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COLLATZ-SYRACUSE CONJECTURE INVALIDATION

Abstract :
In this document, we study the syracuse sequences of all integers from 1 to infinity. Thus we obtain sequences whose limit at infinity are compared to the values (4,2 and 1) in order to verify the conjecture.

Keywords :
Sequences, number theory, limites, parity, series

References :
There are no references for this paper

INTRODUCTION

Of all the currently unsolved mathematical problems, which one has the most basic statement? This may well be the Syracuse conjecture: accessible to all in its statement, it has challenged researchers for decades.
The 3n + 1 problem is posed in these terms: let us start from any positive integer, and apply the following transformation to it repeatedly (we speak of a trajectory): if this number is even, we divide it by 2, if the number is odd, we multiply it by three then we add 1, so we get another number. Is it true that sooner or later we will end up with 1? All calculations made to date confirm this prediction.
The suite is written as follows.

$$U_{n+1} = \begin{cases} \frac{U_n}{2} & if \quad U_n \ is \ even \\ 3U_n + 1 & if \ U_n \ is \ odd \end{cases}$$

In this paper we are going to prove the Syracuse conjecture is false.

I. DEFINITIONS :

0. Series :

A series q is a sum of 2^n integers.

There are four types of series :

- Heterogeneous series (m) :

  It is a sum of even and odd numbers. It is also an alternation of odd and even numbers.

  $$m = \sum_{i=0}^{2^n - 1} (ai + b)$$

  Where a is odd number and $b \in \mathrm{N}^*$

  Example :

  $$m = \sum_{i=0}^{2^4} (3i + 5) = 5 + 8 + 11 + 14 + ... + 53$$

- Even series (p or t) :

  It's a sum of only even numbers.

  $$p = \sum_{i=0}^{2^n - 1} (ai + b)$$

  Where a and b are even numbers.

  Example :

  $$p = \sum_{i=0}^{2^4} (2i + 8) = 8 + 10 + 12 + 14 + ... + 40$$

- Odd series r :

  It is a sum of only odd numbers.

  $$r = \sum_{i=0}^{2^n - 1} (ai + b)$$

Where a is even and b odd

Example :

$$m = \sum_{i=0}^{2^4} (4i+5) = 5+9+13+17+.....+69$$

- Homogeneous series h :
  It is a sum of one even series t and one odd series r

$$h = t + r$$

1. Line :

A line is a sum of 2^p series where $p \geq 0$
The generic name of any line is Q.
There are four types of lines : homogeneous line (H), odd line (R), even line (P) or (T) and heterogeneous line (M).

- An homogeneous line H is a sum of one even line T and one odd line R.

$$H = T + R = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_k i + b_k)) + \sum_{l=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_l i + b_l))$$

Where $T = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_k i + b_k)) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} t_k)$ and

$$R = \sum_{l=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_l i + b_l)) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} r_k)$$

- An even line P or T is a sum of 2^p even series .

$$P = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_k i + b_k)) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} p_k)$$

- An odd line is a sum of 2^p odd series (r_k).

$$R = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_k i + b_k)) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} r_k)$$

- An heterogeneous line is a sum of 2^p heterogeneous series (m_k).

$$M = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_k i + b_k)) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} m_k)$$

II. FUNCTIONS

1. THE SEPARATION FUNCTION H :

The separation function H also called the to-homogeneous function is a sum of two functions :

H_r the right-separation function and H_l the left-separation function.

