



Study the Effect of Climate Change in Nigeria Using Mathematical Models

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ABSTRACT

The study work focused on the effect of climate change in Nigeria using mathematical modeling. Climate change is a global problem that different field of studies are trying to handle it, so mathematician use many models and techniques such as "The Green House Gas Model, Energy Balance Equation, Ice Sheet Models " etc in order to find the best methods to describe and solve these problems, to reduce its harm to human being and all other living things, also providing scientific information towards decision maker in sequence to protect our Planet.

Finally, the results obtained in this project help us to see that from the last few decades the global temperature has increased far above its normal rise based on the increase of human activities and industrial release of excess CO₂. Graphs are also displayed to show average rainfall, minimum temperature, maximum temperature and solar radiation. The graphs gave a clear view of the rise and fall of the temperature and solar radiation depending on season and monthly rainfall in a year.

KEY WORD: Climate Model, Climate Change, Energy Balance Equation, Bifurcation Equation, Climate System, Rainfall.

1.1 INTRODUCTION

The history of the scientific discovery of climate change began in the early 19th century when ice age and other natural changes in the pale climate were first suspected and the natural greenhouse effect first identified. In the late 19th century, scientists first argued that emissions of greenhouse gases could change the climate. Many other theories of climate change were advanced, involving forces from volcanism to solar variation. By the 1990s, as a result of improving fidelity of computer models and observational work confirming the milankovitch theory of the ice ages a consensus position formed: greenhouse gases were deeply involved in most climate changes and human-caused emissions were bringing discernible global warming. Since the 1990s, scientific research on climate change has included multiple disciplines and has expanded. Research has brought our understanding of causal relations, links with historic data and ability to model climate change numerically.

Until around the early 20th century, climate scientists were primarily concerned with the study of past climatic states. This was done by observation of the environment using mostly geological, geographical and botanical methods. By the end of the 1950s, important physical measurement methods were developed. The measurement of weak radioactivity of various isotopes was the basis for the dating of organic material and enabled the determination of flux rates in different environmental systems.

Length scales involved range from planetary for the largest scale motions of the oceans and atmosphere to microscopic scales of such important processes as cloud condensation or dissipation of turbulent energy. Time scales range from hundreds of thousands of years for the cyclic variations of Earth's orbit around the sun to seconds a wave takes to break on the beach. Exchanges of energy, momentum and matter, such as water and carbon, connect the components of the climate system and lead to interactions between them.

The study of climate and weather change in our society and the world at large has been a major field of research for many scientists and researchers over the last century because of its major role in the lives of humans and how it can affect the nature of living, spread of diseases (air borne diseases), production of farm product and many more. This research is going to look into some of the effects and give appropriate solutions (using growth and decay model) to those problems.

1.2 STATEMENT OF THE PROBLEM

Climate change and the release of excess CO₂ leads to global warming, change in the weather and season, is bringing about low or high yield of farm products. Unstable weather and climate conditions can lead to different health challenges in the society, so also national security depends on the stability of the climate change. Consequently, if the climate and weather were to suddenly change, disaster response team may not adapt quickly enough.

1.3 RESEARCH QUESTION

- i. Does the public have adequate knowledge about the dangers of global warming to their society?
- ii. Can climate and weather change cause low or high farm yield?
- iii. Are there other means of reducing the global warming to barest minimal?
- iv. nimal?
- v. Are the preventive measures of global warming affordable by a medium or low income earner?

1.4 OBJECTIVE OF THE STUDY

To bring public awareness on the dangers of excessive use of gases such as CO₂ that leads to global warming and the depletion of the ozone layer. To advice farmer on how to observe the weather before planting, which will increasing their farm yield.

1.5 SIGNIFICANCE OF THE STUDY

The research will provide possible way out, of reducing global warming to our immediate surrounding by informing people about the use of charcoal stoves, gas cookers, electric cooker e.t.c, which emits less CO₂ into the atmosphere. The research will also try to provide appropriate advice to farmers on how to observe the weather during the rainy season, before embarking on their farming activities to provide high yield of crops.

1.6 RESEARCH HYPOTHESIS

It is evident that at the end this research we will find out that the effect of climate, weather and global warming on our environment, is vast spreading to almost every part of the world, thereby affecting our food production, lifestyle, spread of diseases and so on

1.7 RESEARCH METHODOLOGY

Here, the research is going to be based on experimental method of computing data to give results. We are going to use some models such as the energy balance equation, budykos model, ice sheet model as shown below;

$$R \frac{dT}{dt} = Q(1 - \alpha) - \sigma T^4 \text{ Energy balance equation} \quad (1.1)$$

$$R \frac{dT}{dt} = Q(1 - \alpha) - (A - BT) \text{ Ice sheet coupled model} \quad (1.2)$$

$$T^* = \left(\frac{Q(1-\alpha)}{\sigma} \right)^{1/4} \text{ Bifurcation} \quad (1.3)$$

to calculate the data collected from Nigerian Meteorological Agency (NiMet) on the chart of rainfall and climate change in Nigeria. We will also display our graphs to see the impact of the climate change on our society.

1.8 SCOPE AND FRAME WORK OF THE RESEARCH

This research is done as an undergraduate project and it is limited to an approximation of four/five months. The research is going to focus on Nigeria's climate and weather change of the last few decades, and wholly depends on the information collected from the Nigerian meteorological Agency (NiMet) to predict and reach conclusions.

Earth's climate system can be separated into four components: the atmosphere, the oceans, the cryosphere (ice) and the land surface / vegetation (Fig. 1.1). Physical processes, such as electromagnetic radiation, fluid motions, and precipitation, interact with biological processes, such as carbon uptake by growing plants, and chemical transformations to form a complex and dynamical system.

