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THREE CLASSICAL TOPICS UNDER AN ALTERNATIVE MATHEMATICAL MODEL

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ABSTRACT

In previous books and papers, I stated we have been practicing and teaching our students a bad mathematics, which coexists with strange results, weird conventions and other unclear matters. I also stated it is feasible to have an improved math model, free from all these inconsistencies. Current math model is a poor model because it relies on a poor foundation, particularly the concept of numbers, the main pile in the referred foundation, and does not comply with the unbreakable interdependence of geometry and some branches of mathematics, as number theory, algebra and trigonometry. In spite of that, the math community keeps adding concepts and theories to the present math model without any serious initiative to question the fundamentals of its poor foundation. If, as declared by some math experts, applied mathematics is a bad mathematics, pure mathematics is a bad mathematics too, because it relies on the same poor foundation. In its applied version, and in spite of its many limitations, mathematics allows us to handle real world matters. On the other hand, pure mathematics, as an abstract approach, not necessarily committed to solve any real world problem, has no practical use. With the purpose to demonstrate my innovative concepts, I discuss three classical math topics in this paper. Right triangles, rule of signs, and number zero.

PREAMBLE

We may say that numbers are the main pile in the foundation over which humankind has been developing the whole science of mathematics. Centuries ago, we adopted the assumption and understanding that numbers are positive or negative figures, treated negative numbers differently from positive numbers, and accepted that positive numbers are equivalent to absolute values.

Since then, we have added new concepts and theories to the science of mathematics, including quite sophisticate matters based upon the foundation built on the referred concept of numbers. In spite of facing strange results, unproven conjectures, unsolved problems, and the need of weird conventions and poor explanations to coexist with all those inconsistencies, we have not seen any serious attempt to investigate said foundation, over which we built modern mathematics.

Some famous mathematicians only deal with pure mathematics, because they think (and declare) that applied mathematics is a bad mathematics. I agree with that feeling, but it is relevant to recall that pure mathematics (clearly implied as a good mathematics) relies on the same foundation as applied mathematics does. If, as I say, applied mathematics is a bad mathematics because it relies on a poor foundation, pure mathematics is a bad mathematics too. The difference is the fact that the real world reveals the imperfections of applied math, while pure mathematics remains in the realm of abstraction without facing the real world requirements.

That is the thesis I have been working on for about 7 years. I spent time and effort to show that current math (applied and pure mathematics) is a bad mathematics because it relies on a poor foundation, particularly our mistaken understanding about the concept of numbers.

Numbers quantify things of nature, as books, students and dollar amounts, which are neither positive nor negative, but neutral elements in nature. Numbers are mere indicators of quantities, and do not have signs, except when they are terms of arithmetic and algebraic expressions. When dealing with math equalities, we perform algebraic sums (addition and subtraction) of terms commanded by the plus sign or by the minus sign, all of them as absolute values. The plus and the minus signs only mean addition and subtraction of absolute values, the terms that form math equalities.

I became aware of this innovative concept about numbers when I read a book written by Remo Mannarino[1]. I dare to state that Mannarino's understanding about numbers is the most important math concept conceived during the past five centuries, because it will drastically change mathematics.

This new approach to numbers will impose a major shift in almost every aspect of mathematics. In my research, it led me to propose an alternative math model based on a new foundation, free from strange results, weird conventions and poor explanations. An alternative math model, which yields the same useful results current math model does, but is simpler, logic and consistent with the real world observations and measurements.

The concepts behind the new foundation I suggest are sometimes in full disagreement with the concepts presently accepted, and represent my personal opinion, regardless of what other researchers may say or believe. Until proven otherwise, that alternative math model may be a useful contribution within a reasonable doubt. Some concepts

[1] MANNARINO, Remo: ohomemhorizontal.blogspot.com

and conventions presently adopted, and results yielded by current math model are undoubtedly wrong.

To illustrate the application of the alternative math model, I elected three famous topics often seen when dealing with the science of mathematics. In my analysis of these topics, I granted myself the right to “think outside the box” under a free reasoning approach, with full disregard of any prevailing math concept or understanding. My analysis relies on innovative and polemic fundamentals, although duly justified.