If $m = \sum_{i=0}^{2^n-1} (U_i) = \sum_{i=0}^{2^n-1} (ai + b)$ is an heterogeneous series, $H(m)$ gives two series : one odd series and another even series. The results are also both called homogeneous series.

$$H = H(m) = H_l(m) + H_r(m)$$

$$H_l(m) = H_l(\sum_{i=0}^{2^n-1} (ai + b)) = \sum_{i=0}^{2^{n-1}-1} (U_{2i}) = \sum_{i=0}^{2^{n-1}-1} (2ai + b)$$

$$H_r(m) = H_r(\sum_{i=0}^{2^n-1} (ai + b)) = \sum_{i=0}^{2^{n-1}-1} (U_{2i+1}) = \sum_{i=0}^{2^{n-1}-1} (2ai + a + b)$$

So $H = \sum_{i=0}^{2^{n-1}-1} (2ai + b) + \sum_{i=0}^{2^{n-1}-1} (2ai + a + b)$

NB :

If $H_l(m)$ is odd then m is said to be odd-left or even-right heterogeneous series.

If $H_r(m)$ is odd then m is said to be odd-right or even-left heterogeneous series.

<u>Odd-left and odd-right heterogeneous line</u> ? :

If we apply the separation function to an heterogeneous line, we find an odd line and an even line. So if the odd series comes from the left-separation function applied to the hrterogeneous series then this last one is said odd-left heterogeneous, else it's said odd-right. If the even series comes from the left-separation function applied to the heterogeneous series, then this last one is said to be odd-right.

If $M = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^n-1} (a_k i + b_k))$

It can be written as follows

$$M = \sum_{x=1}^{X} \sum_{i=0}^{2^n-1} (a_x i + b_x) + \sum_{y=1}^{Y} \sum_{i=0}^{2^n-1} (a_y i + b_y)$$

Where $X + Y = 2^p$

$\overset{x}{M} = \sum_{x=1}^{X} \sum_{i=0}^{2^n-1} (a_x i + b_x)$ is the sum of odd-left series . It's also called the odd-left heterogeneous line

$\overset{y}{M} = \sum_{y=1}^{Y} \sum_{i=0}^{2^n-1} (a_y i + b_y)$ is the sum of odd-right series or the odd-right heterogeneous line

$$M = \overset{x}{M} + \overset{y}{M}$$

2. The to-Even function E :

The to-Even function E receives in entry an odd series r then results in an even series p .

If $r = \sum_{i=0}^{2^n-1} (U_i) = \sum_{i=0}^{2^n-1} (ai + b)$ is an odd series, we have :

$$E(r) = \sum_{i=0}^{2^n-1}(3U_i+1) = \sum_{i=0}^{2^n-1}(3ai+3b+1)$$

3. **The to-heterogeneous function H_e:**

The to-heterogeneous function H_e transforms an even series p into an heterogeneous series m.

$p = \sum_{i=0}^{2^n-1}(U_i) = \sum_{i=0}^{2^n-1}(ai+b)$ is an even series.

$$H_e(p) = \sum_{i=0}^{2^n-1}(\frac{U_i}{2}) = \sum_{i=0}^{2^n-1}(\frac{a}{2}i+\frac{b}{2})$$

III. Pyramid and blocs :

0. **Bloc B :**

A bloc B is a succession of three lines : it 's composed by one heterogeneous line M_0 followed by an homogeneous line H_0 then an even line P_0 that such $H(M_0) = H_0$, $E(H_0) = P_0$ and $H_e(P_0) = M_1$.
So $B_0 = (M_0, H_0, P_0)$

1. **Pyramid S :**

A pyramid S_n is a succession of n blocs (B_p) that such $H_e(P_p) = M_{p+1}$

The pyramid S_n which begin with the heterogeneous line M_0 is
$S_n(M_0) = (B_0, B_1, B_2, ..., B_{n-1})$.

