16.2}(\frac{1}{6})^2 - \frac{81}{16.2}(\frac{5}{324})^2 + \frac{1}{16.2}sin(0.5735987756) = -9.403489696 \times 10^{-5}$

$x_{31} = \frac{-1}{20}e^{\frac{-1}{6} \times \frac{5}{324}} = -0.04987156442$

Proceeding in a similar manner the first seven terms produced the following approximations relative to the exact solutions;

$x_1 = \psi_{1,7} = x_{10} + x_{11} + \cdots + x_{16} = 0.50002$

$x_2 = \psi_{2,7} = x_{20} + x_{21} + \cdots + x_{26} = 0.000264$

$x_3 = \psi_{3,7} = x_{30} + x_{31} + \cdots + x_{36} = -0.52360$

It can be observed from the solution obtained above that ADM converges faster to the exact solution as compared to the Broyden's Method whose results are given in ([16])

3.2 Numerical Comparison of ADM with Newton's Method

In this subsection we compare the Adomian decomposition method and Newton's method for the following system of nonlinear equations whose exact solution is $x^* = (0, -5)^t$

$$4x_1 - x_2 = 5$$

$$2x_1^2 - x_2 = 5.$$

17

Introducing the decomposition series we have

$$x_1 = \sum_{m=0}^{\infty} x_{1m} = 1.25 + 0.25 \sum_{m=0}^{\infty} x_{2m}$$

$$x_2 = \sum_{m=0}^{\infty} x_{2m} = -5 + 2 \sum_{m=0}^{\infty} x_{1m}^2$$

Application of ADM

Rewriting the system into canonical form

$$x_1 = 1.25 + 0.25x_2$$

$$x_2 = -5 + 2x_1^2$$

Adomian polynomials for the nonlinear term $N(x) = x_{1m}^2$

$A_{10} = x_{10}^2$

$A_{11} = 2x_{10}x_{11}$

$A_{12} = x_{11}^2 + 2x_{10}x_{12}$

$A_{13} = 2x_{10}x_{13} + 2x_{11}x_{12}$

$A_{14} = 2x_{10}x_{14} + 2x_{11}x_{13} + x_{12}^2$

$\vdots$

Applying the recurrence relations (5) and (6), we have decomposed components of x_1

and x_2 given in table 3.

Hence approximating the solutions using equation (8)

$x_1 = \psi_{1,2} = x_{10} + x_{11} = 0$

$x_2 = \psi_{2,3} = x_{20} + x_{21} + x_{22} = -5$ which converges to the exact solution

18

Table 3: x_{im} components

x_{1m}	x_{2m}
$x_{10} = 1.25$	$x_{20} = -5$
$x_{11} = -1.25$	$x_{21} = 3.1250$
$x_{12} = 0.7813$	$x_{22} = -3.1250$
$x_{13} = -0.7813$	$x_{23} = 7.0313$

Results for Newton's Method

The following results in table 4 where obtained by using Newton's method

Table 4: iterations for Newton's Method

k	$x_1^{(k)}$	$x_2^{(k)}$
0	-1	-3
1	-0.25	-5.999
2	-0.0250	-5.1000
3	-0.0012	-5.0012
4	-0.000	-5.0000
5	0.0000	-5.0000

4 Findings

Comparing the results obtained by Broyden's and Newton's methods in this paper, it is clear that ADM converges faster. It is also important to take not that the solution of Newton's method depends entirely on the choice of the initial guess.Adomian decomposition

19

method can be applied directly to systems of nonlinear equations were it automatically provides the initial solution once expressed in canonical form. Hence there is no need of guessing the initial approximation to the exact solution.

5 Conclusion

In this work we applied MATHEMATICA program to find Adomian polynomials for any given nonlinear term of a system of nonlinear equations. The method has been compared with Broyden's and Newton's Methods of solving systems of nonlinear equations and it showed faster convergence to exact solutions.

6 Declaration

I declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

6.1 Study Limitations

None

6.2 Funding source

None

6.3   Competing Interests

None

7   Acknowledgements

References

[1] A. C. Varsoliwala, T. R. Singh, Mathematical modeling of atmospheric internal waves phenomenon and its solution by elzaki adomian decomposition method, Journal of Ocean Engineering and Science 7 (3) (2022) 203–212.

URL https://doi.org/10.1016/j.joes.2021.07.010

[2] M. A. Hussein, A review on algorithms of laplace adomian decomposition method for fpdes, Scientific Research Journal of Multidisciplinary 2 (2022) 1–10.

