



THE MATHEMATICS OF NUMERICAL EXPRESSIONS

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ABSTRACT

Important mathematical problems stem from the fact that calculations are made with numbers considered in isolation, not with numerical expressions. This article aims to point them out and suggest corrections. For this, it is necessary to redefine the concept of “number” and introduce that of “numerical expressions”, in addition to discussing the meaning of “multiplication”, “images and signs”, “equation”, “polynomial” and other issues. The conclusions reached are surprising, not to say disturbing.

INTRODUCTION

A “numeric expression” is made up of a number and the unit to which it refers. Both the number and the unit must participate in the calculations, not just the number, and they do so by authorization from arithmetic, geometry, or scientific mathematics. Indeed, the use of isolated numbers in operations generates misunderstandings, deadlocks, and serious mathematical errors.

The topics presented throughout this article explain the problems and the solutions needed to avoid them.

Substrate of the new concepts and divergences

Numerical expressions: numerical expressions indicate counts, which can be positive or negative, exception of the count of “the number of times”, which cannot be negative.

Multiplication: there are no negative multipliers.

Subtraction and multiplication signal rules: a simple matter of images.

Equations: the useful (= not ludic) equations are used to solve arithmetic problems involving counts of modules and are always of the first degree.

Polynomials: polynomials are used to make geometric representations and solve geometric problems and can be of the first, second and third degrees.

Mathematics in science: numerical expressions in science are imposed on a case-by-case basis, always observing that the units are also involved in the calculations indicated in the formulas and are modified as result of them.

DISCUSSION

Let us make, at first, a distinction between “number” and “numerical expression of counts”, which is of fundamental importance.

Number is a neutral and multiple up to infinity tool that represents the core of counts. Numbers just express a magnitude and are neutral: there are no positive numbers or negative numbers, let alone imaginary numbers.

A numerical expression of a count contains a number and a counting unit. There are three types of counts, namely: count of modules, count of steps and count of number of times.

The count of modules (= count of entities) is the numerical expression used in the equations. For example, the expression “10 oranges” is a count of oranges. The count of modules is the result of an algebraic sum and can be positive or negative.

The count of steps, or measurement, is the numerical expression used to accomplish measurements of length and to define positions as well as in the polynomial study of algebraic functions. The “step” can be a tacit unit (like a house on a grid paper) or a pattern, like the centimeter or the foot or the inch. When defining positions, the count of steps is equally an algebraic sum and can also be positive or negative.

The count of “number of times” (= frequency) is used as a multiplier in algebraic sums for counts of modules and in algebraic sums for counts of steps. The multiplier participates in the algebraic sums as a mechanism for adding a multiple of a positive or negative count, just to avoid repeated additions of said count. The multiplier has no explicit unity and is neutral (that is, it can never be negative) because there is no negative “number of times”.

Subtraction rules

Both the count of modules and the count of steps are considered from a zero point (origin). Thus, a negative count, be of modules or of steps, is the image, in relation to the origin, of a positive count, and vice versa. In an algebraic sum, the removal of a

positive count corresponds to the addition of its image in relation to the zero origin (this operation is called subtraction of the count considered):

$$A - B = A + (-B).$$

A negative sign means subtraction (addition of the image of B).

Conversely, the removal of a negative count corresponds to the sum of the count, for being the image of its negative image:

$$A - (-B) = A + B.$$

Two negative signs mean the sum of the count (image of the image, or counter-image, of B).

Multiplication rules

First rule: the multiplier can multiply any count.

Second rule: there is no negative multiplier.

Third rule: a count of modules can never be used as a multiplier, that is, it cannot multiply itself or multiply another count, be it of modules or of houses. That is why every equation of modules is necessarily of the first degree (x^2 , x^3 et cetera are impossible!).

Fourth rule: a measurement (positive count of steps) can multiply itself or multiply another measurement, obtaining an area, and can multiply an area to obtain a volume. In these two cases of multiplication, the step or unit adopted (house, centimeter, inch et cetera) is raised, respectively, to the square and to the cube, obtaining a numerical expression with derived unit, that is, house^2 , house^3 , centimeter^2 , centimeter^3 et cetera. See that the units also participate in mathematical calculations.