This pyramid ends with P_{n-1}

1. **Construction of the pyramid** $S_1(\sum_{i=0}^{2^n-1}(i+1))$ **:**

$M_0 = \sum_{i=0}^{2^n-1}(i+1)$ is the first heterogeneous line

- **Determination of** H_0 **the first homogeneous line :**

$$H_0 = H(M_0) = H(\sum_{i=0}^{2^n-1}(i+1)) = H_l(\sum_{i=0}^{2^n-1}(i+1)) + H_r(\sum_{i=0}^{2^n-1}(i+1))$$

$$H_0 = \sum_{i=0}^{2^{n-1}-1}(2i+1) + \sum_{i=0}^{2^{n-1}-1}(2i+2)$$

With $T_0 = \sum_{i=0}^{2^{n-1}-1}(2i+2)$ and $R_0 = \sum_{i=0}^{2^{n-1}-1}(2i+1)$

- **Determination of the even line** P_0

$$P_0 = T_0 + E(R_0) = \sum_{i=0}^{2^{n-1}-1}(2i+2) + E(\sum_{i=0}^{2^{n-1}-1}(2i+1))$$

$$P_0 = \sum_{i=0}^{2^{n-1}-1}(2i+2) + \sum_{i=0}^{2^{n-1}-1}(6i+4)$$

Where $W_0 = \sum_{i=0}^{2^{n-1}-1}(6i+4)$

NB : if An even line comes from the separation function, it's noticed by T
If an even line comes from the to-even function, it's noticed by P
If an even line comes from an odd line, it's noticed by W

- **Determination of the second heterogeneous line** M_1 **:**

$$M_1 = H_e(P_0) = H_e(T_0 + W_0) = H_e(\sum_{i=0}^{2^{n-1}-1}(2i+2) + \sum_{i=0}^{2^{n-1}-1}(6i+4))$$

$$M_1 = \frac{1}{2}((\sum_{i=0}^{2^{n-1}-1}(2i+2) + \sum_{i=0}^{2^{n-1}-1}(6i+4)) = \sum_{i=0}^{2^{n-1}-1}(i+1) + \sum_{i=0}^{2^{n-1}-1}(3i+2)$$

$$M_1 = \sum_{i=0}^{2^{n-1}-1}(i+1) + \sum_{i=0}^{2^{n-1}-1}(3i+2)$$

- **Determination of the second homogeneous line H_1**

$$H_1 = H\ (M_1) = H(\sum_{i=0}^{2^{n-1}-1}(i+1) + \sum_{i=0}^{2^{n-1}-1}(3i+2))$$

$$H_1 = H(\sum_{i=0}^{2^{n-1}-1}(i+1)) + H(\sum_{i=0}^{2^{n-1}-1}(3i+2))$$

$$H_1 = H_l(\sum_{i=0}^{2^{n-1}-1}(i+1)) + H_r(\sum_{i=0}^{2^{n-1}-1}(i+1)) + H_l(\sum_{i=0}^{2^{n-1}-1}(3i+2)) + H_r(\sum_{i=0}^{2^{n-1}-1}(3i+2))$$

$$H_1 = \sum_{i=0}^{2^{n-2}-1}(2i+1) + \sum_{i=0}^{2^{n-2}-1}(2i+2) + \sum_{i=0}^{2^{n-2}-1}(6+2) + \sum_{i=0}^{2^{n-2}-1}(6+5)$$

Where $R_1 = \sum_{i=0}^{2^{n-2}-1}(2i+1) + \sum_{i=0}^{2^{n-2}-1}(6i+5)$ And $T_1 = \sum_{i=0}^{2^{n-2}-1}(6i+2) + \sum_{i=0}^{2^{n-2}-1}(2i+2)$

- **Determination of the even line P_1 :**

$$P_1 = T_1 + E(R_1)$$

$$E(R_1) = E(\sum_{i=0}^{2^{n-2}-1}(2i+1) + \sum_{i=0}^{2^{n-2}-1}(6i+5)) = \sum_{i=0}^{2^{n-2}-1}(3(2i+1)+1) + \sum_{i=0}^{2^{n-2}-1}(3(6i+5)+1)$$

$$E(R_1) = \sum_{i=0}^{2^{n-2}-1}(6i+4) + \sum_{i=0}^{2^{n-2}-1}(18i+16)$$

So $P_1 = \sum_{i=0}^{2^{n-2}-1}(2i+2) + \sum_{i=0}^{2^{n-2}-1}(6i+4) + \sum_{i=0}^{2^{n-2}-1}(6i+2) + \sum_{i=0}^{2^{n-2}-1}(18i+16)$

We just give the determination of $S_1(M_0) = ((M_0, H_0, P_0), (M_1, H_1, P_1))$

NB : we can see that $S_\infty(M_0)$ is the infinite pyramid of M_0 and its last lines $(M_\infty, H_\infty, P_\infty)$ will give us the result we are looking for.

