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Semigroup with the singularity

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Abstract: We present some results on semigroup with left singular and right singular. We proved theorems for a semigroup S to be regular, normal, left normal, leftsemi-normal, right semi-regular, left regular and left quasi-normal.

Prelimanaries:

1. Definition: A semigroup (S, .) is said to be left(right) singular if it satisfies the Identity ab = a (ab = b) for all a,b in S

2. Definition: A semigroup (S, .) is rectangular if it satisfies the identity aba = a for all a,b in S.

3. Definition: A semigroup (S, .) is called left(right) regular if it satisfies the identity aba = ab (aba = ba) for all a,b in S.

4. Definition: A semigroup (S, .) is called regular if it satisfies the identity abca = abaca for all a,b,c in S

5. Definition: A semigroup (S, .) is said to be left(right) normal if abc = acb (abc = bac) for all a,b,c in S.

6. Definition: A semigroup (S, .) is said to be normal if satisfies the identity abca = acba for all a,b,c in S.

7. Definition: A semigroup (S, .) is said to be left(right) quasi-normal if it satisfies the identity abc = acbc (abc = abac) for all a,b,c in S.

8. Definition: A semigroup (S, .) is said to be left (right) semi-normal if it satisfies the identity abca = acbca (abca =abcba) for alla,b,c in S.

9. Definition: A semigroup (S, .) is said to be left(right) semi-regular if it satisfies the identity abca =abacabca (abca = abcabaca) if for all a,b,c in S.

10. Theorem: A left singular semigroup S is normal if and only if it is regular.

Proof: Let S be a left singular semigroup then ab = a for all a,b$\in$S.

Assume that S is normal. Thenabca = (a)cba = abcba = a(b)cba= abacba= aba(cb)a = abaca $\Rightarrow$ abca = abaca. $\Rightarrow$ S is regular.

Conversely, let S be a left singular regular semigroup thenabca = abaca = (ab)aca= aaca = (aa)ca = aca = a(c)a= acba $\Rightarrow$ abca = acba $\Rightarrow$ S is normal.

11. Theorem: A left singular semigroup S left (right) regular if and only if it is regular.

Proof: Let S be a left singular semigroup. Assume that S be a left regular then aba = ab $\Rightarrow$ ab(a) = a(b) $\Rightarrow$ abac = abc $\Rightarrow$ aba(c) = ab(c) $\Rightarrow$ abaca = abca $\Rightarrow$ S is regular.

Conversely, let S be a regular then abaca = abca $\Rightarrow$ aba(ca) = ab(ca) $\Rightarrow$ abac = abc $\Rightarrow$ ab(ac) = a(bc) $\Rightarrow$ aba = ab. Hence S is left regular.

12. Theorem: A left singular semigroup is left(right) semi- normal if and only if it is right(left) semi-normal.

Proof: Let S be a left singular semigroup.Assume that S be left semi-normal then abca = acbca = ac(bc)a= acba = (a)cba= abcba $\Rightarrow$ abca = abcba $\Rightarrow$ S is right semi-normal.

Conversely, assume that S is right semi-normal abca = abcba= (ab)cba = acba = ac(b)a = acbca $\Rightarrow$ abca = acbca. Hence S is left semi-normal.

13. Theorem: A left singular semigroup S is left (right) semi-normal if and only if it is regular.

Proof: Let S be a left singular semigroup. Let S be left semi-normal then abca = acbca = ac(bc)a = acba= (a)cba= abcba = a(b)cba= abacba= aba(cb) a = abaca $\Rightarrow$ abca = abaca. Hence S is regular.

Conversely, let S be regular thenabca = abaca = (ab)aca= aaca = (aa)ca = aca = a(c)a = acba = ac(b)a= acbca $\Rightarrow$ abca = acbca. $\therefore$ S is left semi-normal.

14. Lemma: A left singular semigroup S is left(right) semi-normal if and only if it is normal.

Proof: Let S be a left singular semigroup. Let a,b,c$\in$S.Then, S is left semi-normal $\Leftrightarrow$abca = acbca$\Leftrightarrow$abca = ac(bc)a$\Leftrightarrow$abca = acba$\Leftrightarrow$ S is normal.