Multiplication with two negative factors

A multiplication with two negative factors, although impossible in everyday life, can appear in the process of mathematical operations and in the mathematical treatment of polynomials. In both cases, one of the negative signs shows that the product is a negative count. In turn, the other negative sign indicates subtraction of that product, consequently with a positive result.

Two examples shall clarify the matter.

Example 1: in dealing with the trinomials of the second degree, the following square can occur:

$$x^2 = (-7)x(-7)$$

It implies in subtracting a negative product, as follows:

$$x^2 = (-7)x(-7) = -(7)x(-7) = -(-49) = +49$$

Example 2: multiply (a-5) by (a-8). This multiplication, by the way, is only possible in geometry. It is well to see that we are multiplying (a-5) by (a-8), and not (-5) by (-8), which is impossible (because, as we know, there is no negative multiplier).

The same understanding of example 1 shall prevail. The result of the proposed multiplication is an algebraic sum, the last of its terms being the subtraction of a negative count, which, therefore, must be added:

$$(a-5)x(a-8) = a^2 - 8a - 5a - (-40) = a^2 - 13a + 40$$

Equations

The equation is an instrument of arithmetic. It is, in effect, an arithmetic algorithm that confronts two algebraic sums of counts of modules, forced to be equal, to discover an unknown count of modules, "x", present in one or in both the confronted algebraic sums. "x" is an unknown count of modules and has a single value, to be determined. It is important to observe that there are not ready or aprioristic equations, being necessary to build them case by case in accordance with proposed problems, such as calculating the result of a balance sheet or the age of Diophantus.

It is worth noting that an equation of isolated numbers (as opposed to the equation of counts of modules) has only a playful character, with no relevant mathematical benefit. In fact, it is idle to solve a ludic equation to get an answer like this: $x = 84$. With that "84", what can we do? What should we understand? What matters is an equation of counts of modules (for example, years, millions of dollars or rabbits), providing an answer on the following terms: "Diophantus, considering the information in his tomb, died at 84 years" or "this year the profit of the company in view of its accounts was 84 million dollars". We can be interested in knowing that in the yard of chickens and rabbits with 200 heads and 568 feet there are 84 rabbits, and not that in the yard there is a number 84.

Another extremely important point is that all equations are of the first degree. Why? Because "x" is a count of modules (= count of entities). We cannot multiply one count of modules by another count or by itself. If modules are present, there is not any " x^2 " or " x^3 " or any other power of "x", either in everyday life or in equations.

Playful or ludic equations can be of any degree, but they are extremely difficult to build and, if built, are almost impossible to solve. The only playful equation of the second degree known by the author of this paper comes from the problem of the following type: finding two numbers whose sum is 5 and whose product is 6. As we all know, the problem leads to the equation

$$x^2 - 5x + 6 = 0$$

To solve it, we shall use a mathematical artifice, as the age-old Bhaskara formula. We can imagine how difficult would be to solve an equation with x^3 or greater degree.

In addition, there is no place for “y” in equations, which are necessarily horizontal operations. By the way, $y = f(x)$ is a formula, not an equation.

Polynomials

The polynomial (“y”) is an algebraic sum of measurements (counts of steps), areas (square units) or volumes (cubic units), based on a measurement “x”, which can vary arbitrarily. Unlike the equation, the polynomial is not a confrontation of algebraic sums of counts involving a single unknown count, but a formula consisting of a single algebraic sum involving a count “x” that can assume an indeterminate number of arbitrary values, as in all formulas whatsoever. Its “x” (which may assume as much different counts of steps as desired) has nothing to do with the “x” of the equation, which is a solitary count of modules to be determined.

In polynomials, “x” is therefore a unit of measurement (implicit or explicit). “x²” is its square and “x³” is its cube. If “x” is the numerical expression of a dimension in meters:

- in the expression $(x+2)$, “2” means “2 meters”

- in the expression

$$(x + 2)^2 = x^2 + 2x + 8$$

“(x + 2)²”, “x²”, “2x” and “8” are areas given in m²

- in the expression $(x^3 + 80)$, both “x³” and “8” are volumes given in m³

Geometry works with measurements, with areas and with volumes, and for that reason the non-playful polynomial can be of the first, second and third degrees.