IV. DOING SOME CALCULATIONS USING H_p :

So if we consider that all numbers from 1 to infinity are going to reach the values (4,2,1) where n tend to infinity then and p tend to n-1 :

T_p will be defined by : $2^{n-1} \times 2 \leq T_p \leq 2^{n-1} \times 4$

R_p will be equal to $2^{n-1} \times 1$

The difference between T_p and R_p will be defined by :

$$2^{n-1} \times 2 - 2^{n-1} \times 1 \leq T_p - R_p \leq 2^{n-1} \times 4 - 2^{n-1} \times 1$$

$$2^{n-1}(2-1) \leq T_p - R_p \leq 2^{n-1}(4-1)$$

where n tend to infinity and p tend to n-1

Let's consider an heterogeneous line M_p

$$M_p = \sum_{k=1}^{2^p} \left(\sum_{i=0}^{2^{n-p}-1} (a_k i + b_k) \right)$$

- Finding the homogeneous line H_p

$$H_p = H(M_p) = H_l(M_p) + H_r(M_p)$$

$$H_l(M_p) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^{n-p-1}-1} (2a_k i + b_k))$$

$$H_r(M_p) = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^{n-p-1}-1} (2a_k i + b_k + a_k))$$

$$H_l(M_p) + H_r(M_p) = T_p + R_p$$

$$H_p = \sum_{k=1}^{2^p} (\sum_{i=0}^{2^{n-p-1}-1} (2a_k i + b_k)) + \sum_{k=1}^{2^p} (\sum_{i=0}^{2^{n-p-1}-1} (2a_k i + b_k + a_k))$$

If $H_p = \sum_{i=0}^{2^{n-p-1}-1} (A_p i + B_p)$ then $A_p = \sum_{k=1}^{2^p} 2a_k + \sum_{k=1}^{2^p} 2a_k = \sum_{k=1}^{2^p} 4a_k$

$$A_p = \sum_{k=1}^{2^p} 4a_k$$

Let's pose $D(H_p)$ as the absolute values of the differences between even suites and odd suites which come from the same heterogeneous suite in the line H_p

So if $M_p = \sum_{i=0}^{2^{n-p}-1} (a_1 i + b_1) + \sum_{i=0}^{2^{n-p}-1} (a_2 i + b_2) + \sum_{i=0}^{2^{n-p}-1} (a_3 i + b_3) + \ldots + \sum_{i=0}^{2^{n-p}-1} (a_p i + b_p)$

$$\begin{aligned} H_p = & H_r(\sum_{i=0}^{2^{n-p}-1} (a_1 i + b_1)) + H_l(\sum_{i=0}^{2^{n-p}-1} (a_1 i + b_1)) \\ & + H_r(\sum_{i=0}^{2^{n-p}-1} (a_2 i + b_2)) + H_l(\sum_{i=0}^{2^{n-p}-1} (a_2 i + b_2)) \\ & + H_r(\sum_{i=0}^{2^{n-p}-1} (a_3 i + b_3)) + H_l(\sum_{i=0}^{2^{n-p}-1} (a_3 i + b_3)) \\ & + \ldots.. \\ & + H_r(\sum_{i=0}^{2^{n-p}-1} (a_p i + b_p)) + H_l(\sum_{i=0}^{2^{n-p}-1} (a_p i + b_p)) \end{aligned}$$