15. Theorem: Let S be a left singular semigroup then S is left(right) normal if and only if it is normal

Proof: Let S be a left singular semigroup. Assume that S is left normal then abc = acb $\Rightarrow$ ab(c) = acb $\Rightarrow$ abca = acb $\Rightarrow$ abca = ac(b) = acba $\Rightarrow$ abca = acba

$\therefore$ S is normal.

Conversely, let Sbe normal $\Rightarrow$ abca = acba $\Rightarrow$ ab(ca) = ac(ba) $\Rightarrow$ abc = acb (ca = c and ba = b). Therefore S is left normal.

16. Theorem: A left singular semigroup S is left(right) semi-regular if and only if it is right(left) semi regular

Proof: Let S be a left singular semigroup. Assume that S be left semi-regular then

abca = abacabca= a(ba)cabca= abcabca= abca(b)ca = abcabaca $\Rightarrow$ abca = abcabaca.

$\Rightarrow$ S is right semi-regular.

Conversely, let Sbe right semi-regular then

we have abca = abcabaca $\Rightarrow$ abca = a(b)cabaca = abacabaca = abaca(ba)ca= abacabca $\Rightarrow$ abca = abacabca $\Rightarrow$ S is left semi regular.

17. Theorem: A left singular semigroup S is left(right) semi-regular if and only if it is normal.

Proof: Let S be a left singular semigroup. Assume that S be left semi-regular then

abca = abacabca = (ab)acabca = aacabca = (aa)cabca= acabca= a(ca)bca = acbca = ac(bc)a = acba $\Rightarrow$ abca = acba $\Rightarrow$ S is normal.

Conversely, let S be normal then abca = acba = (a)cba= abcba = a(b)cba= abacba = aba(c)ba = abacaba= abaca(b)a = abacabca $\Rightarrow$ abca = abacabca. $\Rightarrow$ S is left semi-regular.

18. Theorem: A left singular semigroup is left(right) semi-regular if and only if it is left(right) semi-normal

Proof: Let S be a left singular semigroup. Assume that S be a left semi-regular semigroup then, we haveabca = abacabca = (ab)acabca = aacabca = (aa)cabca = acabca = a(ca)bca = acbca $\Rightarrow$ abca = acbca $\Rightarrow$ S is left semi-normal.

Conversely, let S be left semi-normal thenabca = acbca= (a)cbca = abcbca = a(b)cbca = abacbca = aba(c)bca = abacabca $\Rightarrow$ abca = abacabca $\Rightarrow$ S is left semi-regular.

19. Theorem: Let S be a left singular semigroup property then S is left(right) quasi-normal if and only if it is normal

Proof: Let S be a left singular semigroup. Assume that S be left quasi-normal then

abc = acbc $\Rightarrow$ ab(c) = acb(c) $\Rightarrow$ abca = acbca = ac(bc)a = acba $\Rightarrow$ abca = acba $\Rightarrow$
S is normal.

Conversely, let S be normal then abca = acba $\Rightarrow$ ab(ca) = acba $\Rightarrow$ abc = ac(ba) $\Rightarrow$ abc = acb $\Rightarrow$ abc = ac(b)= acbc $\Rightarrow$ abc = acbc $\Rightarrow$ S is left quasi-normal.

20. Theorem: A left singular semigroup S is left(right) quasi-normal if and only if it is left(right) semi-regular.

Proof: Let S be a left singular semigroup. Assume that S be left quasi-normal then abc = acbc $\Rightarrow$ ab(c) = acb(c) $\Rightarrow$ abca = acbca $\Rightarrow$ abca = (a)cbca= abcbca = a(b)cbca= abacbca = aba(c)bca = abacabca $\Rightarrow$ abca = abacabca $\Rightarrow$ S is left semi-regular.

Conversely, let S be left semi-regular thenabca = abacabca $\Rightarrow$ ab(ca) = abacab(ca) $\Rightarrow$ abc = abacabc= (ab)acabc = aacabc = (aa)cabc= acabc = a(ca)bc = acbc $\Rightarrow$ abc = acbc $\Rightarrow$ S is left quasi-normal.

21. Note: Similarly, we can prove that,

a) aleft singular semigroup S is left(right) semi-regular if and only if it is right(left) semi-normal.

b) a left singular semigroup S is left(right) quasi-normal if and only if it is any one of the following:

(1) Regular.

(2) Left(right semi-normal.

(3) Left(right) semi-regular.

(4) Left(right) regular.

(5) Left(right) normal.

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