In Topic I, I make a comparison between the geometric view and the mathematical view under which we treat right triangles, certainly one of the most famous geometric figures. I will show that, contrarily to the common understanding, geometry rules math. I will emphasize the unsurpassable limitation of current math in its process to express right triangles.

In Topic II, I discuss the so-called rule of signs, under which the product or the quotient between to values of a same sign yields a positive value, while the product or quotient between two values of different signs yields a negative value. I will show that said rule of signs does not exist. It will also be clear how unnecessary concepts can complicate things, what possibly explains why most people say they hate mathematics.

In Topic III, I introduce a new approach to treat “zero” when performing operations. We will see that a different approach can avoid indeterminate and/or undefined results, as well as the need of weird conventions and poor explanations presently in use.

The interested reader may see full details about my innovative fundamentals and the alternative math model in the published material listed in ‘REFERENCES”, at the end of this text.

TOPIC I

RIGHT TRIANGLES
(Geometric view versus algebraic view)

INTRODUCTION

In this Topic I, I will present complementary argumentation about right triangles[2]:

> *A comparative analysis of Pythagorean and non-Pythagorean right triangles under geometric and algebraic views.*

According to the common understanding, mathematics is an exact and perfect science, and algebra is an independent and self-sufficient field of work. That common

[2] AMUI, Sandoval, *A circunferência, Pitágoras e Fermat*, Editora CataLivros, 2017. Electronic version available upon request.

understanding also considers that geometry is part of mathematics. Contrarily to that widespread belief, mathematics, particularly algebra, has many limitations, and in most cases, it only yields approximate results. Some people even consider mathematics as a science. Besides, geometry is an independent field of work, which in some cases rules mathematics.

The operation with non-terminating decimals is a fundamental arithmetic problem, not yet solved. Arithmetic is the primeval foundation of mathematics, and numbers are the main pile in that foundation. It means that a true equality only exists when dealing with integers. All others are mere approximations, no matter how many digits we employ. It also means that this arithmetic limitation will propagate throughout the entire science of mathematics.

Geometry deals with a pre-existing subject of study, the geometric figures, a perfect creation of Mother Nature, which follow natural laws. Mathematics is an imperfect creation of humankind, which has no pre-existing subject of study, since mathematics only deals with its own premises, methods and rules. There exists an unbreakable interdependence of geometry and some branches of mathematics, as algebra, number theory and trigonometry, materialized by the use of the Cartesian system of coordinates. Whenever there happens a discrepancy between a geometric law and a mathematical concept, geometry must prevail.

This paper refers to the limitations of mathematics in respect of the dependence of algebra (Cartesian system of coordinates) on certain properties of geometric figures, as well as how said limitations reach other math subjects, as Fermat's Conjecture (Fermat's Last Theorem). The right triangle is the perfect choice to accomplish that task.

NUMBERS

Roughly speaking, we may say we deal with two types of numbers: integers and non-integers. Except when dealing with integers, in many cases mathematics requires the use non-terminating decimals, as "2/3", "√3", "π" and many others. It implies that the results yielded by arithmetic operations are often mere approximations. In brief, in practice we deal and accept imperfect equalities, not true equalities.

