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Study on Nonlinear Partial Differential Equation by Implementing MSE Method

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Abstract

This article reflects the exact traveling wave solutions for investigating nonlinear partial Klien-Gordon differential equations by implementing the modified simple equation method. The proposed algorithm has been successfully tested herewith via the hyperbolic and trigonometric function solutions taking some arbitrary parameters. When the parameters are taken as special values, the exact solutions will demonstrate the different types of traveling wave. These waves are widely used in the field of nonlinear complex physical phenomena like as: plasma physics, solid state physics, particle physics etc. The proposed method is highly efficient and a fruitful mathematical scheme with a view to bring out solitary wave solutions of miscellaneous nonlinear evolution equations.

Keywords: The nonlinear partial Klein-Gordon differential equation; modified simple equation method; traveling wave solutions; soliton.

Introduction

The nonlinear evolution equations (NLEEs) are the most important affairs in order to its wide-ranging applications. Most of the real-world phenomena can be modeled by nonlinear partial differential equations. In modern science, the nonlinear wave phenomena are one of the most attractive fields of research. It occurs in numerous branches of science and engineering, such as: fluid mechanics, plasma physics, solid state physics, optical fibers, signal processing, mechanical engineering, gas dynamics, elasticity, electric control theory, relativity, chemical reactions, ecology, biomechanics etc. The NLEEs have frequent appearances for interpretation of the motion of isolated waves. As the availability of solitary wave in natural science is expanding day by day, it is important to seek for exact travelling wave solutions. The exact traveling wave solutions provide us information about the structure of complex physical phenomena. Therefore, exploration of exact traveling wave solutions to NLEEs reduces into an indispensable work in the study of nonlinear physical phenomena. It is notable to observe that there is no unique method to solve all kinds of NLEEs. As a result, a lot of powerful techniques have been developed to obtain exact solutions of nonlinear physical models, like as, the modified simple equation method [1-4], the (G'/G)-expansion method [5-8], the Jacobi elliptic function method [9], the homotopy perturbation method [10-12], the variational method [13], the Exp-function method [14, 15], the asymptotic method [16], the tanh-function method [17], the F-expansion method [18, 19], the ansatz method [20, 21], the perturbation method [22, 23], the Lie symmetry method [24], the method of integrability [25], the homotopy analysis method [26] etc.

Fordy and Gibbons [32] investigated upon the integrable nonlinear Klien-Gordon equations and toda lattices. On the other hand, Dehghan & Shokri [33] determined the numerical solution of the nonlinear Klein-Gordon equation using radial basis functions. Recently, Zin *et al*. [34] established a new trigonometric spine approach to numerical solution of generalized nonlinear Klien-Gordon equation.

The modified simple equation method is effectively used to investigate exact traveling wave solutions of the nonlinear partial Klien-Gordon differential equations by means of hyperbolic and trigonometric function solutions. This method is simple, direct and constructive to find out exact solutions and solitary wave solutions without help of computer algebraic system.

The objective of this work is to find out exact solitary wave solutions of the nonlinear partial Klein-Gordon differential equation by using modified simple equation method. To the best of our knowledge the MSE method has not yet been applied to the above mentioned equation in previous research. It is the fairness and individuality of this work.

The article is oriented as follows: In section 2, we describe the algorithm of the modified simple equation method. In section 3, the method is applied to the nonlinear partial Klein-Gordon differential equation. The physical explanation and graphical representations of the attained solutions are speculated in section 4. Finally, in section 5, we have drawn our conclusions.

3.1 Algorithm of the Modified Simple Equation Method

Let us consider, the nonlinear partial differential equation for $u(x,\ t)$ is in the form

$$F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots\dots) = 0, \tag{1}$$

where, F is a polynomial in $u(x,\ t)$ and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved. For investigating the exact solitary wave solutions of the equations, we have to maintain the following fundamental steps:

Step1: We choose the traveling wave variable

$$u(x,t) = u(\eta), \eta = x - \omega t, \tag{2}$$

where, ω is the speed of the traveling wave. The wave variable (2) permits us to convert Eq. (1) into an ordinary differential equation (ODE) for $u = u(\eta)$:

$$G(u, u', u'', u''', \dots) = 0, \tag{3}$$

where G is a function of $u(\eta)$ and its derivatives wherein prime indicates the derivative with respect to η.