URL https://www.researchgate.net/profile/Mohammed-Hussein-67/publication/363615843_A_Review_on_Algorithms_of_Laplace_Adomian_Decomposition_Method_for_FPDEs/links/6325a54b071ea12e363c493d/A-Review-on-Algorithms-of-Laplace-Adomian-Decomposition-Method-for-FPDEs.pdf

[3] M. Kumar, Recent development of adomian decomposition method for ordinary and partial differential equations, International Journal of Applied and Computational Mathematics 8 (2) (2022) 81.

URL https://doi.org/10.1007/s40819-022-01285-6

[4] H. Thabet, S. Kendre, New modification of adomian decomposition method for solving a system of nonlinear fractional partial differential equations, Int. J. Adv. Appl. Math. Mech 6 (3) (2019) 1–13.

URL https://www.researchgate.net/publication/332014078_New_modification_of_Adomian_decomposition_method_for_solving_a_system_of_nonlinear_

22

fractional_partial_differential_equations?enrichId=

rgreq-1df18f6fed087a7520e83a893e9a5665-XXX&enrichSource=

Y292ZXJQYWdlOzMzMjAxNDA3ODtBUzo3NDA5OTQxMjMzMDA4NjVAMTU1MzY3ODE3NDE1OQ%

3D%3D&el=1_x_2&_esc=publicationCoverPdf

[5] A. A. Hamoud, K. P. Ghadle, The reliable modified of adomian decomposition method for solving integro-differential equations, Journal of the Chungcheong Mathematical Society 32 (4) (2019) 409–409.

URL https://doi.org/10.14403/jcms.2019.32.4.409

[6] E. Babolian, J. Biazar, A. R. Vahidi, Solution of a system of nonlinear equations by adomian decomposition method, Applied Mathematics and Computation 150 (3) (2004) 847–854.

URL https://doi.org/10.1016/S0096-3003(02)00806-8

[7] H. Jafari, V. Daftardar-Gejji, Revised adomian decomposition method for solving a system of nonlinear equations, Applied Mathematics and Computation 175 (1) (2006) 1–7.

URL https://doi.org/10.1016/j.amc.2005.07.010

[8] A. Vahidi, S. Javadi, S. Khorasani, Solving system of nonlinear equations by restarted adomain's method, Appl. Math. Sci 6 (11) (2012) 509–516.

URL http://m-hikari.com/ams/ams-2012/ams-9-12-2012/vahidiAMS9-12-2012-4.pdf

[9] A.-M. Wazwaz, A new algorithm for calculating adomian polynomials for nonlinear operators, Applied Mathematics and computation 111 (1) 33–51.

23

URL 10.1016/S0096-3003(99)00063-6

[10] H.-W. Choi, J.-G. Shin, Symbolic implementation of the algorithm for calculating adomian polynomials, Applied mathematics and computation 146 (1) 257–271.

URL https://doi.org/10.1016/S0096-3003(02)00541-6

[11] J.-S. Duan, Convenient analytic recurrence algorithms for the adomian polynomials, Applied Mathematics and Computation 217 (13) 6337–6348.

URL https://doi.org/10.1016/j.amc.2011.01.007

[12] J.-S. Duan, R. Rach, A.-M. Wazwaz, Oxygen and carbon substrate concentrations in microbial floc particles by the adomian decomposition method, MATCH Commun. Math. Comput. Chem 73 (2015) 785–796.

URL https://match.pmf.kg.ac.rs/electronic_versions/Match73/n3/match73n3_785-796.pdf

[13] M. Azreg-Aïnou, A developed new algorithm for evaluating adomian polynomials, arXiv preprint arXiv:0906.0244 (2009).

URL https://doi.org/10.48550/arXiv.0906.0244

[14] F. Hendi, H. Bakodah, M. Almazmumy, H. Alzumi, A simple program for solving nonlinear initial value problem using adomian decomposition method, Int J Res Rev Appl Sci 12 (3) (2012) 397–406.

URL https://www.researchgate.net/profile/Ho-Bakodah/publication/284179258_A_simple_program_for_solving_nonlinear_initial_value_problem_using_Adomian_decomposition_method/links/565adfa008ae4988a7ba5f8a/

24

A-simple-program-for-solving-nonlinear-initial-value-problem-using-Adomian-dec

pdf

[15] J. D. Faires, R. L. Burden, Numerical methods., Thomson, 2003.

URL http://repository.vnu.edu.vn/handle/VNU_123/83453

[16] C. Remani, Numerical methods for solving systems of nonlinear equations, Lake-

head University Thunder Bay, Ontario, Canada (2013).

URL   https://www.hlevkin.com/hlevkin/60numalgs/Common/

Remani-Numerical%20Methods%20for%20Solving%20Systems%

20ofNonLinearEquations.pdf

25