MATHEMATICAL EXPRESSIONS OF GEOMETRIC FIGURES

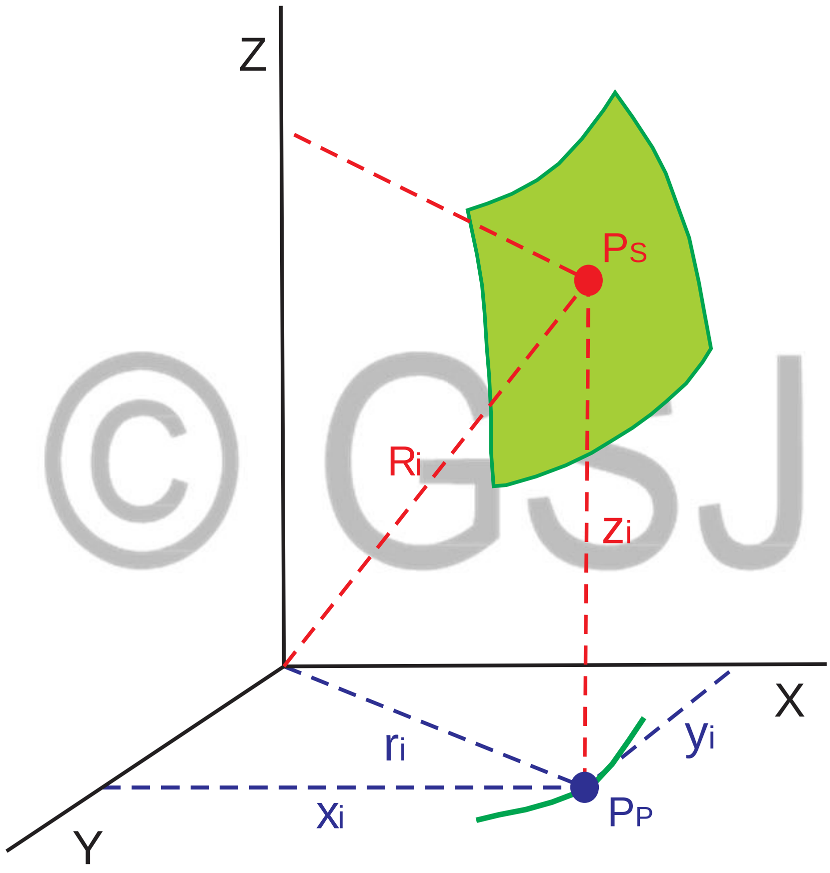
The Cartesian system of coordinates materializes the interdependence of geometry and some branches of mathematics, particularly algebra and trigonometry (I include number theory)[3]. With some limitations, we may express an existing geometric figure in mathematical terms (as an ellipse) or, contrarily, identify the geometric figure that corresponds to a given mathematical expressions. In the latter, we may end up with an abstraction, a geometric figure that does not exist in the real world (as an elliptic curve).

[3] AMUI, Sandoval, Pythagoras' Theorem, Fermat's Conjecture and Beal's Conjecture, AYA Editora, 2023.

In doing so, we cannot violate the applicable geometric laws, as current mathematics sometimes does. Whenever a mathematical expression represents an existing geometric figure, said expression must conform to the geometric law ruling the geometric figure. I will emphasize this matter by handling the right triangles and its unbreakable connection with the circumference (geometric view versus algebraic view).

We use the well-known Theorem of Pythagoras in connection with the Cartesian system of coordinates, as we see in Figure 01. A point "P_P" in a plane is obtained by the relationship "$x_i^2 + y_i^2 = r_i^2$", while a point "P_S" in space is determined by the relationship "$x_i^2 + y_i^2 + z_i^2 = R_i^2$".

Figure 01: Cartesian system of coordinates



The geometric place resulting from the movement of point "P_P" in the plane "xy" when "r_i" is kept constant is the contour line of a circumference, and the geometric figure formed by the movement of point "P_S" in the space "xyz" when "R_i" is kept constant is the surface of a sphere. In both cases, we need to comply with the geometric law ruling right triangles.