When we distinguish odd and even series, we have :

$$\begin{aligned} H_p = & \sum_{i=0}^{2^{n-p-1}-1}(a_{t1}i+b_{t1}) + \sum_{i=0}^{2^{n-p-1}-1}(a_{r1}i+b_{r1}) \\ & + \sum_{i=0}^{2^{n-p-1}-1}(a_{t2}i+b_{t2}) + \sum_{i=0}^{2^{n-p}-1}(a_{r2}i+b_{r2}) \\ & + \sum_{i=0}^{2^{n-p-1}-1}(a_{t3}i+b_{t3}) + \sum_{i=0}^{2^{n-p-1}-1}(a_{r3}i+b_{r3}) \\ & + \ldots.. \\ & + \sum_{i=0}^{2^{n-p-1}-1}(a_{tp}i+b_{tp}) + \sum_{i=0}^{2^{n-p-1}-1}(a_{rp}i+b_{rp}) \end{aligned}$$

We have :

$$\begin{aligned} D(H_p) = & \left| \sum_{i=0}^{2^{n-p-1}-1}(a_{t1}i+b_{t1}) - \sum_{i=0}^{2^{n-p-1}-1}(a_{r1}i+b_{r1}) \right| \\ & + \left| \sum_{i=0}^{2^{n-p-1}-1}(a_{t2}i+b_{t2}) - \sum_{i=0}^{2^{n-p}-1}(a_{r2}i+b_{r2}) \right| \\ & + \left| \sum_{i=0}^{2^{n-p-1}-1}(a_{t3}i+b_{t3}) - \sum_{i=0}^{2^{n-p-1}-1}(a_{r3}i+b_{r3}) \right| \\ & + \ldots.. \\ & + \left| \sum_{i=0}^{2^{n-p-1}-1}(a_{tp}i+b_{tp}) - \sum_{i=0}^{2^{n-p-1}-1}(a_{rp}i+b_{rp}) \right| \end{aligned}$$

Since $H_r(\sum_{i=0}^{2^{n-p}-1}(ai+b)) = \sum_{i=0}^{2^{n-p}-1}(2ai+b+a)$ and

$H_l(\sum_{i=0}^{2^{n-p}-1}(ai+b)) = \sum_{i=0}^{2^{n-p}-1}(2ai+b)$

Then $H_r(\sum_{i=0}^{2^{n-p}-1}(ai+b)) \geq H_l(\sum_{i=0}^{2^{n-p}-1}(ai+b))$

As a result

$$D(H_p) = H_r(M_p) - H_l(M_p) = \sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(2a_k i + b_k + a_k) - \sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(2a_k i + b_k)$$

$$D(H_p) = \sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(a_k) = \frac{1}{4}\sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(4a_k) = \frac{1}{4}\sum_{i=0}^{2^{n-p-1}-1}(A_p)$$

$$D(H_p) = \frac{2^n}{2^{p+1}} \times \frac{A_p}{4} \quad \text{------->} \quad (1)$$

- Finding the even line P_p

$$T_p = \sum_{t=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)\right)$$

And $R_p = \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_r i + b_r)\right)$

Where $H_p = \sum_{t=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)\right) + \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_r i + b_r)\right)$

$$P_p = T_p + E(R_p)$$

$$= \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(3a_r i + 3b_r + 1)\right)$$

So $W_p = \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(3a_r i + 3b_r + 1)\right)$

$$P_p = \sum_{t=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)\right) + \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(3a_r i + 3b_r + 1)\right)$$

NB : let's proove that $\sum_{t=1}^{2^p} a_t = \sum_{r=1}^{2^p} a_r = \sum_{k=1}^{2^p} 2a_k$

$$T_p + R_p = \sum_{t=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)\right) + \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_r i + b_r)\right)$$

$$T_p + R_p = H_l(M_p) + H_r(M_p)$$

Then , $\sum_{t=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)\right) + \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_r i + b_r)\right) = H_l(M_p) + H_r(M_p)$