Step 2: Let us assume that the formation of the solution of Eq. (3) is of the form

$$u(\eta) = \sum_{i=0}^{n} A_i \left[\frac{\psi'(\eta)}{\psi(\eta)}\right]^i, \tag{4}$$

where A_i $(i = 0,1,2,3, \dots)$ are undetermined constants to be calculated, such that $A_i \neq 0$, and $\psi(\eta)$ is an unknown function to be determined later. In the MSE method, ψ is not previously known or not a solution of any known equation, wherever, in the Exp-function method, Jacobi elliptic function method, (G'/G)-expansion method etc., the solutions are presented with respect to some pre-defined functions. Therefore, it is impossible to realize before what kind of solutions are may found by through this method. This is the individuality and fairness of this method.

Step 3: The positive integer n in Eq. (4) can be estimated by taking into account the homogeneous balance between the highest order derivative and the nonlinear terms of the highest order take place in Eq. (3).

Step 4: Substituting Eq. (4) into (3) and calculating the necessary derivatives $u', u'', u''', \dots$ *so on*, we interpret the function $\psi(\eta)$. In addition to, we have a polynomial in $(\psi'(\eta)/\psi(\eta))$ and its derivatives. Equating the coefficients of like power of this polynomial to zero, we obtain a determined set of equations which can be solved for finding A_i $(i = 0, 1, 2, 3, \dots)$ and $\psi(\eta)$. This will fulfill the determination of the solution of Eq. (1).

4.1 Applications of the MSE method

The nonlinear Klein-Gordon differential equation plays an important role in the nonlinear nuclear and particle physics over the decades. In this sub-section, we apply the Modified Simple Equation (MSE) method to solve the nonlinear Klein-Gordon differential equation. Let us consider the nonlinear Klein-Gordon equation is in the from

$$u_{tt} - u_{xx} - au - cu^3 = 0, \tag{5}$$

where a and u are the real valued constants. Using the traveling wave variable $\eta = x - \omega t$, the Eq. (5) is transformed into the following Ordinary Differential Equation (ODE) for $u = u(\eta)$:

$$(\omega^2 - 1)u'' - au - cu^3 = 0. \tag{6}$$

Balancing the highest–order derivative u'' and the nonlinear term of the highest order u^3, we obtain $N = 1$. Therefore, the solution of Eq. (5) becomes the following form:

$$u(\eta) = A_0 + A_1\left(\frac{\psi'}{\psi}\right),$$

(7)

where A_0 and $A_1$are arbitrary constants such that $A_1 \neq 0$ and $\psi(\eta)$ is an unknown function to be determined later.

Substituting Eq. (7) into Eq. (6) yields a polynomial in $\frac{1}{\psi^j}$ $(j = 0, 1, 2, 3, \dots)$ and equating the coefficients of $\psi^0, \psi^{-1}, \psi^{-2}, \psi^{-3}, \psi^{-4}$ to zero yields

$$-aA_0 - cA_0 = 0, \tag{8}$$

$$(k^2-1)A_0\psi''-aA_1\psi'-3cA_0{}^2A_1\psi'=0,$$

(9)

$$-3A_1(k^2-1)\psi'\psi''-3cA_0A_1{}^2\psi'^2=0, \tag{10}$$

$$2A_1(k^2-1)\psi'^3-cA_1^3\psi'^3=0. \tag{11}$$

Solving Eq. (8), we obtain

$$A_0=0,\pm\sqrt{-\frac{a}{c}}, \tag{12}$$

since $A_1\neq 0$, from Eq. (11) we obtain

$$A_1=\pm\sqrt{\frac{2(k^2-1)}{c}}. \tag{13}$$

Solving Eq. (9) and (10), we obtain

$$\psi'=\frac{-c_0(k^2-1)}{cA_0A_1}e^{-l\eta}, \tag{14}$$

where, $l=\frac{a+3cA_0^2}{cA_0A_1}$.

Integrating Eq. (14), we obtain

$$\psi=c_{00}+ne^{-l\eta}, \tag{15}$$

where $n=\frac{c_0(k^2-1)}{a+3cA_0^2}$. Therefore, after substitution the solution of Eq. (5) is,

$$u(\eta)=A_0-\frac{1}{cA_0}\left[\left\{\frac{c_0(k^2-1)e^{-l\eta}}{c_{00}+ne^{-l\eta}}\right\}\right]. \tag{16}$$

If $A_0=0$, the solution of Eq. (16) is undefined. Therefore, we neglect this case.