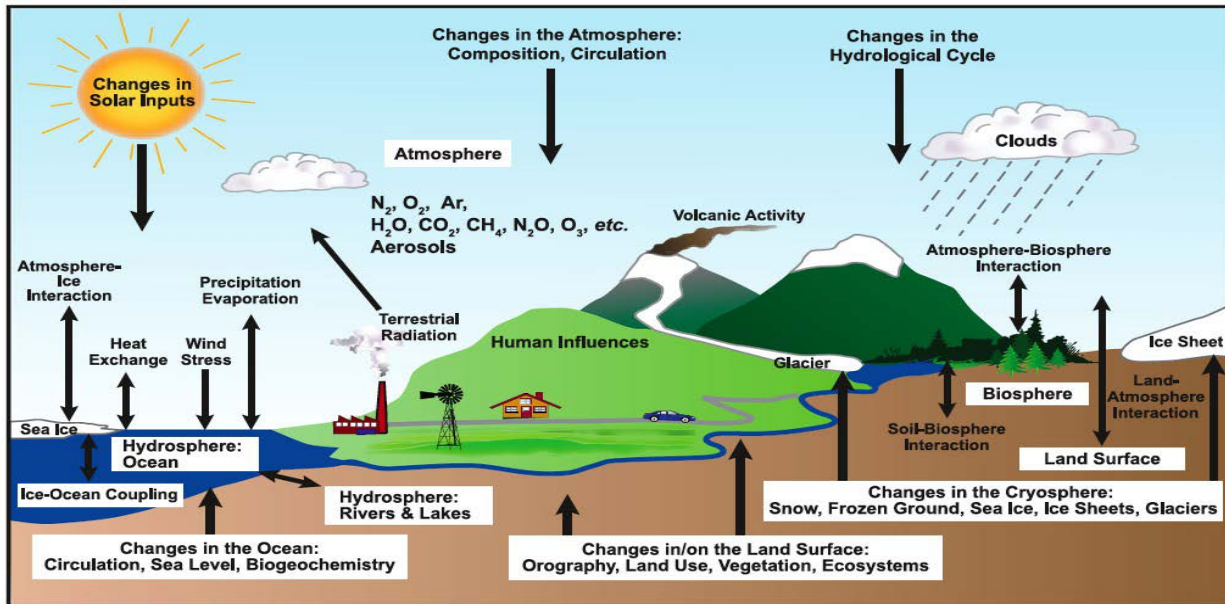


Figure 1.1: Components of the climate system, their interactions and processes. From IPCC (2007).

2.1 DEFINITION OF TERMS

2.1.1 CLIMATE

This is a word from ancient Greek “klima”, meaning inclination. Climate is commonly defined as the weather averaged or the statistics of weather over a long period. The standard averaging period is 30 years, but other period may be used depending on the purpose. It is measured by assessing the patterns of variation in temperature, humidity, atmospheric pressure, wind, precipitation, atmospheric particle count and other meteorological variables in a given region over long period of time.

2.1.2 WEATHER

This is the state of the atmosphere with respect to heat or cold, wetness or dryness, calm or storm, clearness or cloudiness of an environment. This refers to day-to-day temperature and precipitation activity and it is driven by air pressure, temperature and moisture differences between one place and another. On earth, the common weather phenomena include wind, cloud, rain, snow, fog and dust storms while the less common events include natural disasters such as tornadoes, hurricanes, typhoons and ice storms.

2.1.3 CLIMATE MODEL

A climate model is essentially a representation of the many interactions and dynamics within the climate which includes the atmosphere, ocean, land surface, and ice to make predictions of possible climate change for the future. Climate models typically quantitative in nature and range

from simple depictions of the Earth’s climate to very complex ones. Virtually all climate models will take into consideration the incoming visible light and infrared radiation from the sun as well as the outgoing infrared radiation (of longer wavelength) that is sent back into space. Although scientists disagree as to how much exactly temperature will raise, the great majority of scientists do agree that anthropogenic global warming is a reality and a serious issue. For zero-dimensional models, a very simple model of the radioactive equilibrium of the Earth is shown below

Where

- The left hand side represents the incoming energy from the sun
- The right hand side represents the outgoing energy from the earth.

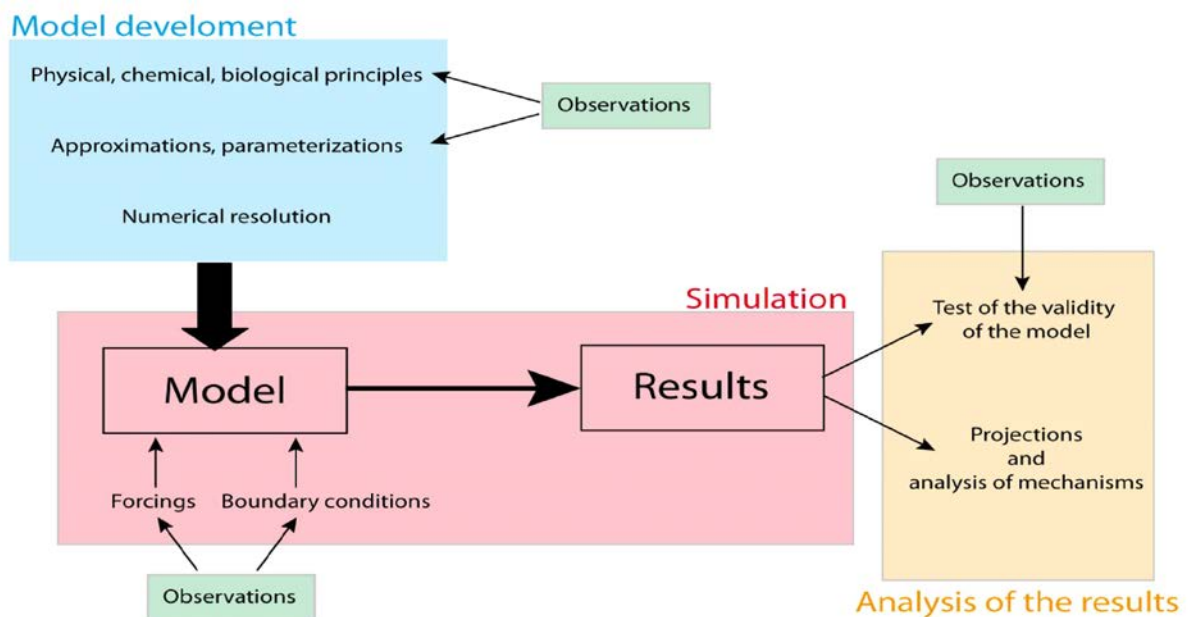


Figure1.2 Model development Goosses et,al 2010

2.1.4 GLOBAL WARMING

Global warming is the increase in the average temperature of the Earth’s near-surface air and the oceans, it can also be defined as a gradual increase in the overall temperature of the earth’s atmosphere generally attributed to the greenhouse effect caused by increased levels of carbon dioxide CO₂, chlorofluorocarbons CFCs, and other pollutants. Climate scientists have since the mid-20th century gathered detailed observations of various weather phenomena (such as temperature, precipitation, and storms) and of related influences on climate. These data indicate that Earth’s climate has changed over almost every conceivable timescale since the beginning of

geologic time and that the influence of human activities since at least the beginning of the industrial revolution has been deeply woven into the very fabric of climate change. Global warming is related to the more general phenomenon of climate change, which refers to changes in the totality of attributes that define climate. In addition to changes in air temperature, climate change involves changes to precipitation patterns, winds, ocean currents. Normally, climate change can be viewed as the combination of various natural forces occurring over diverse timescale.