Circumference contour line: $x^2 + y^2 = r^2$

Sphere surface: $x^2 + y^2 + z^2 = R^2$

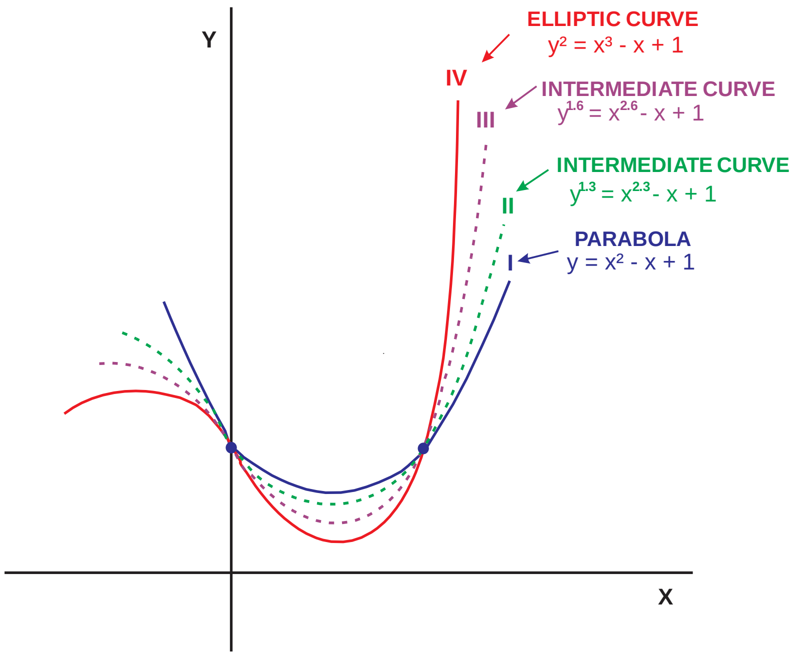
NUMBER OF VARIABLES AND EXPONENTS

As a side comment, let us keep in mind that in any algebraic expression[4]:

(i) The "number of variables" only indicates if we are dealing with a flat geometric figure (two variables, "x" and "y") or a spatial geometric figure (three variables, "x", "y" and "z"); that number determines the "geometric order" of math expressions. Geometry (and the real world) does not accept more than three dimensions. We use algebraic expressions with more than three variables in connection with sciences (formulas) or as ludic algebraic exercises (abstractions of an imaginary field of work).

(ii) The "variable exponents" (the "algebraic degree" of math expressions) inform if we deal with a classical geometric figure (follows a geometric law and a math expression, as a parabola) or with a modified geometric figure (only follows a math expression, as an elliptic curve[5]). When we deal with exponents different from "1" and "2", we face imperfect geometric figures or mere abstractions (Figure 02).

Figure 02: Classical and modified geometric figures
(Elliptic curve as a distorted parabola)

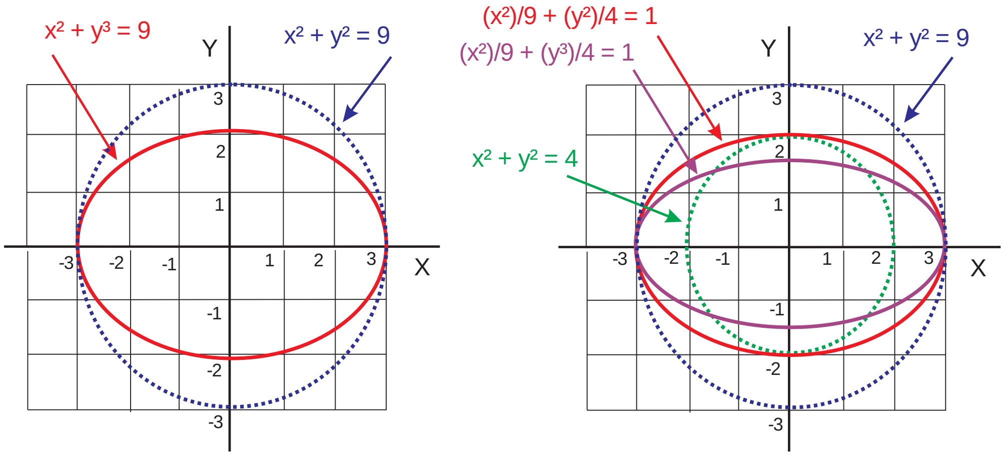


The expression "$x^2 + y^2 = 9$" represents a circumference, while the expression "$(x^2)/9 + (y^2)/4 = 1$" represents an ellipse, both are classical geometric figures that obey geometric laws, in addition to well-known math formulas. Their respective modified math expressions "$x^2 + y^3 = 9$" and "$(x^2)/9 + (y^3)/4 = 1$" are modified geometric figures[6], oval-