Substituting the values of A_0, l and n into Eq. (16), we obtain

$$\boldsymbol{u(\eta) = \pm\sqrt{-\frac{a}{c}}\left[1+\frac{1}{a}\left\{\frac{2c_0(k^2-1)e^{\sqrt{-\frac{2a}{k^2-1}}\,\eta}}{2ac_{00}-c_0(k^2-1)e^{\sqrt{-\frac{2a}{k^2-1}}\,\eta}}\right\}\right]} \quad (17)$$

Since $\boldsymbol{c_0}$ and $\boldsymbol{c_{00}}$ are constants of integration, we can randomly choose their values.

If we choose $\boldsymbol{c_0 = \frac{1}{k^2-1}}$ and $\boldsymbol{c_{00} = \frac{1}{2a}}$, then the solution of Eq. (17) becomes,

$$\boldsymbol{u_1(x,t) = \pm\sqrt{-\frac{a}{c}}\ \tanh\left\{\frac{1}{2}\sqrt{\frac{-2a}{k^2-1}}(x-\omega t)\right\}} \quad (18)$$

Again, if we choose $\boldsymbol{c_0 = -\frac{1}{k^2-1}}$ and $\boldsymbol{c_{00} = \frac{1}{2a}}$, then the solution of Eq. (17) becomes,

$$\boldsymbol{u_2(x,t) = \mp\sqrt{-\frac{a}{c}}\ \coth\left\{\frac{1}{2}\sqrt{\frac{-2a}{k^2-1}}(x-\omega t)\right\}} \quad (19)$$

Utilizing hyperbolic function identities, the Eqs. (18) and (19) can be rewritten as

$$\boldsymbol{u_3(x,t) = \mp i\sqrt{-\frac{a}{c}}\ \tan\left\{\frac{1}{2}\sqrt{\frac{-2a}{k^2-1}}(x-\omega t)\right\}} \quad (20)$$

$$\boldsymbol{u_4(x,t) = \pm i\sqrt{-\frac{a}{c}}\ \cot\left\{\frac{1}{2}\sqrt{\frac{-2a}{k^2-1}}(x-\omega t)\right\}}$$

(21)

Remark: The solutions (16) to (21) have been verified by transforming them back into the original equation and found them correct.

4.2 Physical explanations and graphical Representations

In this section, we will discuss the physical explanations and graphical representations of the solutions of the nonlinear partial Klien-Gordon differential equation. By applying MSE method the nonlinear partial Klien-Gordon differential equation affords the traveling wave solutions examined underneath from the Eq. (18)-(21) respectively by means of the symbolic computation software Maple.

The solution $u_1(x,t)$ is shown in the Fig.1 which is the shape of singular kink type traveling wave solution with wave speed $\omega = 2, c = 1, k = 2, a = -4$ within the interval $-10 \le x, t \le 10$. On the other hand, the Fig. 2 is found from the solution $u_2(x,t)$ represents the shape of periodic wave soliton solution with wave speed $\omega = 2, c = 1, k = 5, a = -3$ within the interval same interval. Further, the solution $u_3(x,t)$ is indicated by the Fig. 3 which exhibits the shape of double kink wave soliton solution with wave speed $\omega = 5, c = -4, k = 3, a = -2$ within the interval $-10 \le x, t \le 10$. Finally, the solution $u_4(x,t)$ is represented by Fig. 4 and it extracts the shape of periodic wave soliton solution with $\omega = 7, c = -3, k = 4, a = -5$ within the unaltered range $-10 \le x, t \le 10$.

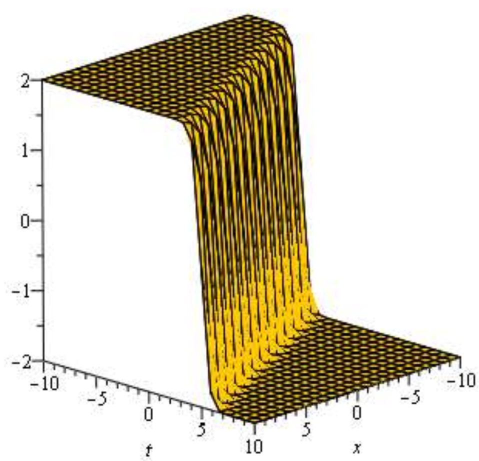

Figure 01. Singular kink wave soliton solution for $\boldsymbol{u_1(x,t)}$.