2.1.5 GREENHOUSE GASES

Greenhouse gases are thought to be the main contribution to climate change (The greenhouse effect). They are very efficient in trapping heat into the atmosphere; therefore, it results in the greenhouse effect.

The solar energy is absorbed by the earth's surface and then reflected back to the atmosphere as heat. Then as the heat goes out to space, greenhouse gases absorb a part of the heat. After that, they radiate the heat back to the earth's surface, to another greenhouse gas molecule, or to space (The Green Effect). Daniela Burghila et al. stated in "climate change effect- where to next", the biggest concern scientists have is about the emission of CO₂ since it is about 75% of the total global emission of greenhouse gases..

The major atmospheric constituents, Nitrogen (N₂), Oxygen (O₂), and Argon (Ar) are not greenhouse gases because molecules containing two atoms of the same element such as N₂ and O₂ have no net change in the distribution of their electrical charges when they vibrate, and monatomic gases such as Ar do not have vibrational modes. Hence they are almost totally unaffected by infrared radiation.

The major greenhouse gases in the earth's atmosphere are:

- Water vapour (H₂O)
- Carbon dioxide (CO₂)
- Methane (CH₄)
- Nitrous oxide (N₂O)
- Ozone (O₃)
- Chlorofluorocarbons (CFCs)
- Hydrofluorocarbons (HFCs)

2.1.6 ATMOSPHERE

An Atmosphere (from ancient greek (atmos), meaning 'vapour', and (sphaira), meaning 'ball' or 'sphere'). The atmosphere is a layers of gases surrounding a planet or other material body, that is held in place by the gravity of that body. An atmosphere is more likely to be retained if the gravity it is subject to is high and the temperature of the atmosphere is low. The atmosphere of Earth is composed of nitrogen (about 78%), oxygen (about 21%), argon (about 0.9%), carbon dioxide (0.04%) and other gases in trace amounts.

2.1.7 HYDROSPHERE

The hydrosphere is the part of a planet that's made of water and watery layers of the Earth. Oceans, rivers, lakes and clouds are all typically included in the hydrosphere. Saltwater accounts for 97.5% of Earth's water bodies whereas fresh water accounts for only 2.5%. Of this fresh water, 68.9% is in the form of ice and permanent snow cover in the Arctic, the Antarctic, and mountain glaciers; 30.8% is in the form of flesh groundwater; and only 0.3% of the freshwater on Earth is in easily accessible lakes, reservoirs and river systems.

2.1.8 CRYOSPHERE

According to the oxford dictionary, the cryosphere is the part of the Earth's surface where water as ice; the entire region of the natural environment that is below 0c, especially permanently. It is also the portion of Earth's surface where water is in solid form, including sea ice, ice sheets, and frozen ground

2.2 REVIEW OF RELATED WORK

Vilhelm Bjerknes, (1904) proposed that weather prediction could be seen as an initial value problem in mathematics since the equations govern how metrological variables change with time, if we know the initial condition of the atmosphere, we can solve the equations to obtain new values of those variables at a later time. Future values of meteorological variables are solved for by finding their initial values and then adding the physical forcing that acts on the variables over the time period of the forecast. This is stated as

$$A(\text{forecast}) = A(\text{initial}) + F(A)$$

Where F(A) stands for the combination of all of the kind of forcing that can occur.

L.F Richardson, (1922) in his book weather prediction by numerical process showed how the differential equations governing atmospheric motions could be written approximately as a set of algebraic difference equations for values of the tendencies of various field variables at a finite

number of points in space. By extrapolating the computed tendencies ahead a small increment in time, an estimate of the fields at a short time in the future could be obtained even for a short-range forecast over a small area of the earth this procedure requires an enormous number of arithmetic calculations. He tried a “forecast” by hand- unfortunately the results were poor due to poor initial data and the inclusion of fast waves like sound and gravity waves.

J.B Charney (1948), showed how dynamical equations could be simplified using the geostrophic and hydrostatic approximations so that sound and gravity waves were filtered out (especially the quasi-geostrophic model).

Gethner, Robert (1998), A planet’s albedo is the percent of incoming solar radiation that is immediately reflected back into space due to coloring of the planet. The earth’s albedo, denoted by, is 0.3, so 30% of incoming solar radiation is immediately reflected back into space. i.e. And the total amount of incoming solar radiation flux is equal to

According to **Holli Riebeek**, the author of “Global Warming,” nature also contributes to climate change by emitting CO₂ from volcanos.

The Gravity Recovery and Climate Experiment (GRACE) mission, launched in 2002, allows monitoring of changes hydrology and cryosphere with terrestrial and ocean applications. This review article focuses on its contribution to the detection and quantification of climate change signals. Climate change signals begin to manifest slowly on land, sea and even the atmosphere. The ocean mass changes are derived from GRACE Level 2 data of three processing centers using an averaging kernel method and scaling.

Prediction of the weak **El Nino, (2002/2003)** also known as the irregular warming of waters in the tropical Eastern Pacific, strongly affects the tropical climate and in particular the water cycle. The formation of atmospheric pressure and temperature anomalies also causes deviations from the usual climate around the globe. These changes, which may last some months up to around 1.5 years, cause severe economic damage. The prediction evolving was available in August 2002. A moderate increase in SST (sea surface temperature) in the tropical Eastern pacific (right) was expected. It is important to note that the single models differ in their quantitative prediction. Hence, the prediction bears an uncertainty, analoguos to the daily weather forecast in which the occurrence of rain is also given with a probability.

According to **NASA, (as of 2005)** on average, volcanoes emit between 130 and 230 million tons of CO₂ per year. However, by burning fossil fuels, people release in excess of 100 times more, about 26 billion tons of CO₂, into the atmosphere every year.