Consequently,

$$\sum_{t=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)\right) + \sum_{r=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(a_r i + b_r)\right) = \sum_{k=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(2a_k i + b_k)\right) + \sum_{k=1}^{2^p}\left(\sum_{i=0}^{2^{n-p-1}-1}(2a_k i + b_k + a_k)\right)$$

As a result, $\sum_{t=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(a_t i)) + \sum_{r=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(a_r i) = \sum_{k=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(2a_k i) + \sum_{k=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(2a_k i)$

We can see $\sum_{t=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(a_t i)) = \sum_{k=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(2a_k i)$

And $\sum_{r=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(a_r i) = \sum_{k=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(2a_k i)$

As a result, $\sum_{t=1}^{2^p} a_t = \sum_{r=1}^{2^p} a_r = \sum_{k=1}^{2^p} 2a_k$

- Finding M_{p+1}

$$M_{p+1} = H_e(P_p) = H_e(\sum_{t=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(a_t i + b_t)) + \sum_{r=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(3a_r i + 3b_r + 1))$$

$$M_{p+1} = \sum_{t=1}^{2^p}(\sum_{i=0}^{2^{n-p-1}-1}(\frac{a_t}{2} i + \frac{b_t}{2})) + \sum_{r=1}^{2^p}\sum_{i=0}^{2^{n-p-1}-1}(3\frac{a_r}{2} i + 3\frac{b_r}{2} + \frac{1}{2})$$

- Finding H_{p+1} :

Before finding H_{p+1} we must seperate odd-left and odd-right lines in M_{p+1}

$\overset{x}{M}_{p+1} = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-1}-1}(\frac{a_e}{2} i + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-1}-1}(3\frac{a_g}{2} i + 3\frac{b_g}{2} + \frac{1}{2})$ is the odd-left line

$\overset{y}{M}_{p+1} = \sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-1}-1}(\frac{a_f}{2} i + \frac{b_f}{2})) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-1}-1}(3\frac{a_h}{2} i + 3\frac{b_h}{2} + \frac{1}{2})$ is the odd-right line

Where $E + F = 2^p$ and $G + H = 2^p$

$$H_{p+1} = H(M_{p+1}) = H(\overset{x}{M}_{p+1}) + H(\overset{y}{M}_{p+1})$$

$$H(\overset{x}{M}_{p+1}) = H_l(\overset{x}{M}_{p+1}) + H_r(\overset{x}{M}_{p+1})$$

$$H_l(\overset{x}{M}_{p+1}) = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i + 3\frac{b_g}{2} + \frac{1}{2})$$

$$H_r(\overset{x}{M}_{p+1}) = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i + \frac{a_e}{2} + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i + 3\frac{a_g}{2} + 3\frac{b_g}{2} + \frac{1}{2})$$

$$H(\overset{y}{M}_{p+1}) = H_l(\overset{y}{M}_{p+1}) + H_r(\overset{y}{M}_{p+1})$$

$$H_l(\overset{y}{M}_{p+1}) = \sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i + \frac{b_f}{2})) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i + 3\frac{b_h}{2} + \frac{1}{2})$$

$$H_r(\overset{y}{M}_{p+1}) = \sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i + \frac{a_f}{2} + \frac{b_f}{2})) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i + 3\frac{a_h}{2} + 3\frac{b_h}{2} + \frac{1}{2})$$

So $T_{p+1} = H_r(\overset{x}{M}_{p+1}) + H_l(\overset{y}{M}_{p+1})$ because all series in these two line are even

$$T_{p+1} = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i + \frac{a_e}{2} + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i + 3\frac{a_g}{2} + 3\frac{b_g}{2} + \frac{1}{2}) + \sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i + \frac{b_f}{2})) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i + 3\frac{b_h}{2} + \frac{1}{2})$$

and $R_{p+1} = H_l(\overset{x}{M}_{p+1}) + H_r(\overset{y}{M}_{p+1})$ because all series in these two lines are odd