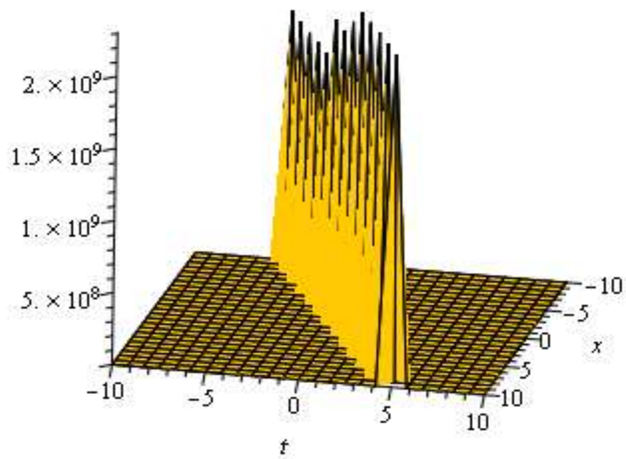

Figure 02. Periodic wave soliton solution for $\boldsymbol{u_2(x,t)}$.

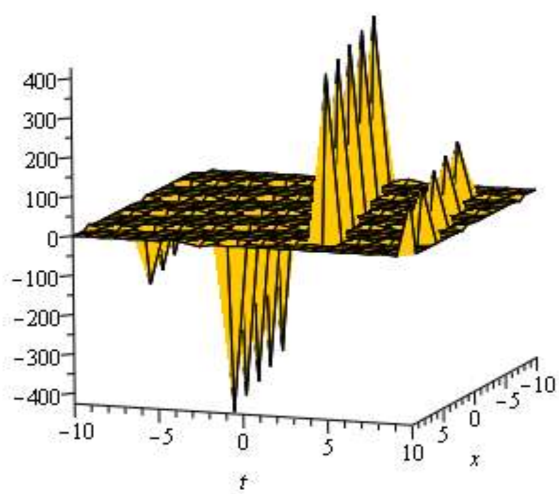

Figure 03. Double kink wave soliton solution for $\boldsymbol{u_3(x,t)}$.

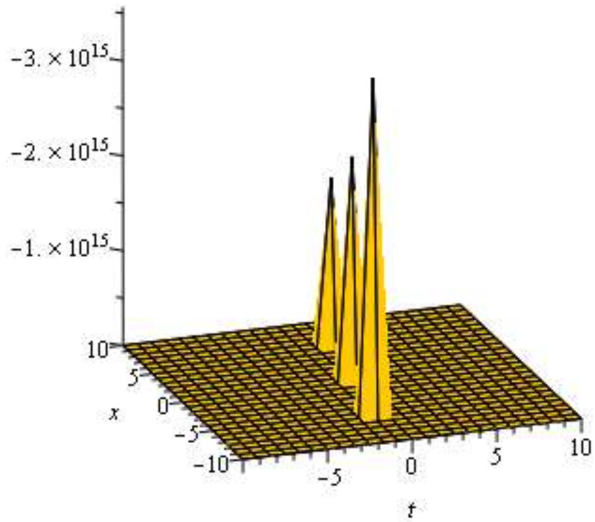

Figure 04. Periodic wave soliton solution for $\boldsymbol{u_4(x,t)}$.

Conclusion

In this paper, the MSE method has been implemented to find out the exact traveling wave solutions and then solitary wave solutions of the nonlinear partial Klien-Gordon differential equation. It is important to note that, the MSE method is easier and simpler than the other currently proposed methods. It has wide-ranging applications in the field of nonlinear complex phenomena, plasma physics, solid state physics, particle physics etc. Here, we have achieved the value of the coefficients without using any symbolic computation software such as Maple, Mathematica etc. It is happened here because this method is very easy, concise and straightforward. Also, it is quite capable and almost well suited for finding exact solutions of other nonlinear evolution equations in mathematical physics.

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