Don Wuebbles, (2007) a coordinating lead author and contributor to a number of the reports of the International Intergovernmental Panel on Climate Change (IPCC), which was awarded the Nobel Peace Prize in 2007, and a Professor of Atmospheric Sciences at the University of Illinois at Urbana-Champaign, stated, “Volcanos used to release CO₂ many millions years ago. Back where dinosaurs existed, we had levels of CO₂ that is approximately similar to what we have now because of the CO₂ emitted by volcanos. But, volcanos release a small amount of CO₂ and they can’t explain the increase of CO₂ that we had in the last century” (Phone interview). Volcanos do contribute to climate change by emitting CO₂. However, the amount of CO₂ they emit is relatively small if we compare it to the amount of CO₂ that is being released by human activities.

Burghila et al. (2007) stated in “Climate Change Effects- Where to Next?”, that the country’s 2007 drought was the severest in 60 years. By increasing the concentration of the greenhouse gases, we are increasing the amount of heat that is in our atmosphere (NASA). Hurricanes have also become more aggressive largely because of warmer temperatures that mainly resulted from the emission of greenhouse gases. Warmer temperatures result in warmer water in the oceans. As the result of warmer oceans, hurricanes and tornados become more intense.

Wuebbles stated, “Warmer atmosphere result in more energy in the atmosphere. When hurricanes start, they usually pick up energy from the oceans and as the result of warmer water in the oceans because of greenhouse effect, hurricanes have more energy. Therefore, hurricanes become more intense. Now if the water was colder that gives less energy to hurricanes and make it less intense” (phone interview). Also, warmer temperature means the atmosphere holds more water vapor and that makes rainfalls more extreme and intense.

The Mathematical Science Research Institute (MSRI), April 11 to April 13, 2007, convened a symposium, to access how mathematicians can address the broader issues of climate change and the narrower issues of methodology lying behind the climate model. Climate models and their projections for the future- especially extended out to 2100- are subject to a variety of uncertainties. These include imperfections in the climate models, the limitations of our computing power, and the inherently unpredictable nature of nonlinear equations.

“Humans, drivers of national greenhouse-gas emission June 2012” The article shows that human stress on the environment has long been debated and different views about the human drivers of greenhouse-gas emissions have emerge. Now research synthesizes the debate by looking at empirical evidence and offers new insights on the role of human population, affluence, urbanization, trade, culture and institutions on greenhouse-gas emission trend. Centuries of speculation about the causes of human stress on the environment is now being disciplined with empirical evidence, including analyses of differences in greenhouse-gas emissions across contemporary nation states. The cumulative results can provide useful guidance for both climate projections and for policy design. Growing human population and affluence clearly contribute to enhanced environmental stress. Evidence does not support the argument for amelioration of greenhouse-gas emissions at the highest levels of affluence. However, the role of other factors, such as urbanization, trade, culture and institutions remains ambiguous.

In 2013, an article of nature climate change wrote that Earthworms play an essential part in determining the greenhouse-gas balance of soils worldwide, and their influence is expected to grow over the next decades. They are thought to stimulate carbon sequestration in soil aggregates, but also to increase emissions of the main greenhouse gases carbon dioxide and nitrous oxide. Hence, it remains highly controversial whether earthworms predominantly affect soils to act as a net source or sink of greenhouse gases. Here, we provide a quantitative review of the overall effect of earthworms on the soil greenhouse-gas balance. Our results suggest that although earthworms are largely beneficial to soil fertility, they increase net soil greenhouse-gas emissions.

Nigerian Meteorological Agency (NiMet) 2017, gave a prediction of the SRP (seasonal rainfall prediction) which comprises the following components:

- Onset and cessation dates of the growing season
- Length of the growing season
- Seasonal amount of rainfall
- Little dry season and dry spells
- Human thermal stress January-December

At the time of making this forecast in January 2017, NiMet forecasts that the neutral phase is mostly likely to dominate the January to June weather systems across the country. Thereafter, it is expected to give way to a dominant El Nino phase through the end of 2017.

Nigerian Meteorological Agency (NiMet) (2018), predicted the SRP (seasonal rainfall predictions) based on two different phases of ENSO (El-Nino southern Oscillation), starting with a La-Nina (cold) phase in the first quarter and a neutral phase in the second and third quarter. These phases are favorable for moderately high to normal rainfall in the country. The prediction of 2018 rainfall season was expected to start by early march over the southern coastal cities and end in December over the coast. A normal length of season is expected with normal rainfall amounts. That should not present any major threat to agriculture.

2.3 THEOREMS:

2.3.1 Chaos Theory

Climate is chaotic and cannot be predicted Lorenz (1963), in the landmark paper that founded chaos theory, said that because the climate is a mathematically-chaotic object, accurate long term prediction of the future evolution of the climate is not possible “by any method”. One of the defining traits of a chaotic system is ‘sensitive dependence to initial conditions’. This means that even very small change in the state of the system can quickly and radically change the way that the system develops over time. Although it generally not possible to predict a specific future state of a chaotic system, it is still possible to make statistical claims about the behavior of the system as a whole (it is very likely that Oregon’s December 2012 temperatures will be colder than its July 2012 temperatures). There are chaotic components to the climate system, such as El Nino and fluid turbulence, but they all have much less long-term influence than the greenhouse effect.

2.1.2 Milankovic Theory

The Milankovitch theory of climate change states that “as the earth travels through space around the sun cyclical variations in three elements of Earth’s-sun geometry combine to produce variations in the amount of solar energy that reaches Earth (Kaufman 2002)”. These three elements that have cyclic variations are eccentricity, obliquity, and precession.

3. MATHEMATICAL TECHNIQUES

The study is going to be concentrating on the global average temperature and the effect of greenhouse gases which can be modeled using the energy balance equation (EBM)[Kaper and Engler 2013, 16]. We make basic model of the temperature of the Earth by assuming that it receives incoming solar energy but radiates some of it back into space.