[4] AMUI, Sandoval, You may not enjoy mathematics (but you do not have to hate it), AYA Editora, 2022.
[5] It is my view that the elliptic curve "$y^2 = x^3 – x + 1$" is an abstraction, resulting from the modification of the parabola "$y = x^2 – x + 1$", when we increase the exponent of "y" from "1" to '2", and at the same time we increase the exponent of "x^2" from "2" to "3", as Figure 02 illustrates.
[6] Not necessarily perfect forms, with a perfect contour line.

shaped forms, which (although following a math expression) do not obey geometric formation laws (Figure 03).

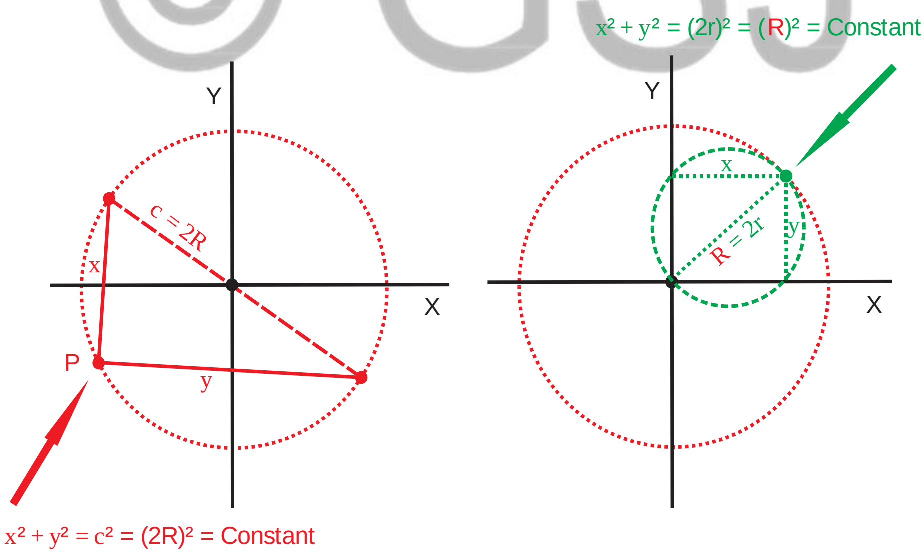
Figure 03: Classical and modified geometric figures
(Circumference and ellipse, and resulting oval-shaped forms)



RIGHT TRIANGLE, CIRCUMFERENCE AND SPHERE

To illustrate the point, let us consider the geometric view and the corresponding mathematical formula of two well-known geometric figures, circumference and sphere (Figures 04 and 05).

Figure 04: Circumference and right triangle

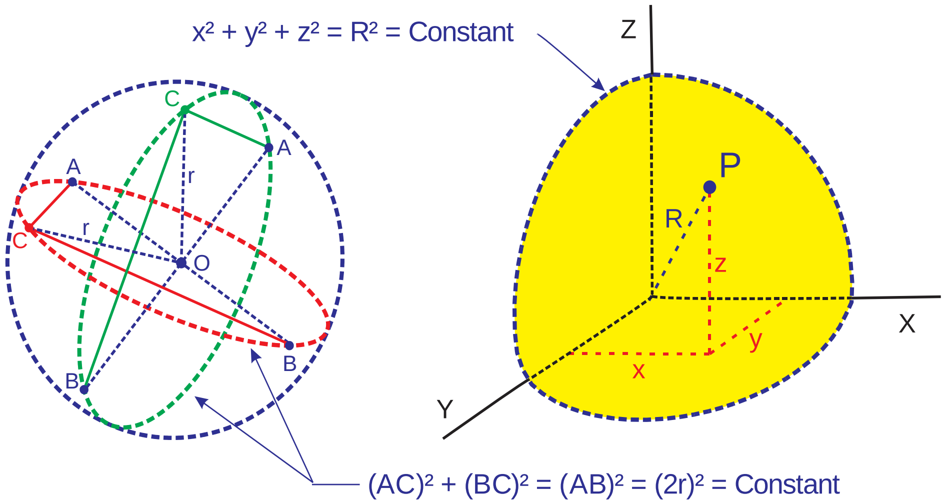


Geometry Cartesian system

Both geometric figures, circumference and sphere, obey geometric laws of formation and follow algebraic formulas.