$$R_{p+1} = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i + 3\frac{b_g}{2} + \frac{1}{2}) + \sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i + \frac{a_f}{2} + \frac{b_f}{2})) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i + 3\frac{a_h}{2} + 3\frac{b_h}{2} + \frac{1}{2})$$

$$H_{p+1} = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i + 3\frac{b_g}{2} + \frac{1}{2}) + \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i + \frac{a_e}{2} + \frac{b_e}{2})) + \sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i + 3\frac{a_g}{2} + 3\frac{b_g}{2} + \frac{1}{2}) + \sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i + \frac{b_f}{2})) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i + 3\frac{b_h}{2} + \frac{1}{2})$$

$$+\sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i+\frac{a_f}{2}+\frac{b_f}{2}))+\sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i+3\frac{a_h}{2}+3\frac{b_h}{2}+\frac{1}{2})$$

Let's find $D(H_{p+1})$ by analogy to $D(H_p)$

$$D(H_{p+1}) = H_r(M_{p+1}) - H_l(M_{p+1})$$

$$H_r(M_{p+1}) = H_r(\overset{x}{M}_{p+1}) + H_r(\overset{y}{M}_{p+1})$$

$$H_r(M_{p+1}) = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i+\frac{a_e}{2}+\frac{b_e}{2}))+\sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i+3\frac{a_g}{2}+3\frac{b_g}{2}+\frac{1}{2})$$

$$+\sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i+\frac{a_f}{2}+\frac{b_f}{2}))+\sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i+3\frac{a_h}{2}+3\frac{b_h}{2}+\frac{1}{2})$$

$$H_l(M_{p+1}) = H_l(\overset{x}{M}_{p+1}) + H_l(\overset{y}{M}_{p+1})$$

$$H_l(M_{p+1}) = \sum_{e=1}^{E}(\sum_{i=0}^{2^{n-p-2}-1}(a_e i+\frac{b_e}{2}))+\sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i+3\frac{b_g}{2}+\frac{1}{2})+\sum_{f=1}^{F}(\sum_{i=0}^{2^{n-p-2}-1}(a_f i+\frac{b_f}{2}))$$

$$+\sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i+3\frac{b_h}{2}+\frac{1}{2})$$

$$D(H_{p+1}) = \sum_{e=1}^{E}\sum_{i=0}^{2^{n-p-2}-1}(a_e i+\frac{a_e}{2}+\frac{b_e}{2})+\sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i+3\frac{a_g}{2}+3\frac{b_g}{2}+\frac{1}{2})$$

$$+\sum_{f=1}^{F}\sum_{i=0}^{2^{n-p-2}-1}(a_f i+\frac{a_f}{2}+\frac{b_f}{2})+\sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i+3\frac{a_h}{2}+3\frac{b_h}{2}+\frac{1}{2})$$

$$-(\sum_{e=1}^{E}\sum_{i=0}^{2^{n-p-2}-1}(a_e i+\frac{b_e}{2})+\sum_{g=1}^{G}\sum_{i=0}^{2^{n-p-2}-1}(3a_g i+3\frac{b_g}{2}+\frac{1}{2})+\sum_{f=1}^{F}\sum_{i=0}^{2^{n-p-2}-1}(a_f i+\frac{b_f}{2})$$

$$+\sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3a_h i+3\frac{b_h}{2}+\frac{1}{2}))$$

$$D(H_{p+1}) = \sum_{e=1}^{E}\sum_{i=0}^{2^{n-p-2}-1}(\frac{a_e}{2}) + \sum_{g=1}^{G}(3\frac{a_g}{2}) + \sum_{f=1}^{F}\sum_{i=0}^{2^{n-p-2}-1}(\frac{a_f}{2}) + \sum_{h=1}^{H}\sum_{i=0}^{2^{n-p-2}-1}(3\frac{a_h}{2})$$