3.1.1 GLOBAL AVERAGE TEMPERATURE MODELS

$$R \frac{dT}{dt} = Q(1 - \alpha) - \sigma T^4 \quad \text{Energy balance equation} \quad (3.2.1)$$

$$R \frac{dT}{dt} = Q(1 - \alpha) - \epsilon \sigma T^4 \quad (3.2.2)$$

3.1.2 BUDYKO'S MODEL

This model approach was first suggested in the 1960s by Budyko, who proposed the simple linear model which accounts average cloudiness condition, effect of infrared absorbing gases, variability of water vapour etc.

$$E_{\text{out}}: T \rightarrow E_{\text{out}}(T) = a + bT - (a_1 + b_1 T)n \quad (3.2.3)$$

(Where $a, b, a_1 \& b_1$ are observational data and n is a cloudiness coefficient)

$$E_{\text{out}}: T \rightarrow E_{\text{out}}(T) = A + BT \quad (\text{Where the values of } A \text{ and } B \text{ vary with temperature})$$

$$R \frac{dT}{dt} = Q(1 - \alpha) - (A + BT) \quad \text{Global surface temperature model} \quad (3.2.4)$$

This leads to the energy balance equation

$$(1 - \alpha(T))Q = A + BT \quad (3.2.5)$$

From the equation 3.2.4 when the value of B is negative

$$R \frac{dT}{dt} = Q(1 - \alpha) - (A - BT) \quad \text{Ice sheet coupled model} \quad (3.2.6)$$

3.1.3 BIFURCATION

We know that S_0 where $Q = \frac{1}{4}S_0$ is the solar constant. If the solar constant decreases, the equilibrium states merge and then disappear, leaving only the deep-freeze state. The disappearance and emergence of solutions indicate qualitative changes in the climate system, such changes is as a result of small smooth variation of a parameter of the problem, hence we can say that bifurcation has occurred.

$$R \frac{dT}{dt} = Q(1 - \alpha(T)) - \epsilon \sigma T^4 = F(T) \quad (3.2.7)$$

$$R \frac{dT}{dt} = E_{\text{in}}(T) - E_{\text{out}}(T) \quad (3.2.8)$$

Where $E_{\text{in}} = (1 - \alpha)Q$, $Q = \frac{1}{4}S_0$ and $E_{\text{out}} = \sigma T^4$, $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

$$R \frac{dT}{dt} = (1 - \alpha)Q - \sigma T^4 \quad (3.2.9)$$

Which reduces at equilibrium to the energy balance equation

$$(1 - \alpha)Q = \sigma T^4$$

This equation can be solved for the temperature T ; the solution is

$$T^* = \left(\frac{Q(1-\alpha)}{\sigma}\right)^{1/4} \quad (3.2.10)$$

A typical value for the earth's albedo is $\alpha = 0.30$, $S_0 = 1,368\text{Wm}^{-2}$, so $Q = 342\text{Wm}^{-2}$. Thus the basic model gives the equilibrium temperature $T^* = 254.8\text{K}$

3.1.4 GREENHOUSE EFFECT

We know that the sun and the Earth emits temperatures that differ by an order of magnitude, their spectra barely overlap (measurements can shows that). Due to many chemical properties, greenhouse gases increase the opacity of the atmosphere infrared, hence affecting the quantity E_{out} but not E_{in} therefore leading to increase in temperature.

$$\Delta T = Q(1 - \alpha(T)) - \epsilon\sigma T^4 \quad (3.2.11)$$

$$(1 - \alpha)Q = \epsilon\sigma T^4$$

Dividing both sides with $\epsilon\sigma$

$$\frac{(1-\alpha)Q}{\epsilon\sigma} = T^4$$

Taking the fourth root

$$T^* = \left(\frac{(1-\alpha)Q}{\epsilon\sigma}\right)^{1/4}$$

If we choose $\epsilon = 0.62$, with a 38% reduction we can recover the global mean surface temperature for any year.

3.2 VARIABLE DESCRIPTION AND THEIR UNITS

- T (K, Kelvins) is the average temperature in the Earth's photosphere (upper atmosphere, where the energy balance occurs in the model) (1kelvin = 1°c);
- t (years) is time;
- $R = (\text{W-yr}/\text{m}^2\text{K})$ is the average heat capacity of the Earth/ atmosphere system (heat capacity is the amount of heat required to raise the temperature of an object or substance 1kelvin (= 1°C));
- $Q = (\text{W}/\text{m}^2)$ is the annual global mean incoming solar radiation per square meter of the Earth's surface;
- $\epsilon = \text{ORL emissivity factor}$
- A and $B =$ are empirically determined parameters
- $\alpha =$ is planetary albedo (dimensionless)
- $\sigma = (\text{w}/\text{m}^2\text{K}^4)$ is a constant of proportionality (Stefan-Boltzmann constant)[16]
- $E_{\text{in}} =$ average amount of solar energy reaching one square meter of the Earth's surface per unit time

- E_{out} = average amount of energy emitted by one square meter of the Earth's surface per unit time

3.3 CONSTANT VALUES OF SOME VARIABLES

Values for the parameters are:

$R = 2.912 \text{ w-yr/m}^2\text{K}$ [Ichii et al. 2003]; $Q = 342 \text{ W/m}^2$, $\alpha = 0.30$, $\epsilon = 0.6$ and $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

3.4 DATA COLLECTION AND DISPLAY

All the data's used in this study were collected from the Nigerian meteorological agency (NiMet), and few others from Hans Kaper, Hans Engler mathematics & climate textbook 2013.

Graphs also displayed to show the increase in the amount of greenhouse gases used and others in next chapter as indicated.

Here we have seen different climate models which include; the energy balance equation, ice sheet couple model, bifurcation model, equilibrium solution, and the greenhouse gas model. All of these models will help us calculate the global average temperature of Nigeria as a case study in this research. All the models provided in 3.1 are similar to each other but all of them are important depending on the initial variables available.

It was observed that there was an increase in the of daily amount of rainfall and the extension of rainy season which is usually 8 months (March-October) to 10 months(February-November) this was evident even in some states of the northern part of the country (potiskum L.G.A, Yobe State) for the last few years (2017,2018). The dry season had been observed shorter and hotter as the year's progress. These phenomena are conformity with the consequences of global warming resulting from increased anthropogenic activities. (Tamunoberetonaria et. al. 2013).