Moreover, it is clear the unbreakable relationship between the circumference and the right triangle. There is no right triangle outside a circumference, and any circumference with a constant diameter "d" contains a group of an infinite number of right triangles with variable legs "x" and "y" and a common hypotenuse, which is the diameter "d" of the circumscribing circumference (Theorem of Infinite Right Triangles[7]).

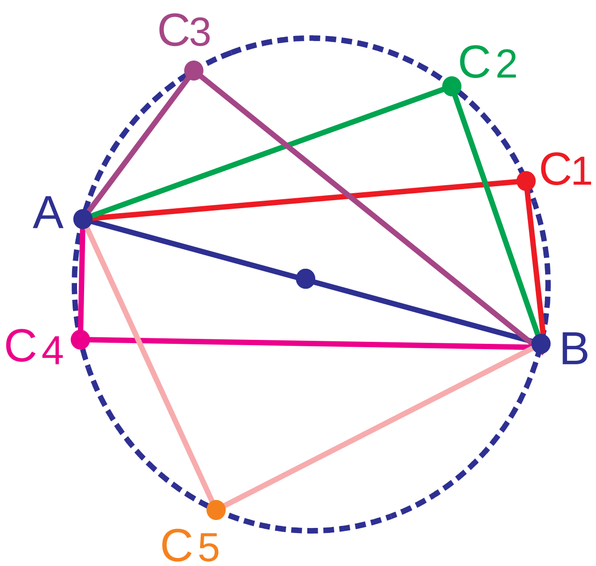
Figure 05: Sphere



Geometry Cartesian system

Figure 06 illustrates my Theorem of Infinite Right Triangles. In terms of geometry, the circumference of diameter "AB" circumscribes a group of an infinite number of right triangles with points "C_1, C_2, C_3 …" located in the contour line of that circumference.

Figure 06: Theorem of Infinite Right Triangles



[7] AMUI, Sandoval, *A circunferência, Pitágoras e Fermat*, Editora CataLivros, Rio de Janeiro, 2017. Electronic version available upon request.

The geometrical place resulting by the right angle corners (C_1, C_2, C_3 …) of the group of all possible right triangles with a same and constant hypotenuse (AB) is a perfect circumference, whose diameter is equal to the given hypotenuse (AB). The right triangles in that group (inscribed in a same circumference) may have variable legs, but a same hypotenuse, whatever their values. The Theorem of Infinite Right Triangles states that.

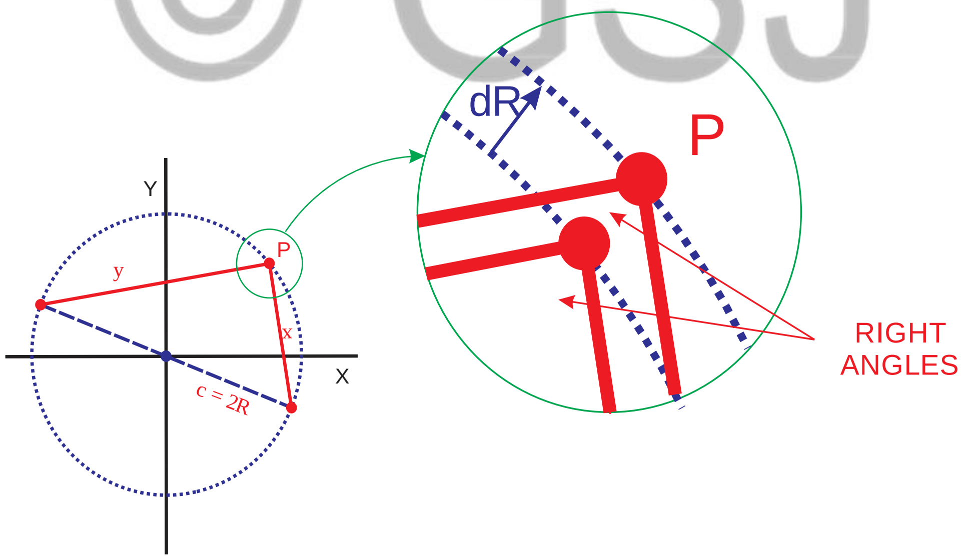
For each circumference, "$x^2 + y^2 = d^2$ = constant", each pair of "x" and "y" that obey said relationship should determine a specific right triangle of the group of infinite right triangles comprised by that circumference of diameter "d". I will show that it does not happen in terms of mathematics.