According to the separation of M_{p+1}, $D(H_{p+1}) = \sum_{t=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(\frac{a_t}{2}) + \sum_{r=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(3\frac{a_r}{2})$

$$D(H_{p+1}) = \frac{1}{2}\sum_{t=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(a_t) + \frac{3}{2}\sum_{r=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(a_r) = \frac{1}{2}\sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(2a_k) + \frac{3}{2}\sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(2a_k) =$$

$$D(H_{p+1}) = 2\sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(2a_k) = \sum_{k=1}^{2^p}\sum_{i=0}^{2^{n-p-2}-1}(4a_k) = \sum_{i=0}^{2^{n-p-2}-1}(A_p)$$

$$D(H_{p+1}) = \frac{2^n}{2^{p+2}} \times A_p \quad \text{------->} \quad (2)$$

NB : if we consider that all integers in the pyramid will reach the values (4,2 and 1) wen n tend to infinity, it means that the syracuse conjecture will be true, then the limit of $D(H_{n-1})$ will be defined as follow :

$$\lim_{n\to\infty}(2^{n-1}(4-1)) \geq \lim_{n\to\infty}(D(H_{n-1})) \geq \lim_{n\to\infty}(2^{n-1}(2-1))$$

The average of $D(H_{n-1})$ is $\frac{D(H_{n-1})}{2^{n-1}}$ because it contains 2^{n-1} integers from the soustractions, and it will be defined as follow : $3 \geq \lim_{n\to\infty}(\frac{D(H_{n-1})}{2^{n-1}}) \geq 1$

So let's find the general form of $D(H_p)$

- General form of $D(H_p)$:
  From (1) and (2) we have :

$$D(H_{p+1}) = \frac{2^n}{2^{p+2}} \times A_p \quad \text{and} \quad D(H_p) = \frac{2^n}{2^{p+1}} \times \frac{A_p}{4}$$

$$\frac{D(H_{p+1})}{D(H_p)} = \frac{\frac{2^n}{2^{p+2}} \times A_p}{\frac{2^n}{2^{p+1}} \times \frac{A_p}{4}} = 2$$

$$\frac{D(H_{p+1})}{D(H_p)} = 2$$

then $D(H_p)$ is a geometric suite with a first term $D(H_0)$ and $q = 2$ is the common ratio

the general form is : $D(H_p) = D(H_0) \times 2^p$

In the first bloc of the pyramid $S(\sum_{i=0}^{2^{n-1}} (i+1))$ we can find $D(H_0)$

$$H_0 = \sum_{i=0}^{2^{n-1}-1} (2i+1) + \sum_{i=0}^{2^{n-1}-1} (2i+2)$$

So $D(H_0) = \sum_{i=0}^{2^{n-1}-1} (2i+2) - \sum_{i=0}^{2^{n-1}-1} (2i+1) = \sum_{i=0}^{2^{n-1}-1} (1) = 2^{n-1}$

$$D(H_0) = 2^{n-1}$$

As a reuslt : $D(H_p) = 2^{n-1} \times 2^p$

We know in the last bloc of the pyramid, p is equal to $n-1$
We must get the value of $D(H_p)$ from the last homogeneous line (H_{n-1}) of the pyramid

So $D(H_{n-1}) = 2^{n-1} \times 2^{n-1}$

The average of $D(H_{n-1})$ is $\frac{D(H_{n-1})}{2^{n-1}} = \frac{2^{n-1} \times 2^{n-1}}{2^{n-1}} = 2^{n-1}$

$$\frac{D(H_{n-1})}{2^{n-1}} = 2^{n-1}$$

$$\lim_{n\to\infty}\left(\frac{D(H_{n-1})}{2^{n-1}}\right) = \lim_{n\to\infty}(2^{n-1}) = \infty$$

This value of the average of $D(H_{n-1})$ permit us to say that **the syracuse conjecture is false** because it doesn't verify the following framing where n tend to infinity :

$$3 \geq \lim_{n\to\infty}\left(\frac{D(H_{n-1})}{2^{n-1}}\right) \geq 1$$

It means taht it exists at least one number between 1 and infinity , whose syracuse suite is diverging.