4.3 Table of values

Table 1: Shown average monthly temperature anomalies

Month	Rainfall (mm)	Minimum Temperature (°C)	Maximum Temperature (°C)	Minimum Humidity (%)	Maximum humidity (%)	Solar Radiation (mJm ⁻²)	Evaluation (mm)
January	19.06	21.92	32.40	42.55	93.56	10.61	114.80
February	45.75	22.49	33.08	46.43	94.32	11.40	119.85
March	141.43	21.38	32.48	56.82	95.41	11.10	120.32
April	154.9	21.26	28.90	59.63	95.02	11.14	95.15
May	209.81	23.14	31.15	59.48	87.84	11.34	100.33

June	222.39	21.78	25.71	64.70	88.31	8.81	81.93
July	460.22	22.54	29.00	68.15	95.40	8.33	80.53
August	361.7	20.84	29.16	69.46	95.50	7.70	65.84
September	361.44	18.32	28.32	59.90	93.25	4.71	72.21
October	293.98	22.62	30.34	63.13	95.78	5.45	82.76
November	142.71	22..97	31.27	52.37	9528	8.71	87.12
December	22.94	22.23	31.13	48.27	95.04	6.15	92.29
Average	203.03	21.79	30.25	50.82	93.73	8.79	92.76

Table 2: Shown average yearly temperature, humidity and evaporation (1999-2019)

Year	Maximum Temperature (°C)	Minimum Temperature (°C)	Maximum Humidity (%)	Minimum Humidity (%)	Rainfall (mm)	Evaporat ion (mm)	Solar Radiation (mJm-2day-1)
1999	31.48	23.48	94.50	58.97	200.45	97.27	14.24
2000	31.24	23.17	95.17	54.54	193.88	102.24	17.9
2001	30.97	22.37	93.66	56.44	216.06	102.89	15.3
2002	30.85	22.63	95.43	59.22	185.2	96.16	17.9
2003	32.29	22.66	95.21	56.85	205.28	91.72	15.98
2004	30.94	22.25	95.17	54.54	213.8	98.33	15.35
2005	30.67	22.72	91.11	65.53	209.14	94.75	14.24
2006	30.71	22.51	94.82	63.83	184.49	96.82	13.78
2007	30.74	22.2	94.94	61	197.82	93.64	12.9
2008	30.42	21.44	93.35	59.37	211.68	96.41	5.32
2009	31.24	23.17	96.18	58.35	193.88	99.73	-10.8

2010	31.48	23.48	95.17	54.54	187.18	102.24	5.47
2011	30.42	21.44	96.18	58.35	211.68	97.27	15.3
2012	30.74	22.2	93.35	59.37	187.18	99.73	9.25
2013	30.71	22.51	94.94	61	184.49	96.41	5.47
2014	30.67	22.72	94.82	63.83	197.82	93.64	15.35
2015	30.9	22.25	91.11	65.53	213.8	96.82	14.24
2016	32.29	22.66	95.17	54.54	209.14	94.75	13.78
2017	30.85	22.63	95.21	56.85	185.2	98.33	12.9
2018	30.97	22.37	95.43	59.22	205.28	91.72	5.32
2019	32.16	22.50	93.66	56.44	216.06	96.16	-10.8
Average	31.03	22.54	94.50	58.97	200.45	97.27	9.25

The above table will be used to show a pictorial representation of itself. The graphs will also show the effect of global warming, rainfall measure etc and hence drawing conclusions will be easy based on the characteristics of the graphs.

4.4 IMPLIMENTATION

The models developed here will be used to solve few examples as seen below

$$1.R \frac{dT}{dt} = Q(1 - \alpha) - \sigma T^4$$

Data

$R = 2.912 \text{ W-yr/m}^2\text{K}$ [Ichii et al 2003], $Q = 342 \text{ W/m}^2$ [Kaper and Engler 2013],

$\alpha = 0.30$ [Kaper and Engler 2013], $\sigma = 5.67 \times 10^{-8}$, $T = 30.71$

$$\begin{aligned} \frac{dT}{dt} &= \frac{Q(1-\alpha) - \sigma T^4}{R} \\ &= \frac{342(1-0.30) - 5.67 \times 10^{-8} (30.71)^4}{2.912} = \frac{342(0.7) - 0.0543155}{2.912} = \frac{239.4 - 0.0543155}{2.912} = \frac{239.3457}{2.912} = 82.1929 \end{aligned}$$

The above model does not take into consideration the earth's atmosphere (global warming), variation of isolation with changes is Earth's orbital parameter etc.

$$2. R \frac{dT}{dt} = Q(1 - \alpha) - (A + BT) \Rightarrow \frac{dT}{dt} = \left(\frac{Q(1-\alpha)-(A+BT)}{R} \right)$$

(The values parameter T does not affect the other parameters in the model). Hence the value of

$$A = 202 \text{ and } B = 1.90$$

$$\frac{dT}{dt} = \frac{342(1-0.3)-(202+1.90)}{2.912} \Rightarrow \frac{dT}{dt} = \frac{342(0.7)-(203.90)}{2.912} = \frac{35.1}{2.912} \Rightarrow \frac{dT}{dt} = 12.0536 \approx 12^\circ\text{C} \approx 44^\circ\text{F}$$

$\frac{dT}{dt}$ = This temperature is close to 15.4°C (though a bit colder). This makes sense since the model attempts to take into consideration the role our atmosphere plays in determining surface temperature.

$$3. T^* = \left(\frac{Q(1-\alpha)}{\sigma} \right)^{1/4} \text{ Equilibrium temperature}$$

$$T^* = \left(\frac{342(1-0.30)}{5.67 \times 10^{-8}} \right)^{1/4} = \left(\frac{342(0.70)}{5.67 \times 10^{-8}} \right)^{1/4} = \left(\frac{239.4}{5.67 \times 10^{-8}} \right)^{1/4} = (4.22222)^{1/4} = 1.4335 \approx 1^\circ\text{C}$$

Hence the equilibrium temperature is increasing at a very slow pace but yet its impact to the surrounding can noticed with time.