First-degree polynomial

A first-degree polynomial is an algebraic sum of measurements that allows us to construct a straight line in a Cartesian representation. Example:

$$y = ax + B,$$

where “x” and “B” are measurements and “a”, a multiplier of measurements. In the Cartesian graph, the straight line connects points defined by ordinates, “y”, resulting from the first-degree polynomial with abscissas “x” considered in sequence.

In the first-degree polynomial, “x” and “y” are measurements, that is, counts of steps.

Second-degree polynomial

A polynomial of the second degree is an algebraic sum of areas that allows the construction of a parable. Example:

$$y = ax^2 + Bx + C,$$

where “y” and “C” are areas (as much as “x²”), “B” and “x” are measurements and “a”, a multiplier of areas. The parable links values corresponding to ordinates “y”, that are as calculated by means of the second-degree polynomial.

Illustrative exercise: calculate $(x - 3)^2$, using the polynomial expression for $x = 5m$. We know that the answer is $4 m^2$. In fact,

$$\begin{aligned} y &= (x - 3)^2 = x^2 - 6x + 9 = \\ 25m^2 - 30m^2 + 9m^2 &= 4 m^2 \end{aligned}$$

Third-degree polynomial

A third-degree polynomial is an algebraic sum of volumes. For example:

$$y = ax^3 + Bx^2 + Cx + D$$

where “a” is a volume multiplier, “B” and “x” are measurements, “C” is an area, as much as “x²”, while “y” and “D” are volumes (as much as “x³”, “Bx²” and “Cx”).

Polynomial equalized to zero

The polynomial is equivalent to a scientific formula. Equalizing the polynomial to zero does not give rise to an equation, as this one is a confrontation of two algebraic sums. For example:

$$y = x^2 - 5x + 6$$

is a polynomial that gives way to the construction of a parable in Cartesian representation, as we have already mentioned. Equalizing it to zero only allows the calculation of the points in the parable where it crosses the “x” axis, which is far from being the comparison of two algebraic sums. We can equalize it to any count of areas, from (-) infinite to (+) infinite, without this characterizing the existence of infinite number of equations. Discovering the “x” corresponding to an arbitrated “y” can even be a mathematical exercise (at first, without any interest), without that it means solving an equation.

Once more, there are no ready or aprioristic equations. To have an equation is necessary a problem that leads to a confrontation of two algebraic sums forced to be equal.

Mathematics in science

The formula is a resource of science, specially physics, which uses numerical expressions created on a case-by-case basis, imposing physical quantities, respective units, and the relevant calculations. Physics indicates, through the formula, whether a numerical expression of a physical quantity may be multiplied by itself or by the numerical expression of another physical quantity. For example, numbers and units are involved in the formula that expresses the law of gravity: the numerical expression of a force, in newtons, results from calculation with numerical expressions in meters and kilograms, as below:

$$F = G. m_1.m_2 / r^2,$$

F = gravitational force given in “newtons”

m₁ and m₂ = masses, given in “kilograms”

r = distance, given in “meters”

G = universal gravitation constant, given in “(n.m² / k²)”, a unit generated by the formula.

Science works with formulas, as above commented, and the author is not aware of any equality imposed on two scientifically constructed algebraic sums. In other words, we understand that the word “equation” is used in science with the meaning of “formula”, metaphorically.

See, finally, that in scientific calculations the physical quantities can be raised to any power if required by the formula. For example, Stefan-Boltzmann’s law states that the flow on the surface of a black body is proportional to the fourth power of the body’s temperature.

CONCLUSIONS

Numbers are neutral. There are not positive, negative, or imaginary numbers.

Calculations should be made with numerical expressions of counts.

Counts of modules are for arithmetic. Counts of steps are for geometry,

Counts can be positive or negative (except the count of the “number of times”, that is always neutral).

The images control the signals in the mathematical operations.

There are not negative multipliers.

Equations that matter are always of the first degree and their “x” has a unique value.

Polynomials may be of first, second and third degrees and their “x” may have any arbitrary value.

A polynomial equalized to zero does not give rise to an equation.

Science imposes on a case-by-case basis the formulas, their units, and the rules to be applied.

In everyday life, no one will be ever called upon to absurdly multiply two negative factors or to absurdly extract the square root of a “negative” number. Never!