4.5 Graphical Representations

Table 3. Shown the graph of months against rainfall is displayed below

Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall	19.06	45.75	141.43	154.9	209.81	222.39	460.22	361.7	361.44	293.98	142.71	22.94

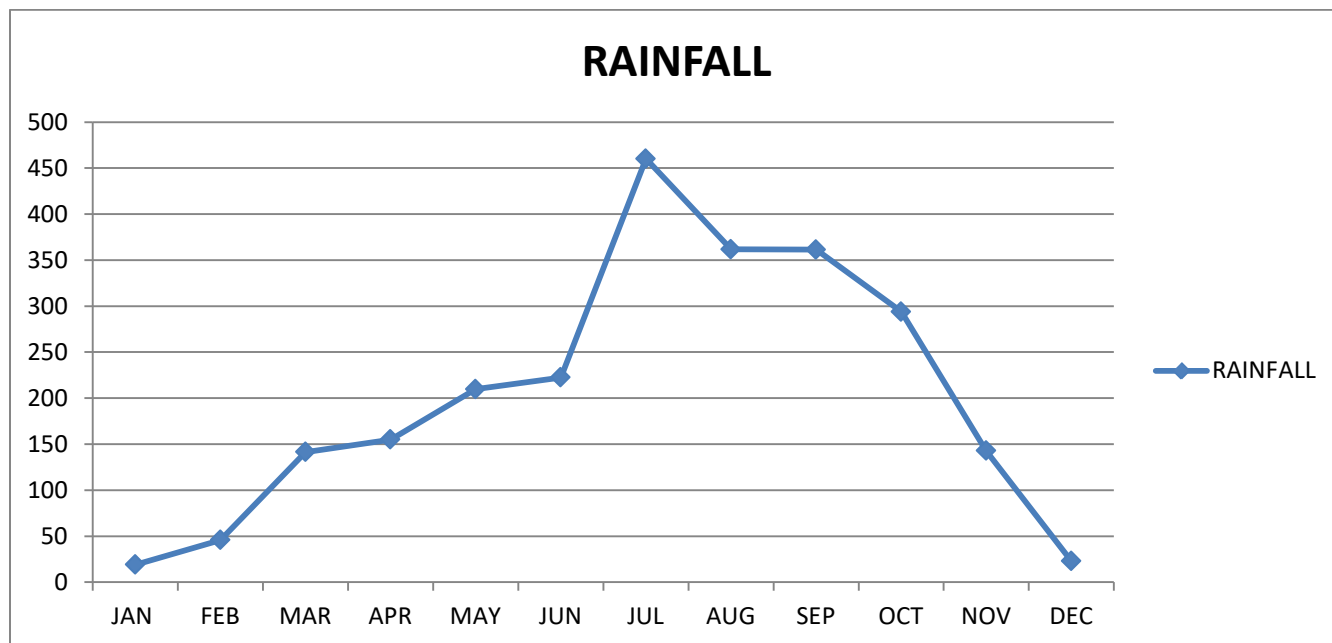


Fig 4g: Graph of Average monthly Rainfall in a year

The above graph shows the amount of rainfall in each month of the year. The figure shows how rainfall varies over time. A control chart is sometimes called shewhart or process behavior chart.

Table 4. Shown the graph of years against maximum temperature is shown below with some few years not shown

Years	1999	2004	2007	2011	2013	2015	2017	2018	2019
Max. Temp	31.5	30.94	30.74	30.42	30.71	30.94	30.85	30.97	32.16

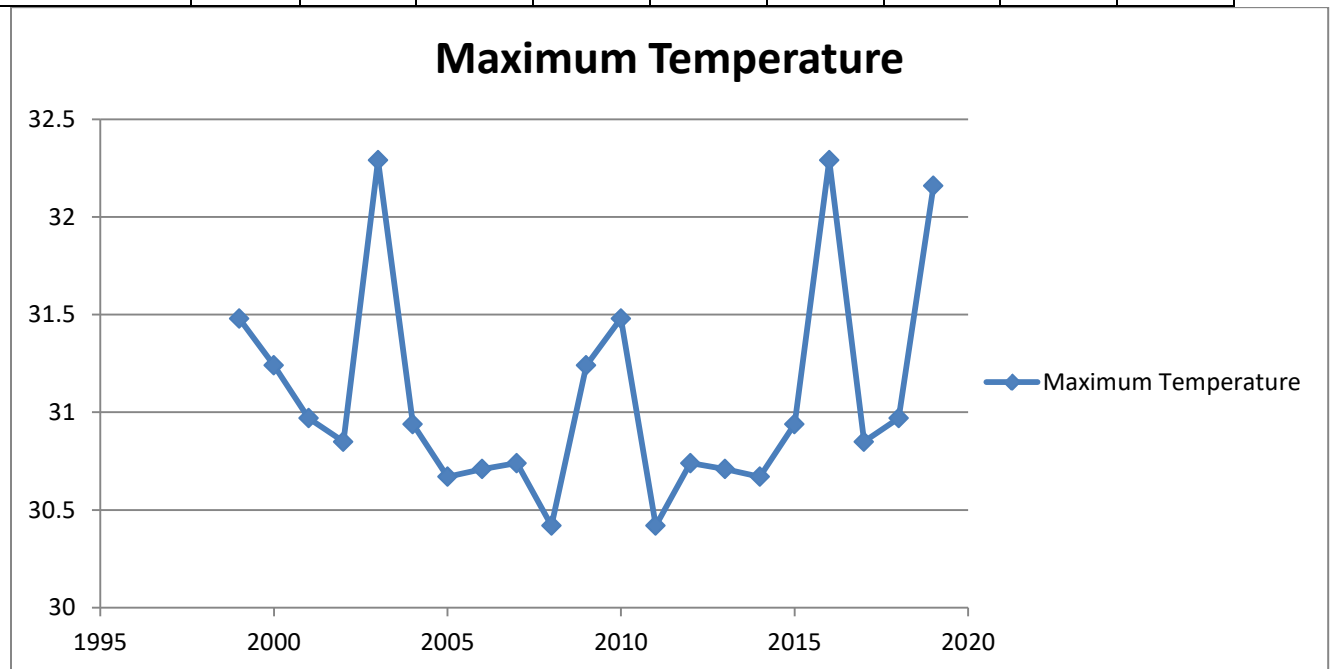


Fig 4h The graph of maximum temperature for the period (1999-2019)

The above graph is showing the average maximum temperature over the nation

From Table 2 the graph of years against minimum temperature is shown below with some few years not shown.

Table 5. Shown the graph of years against minimum temperature

Years	1999	2004	2007	2011	2013	2015	2017	2018	2019
Min. Temp	23.48	22.25	22.2	21.44	22.51	22.25	22.63	22.37	22.50

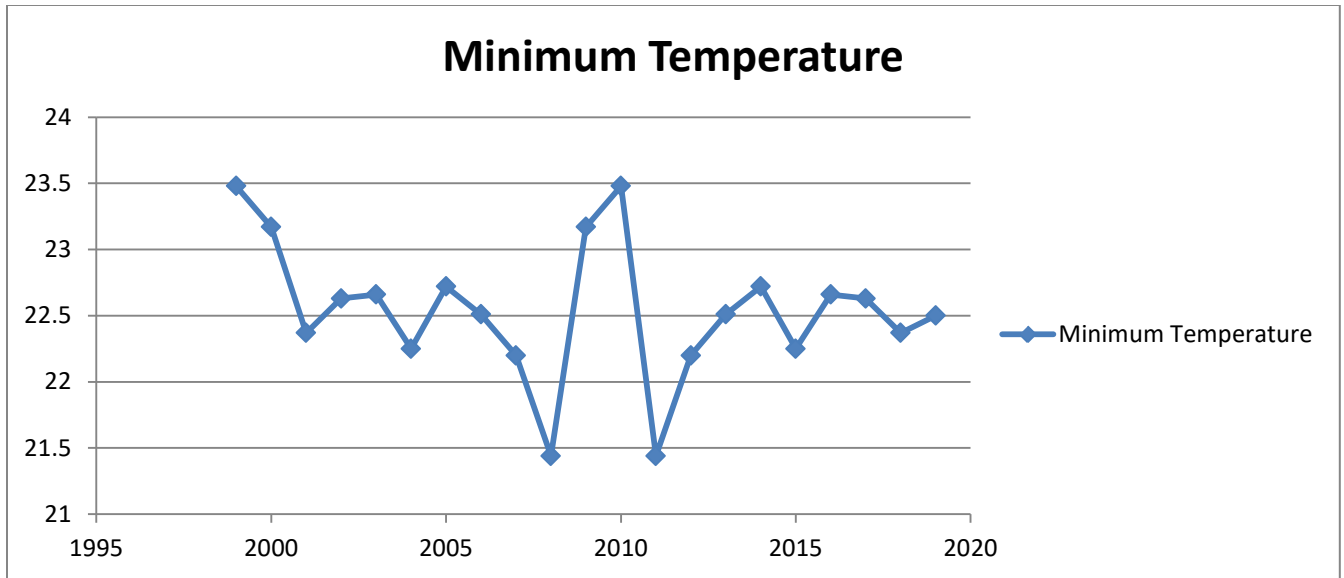


Fig 4: The graph of minimum temperature for the period (1999-2019)

Table 6: The graph of years again minimum temperature

Years	1999	2004	2007	2011	2013	2015	2017	2018	2019
solar rad.	14.24	15.335	12.9	15.3	5.47	14.24	12.9	5.32	-10.8

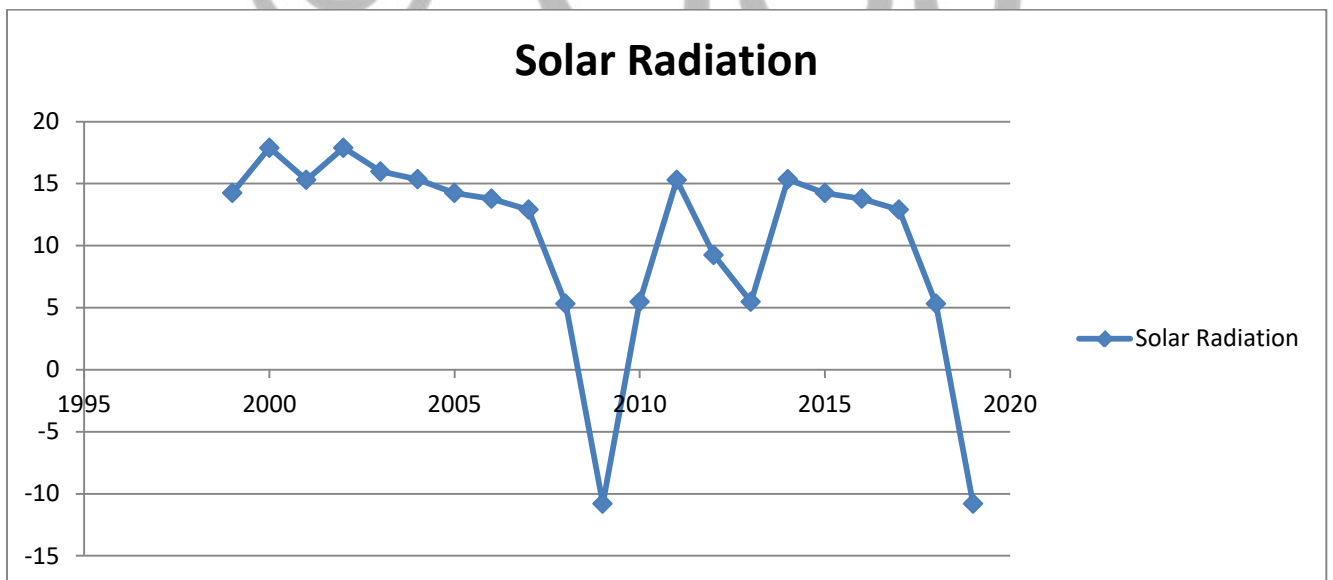


Fig 4j The graph of solar radiation for the period (1999-2019)

4.6 Discussion

Now it clear that the global climatic condition is changing and the Earth is becoming warmer and warmer mostly because of human activities that deals with the release of greenhouse gases. Although the change of the climate is slow, research has shown that from the last 50years the temperature of the Earth has increased to about 1.3°F. This amount may seem to be small but if measures are not taken to enlighten the general public (both rich and poor) to reduces the use machineries that releases much greenhouse gases, the generated heated which is trapped at the bottom of the atmosphere will caused ice melted down and hence increasing the level of the sea and lead to destruction of lives and properties. The continuous release of the greenhouse gases also causes the depletion of the Ozone layer which is also a danger trait to humanity. The change in the climate and weather system is also responsible for the unstable yield of most farm product. Many scientists have suggested different ways of reducing global warming some include: the use of electric cars, less bush burning, use of electric stoves for cooking, eating less meat etc. A medium or low income earner can also use charcoal stove, solar or gas cooker to cook rather than fire wood which releases more CO₂ than the others.

We observe the effect of climate change through the amount of rain, temperatures and the emission of solar radiation which in turn affected agricultural and livestock production.

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