GEOMETRIC VIEW VERSUS MATH VIEW OF RIGHT TRIANGLES

There is a significant difference between the geometric view and the math view of right triangles.

The difference between the radiuses of two circumferences, "dR", can be as small as anyone can imagine (Figure 07). Nevertheless, geometrically speaking we may conceive a perfect circumference and a perfect sphere, without any limitation, whatever the initial radius and the radius increment. It implies that the triangle formed by connecting any point "P" (in the contour line of any circumference) to the ends of any diameter "c" of that circumference will always be a perfect right triangle.

Figure 07: Geometric view of right triangles



In terms of mathematics, all right triangles formed (either explicitly or implicitly[8]) follow the quadratic relationship "$x^2 + y^2 = c^2$ = constant". However, true circumferences and true spheres only occur when dealing with integers and exponent equals to "2" in that quadratic relationship (Pythagorean right triangles). Otherwise, we deal with approximate equalities, representing imperfect geometric figures (the so-called non-

[8] AMUI, Sandoval, The interdependence of geometry and some branches of mathematics, Global Scientific Journal - GSJ, 2022.

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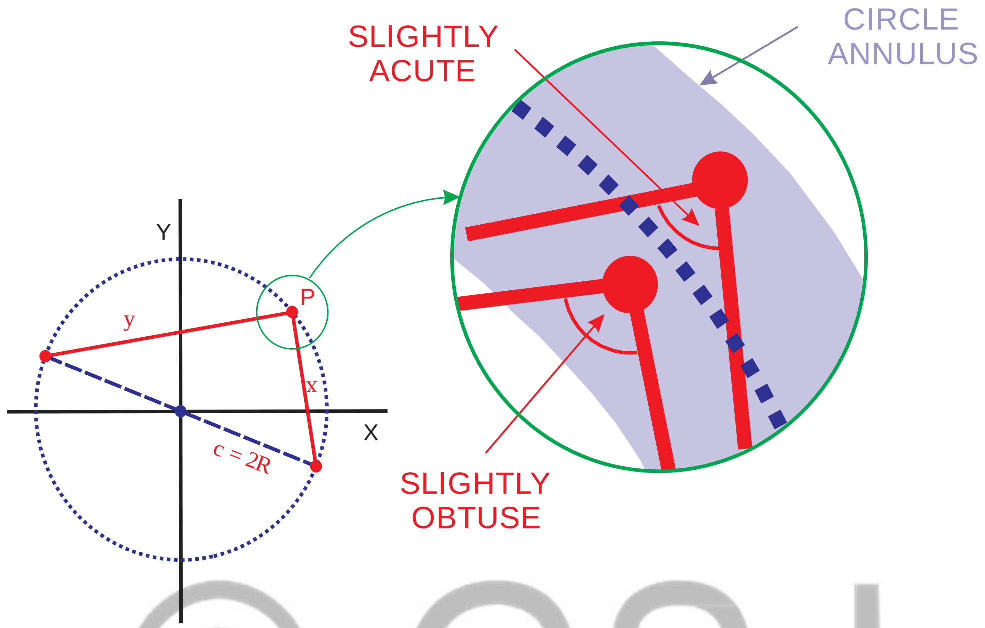
Pythagorean right triangles). The approach is acceptable for practical purpose, since we may use as many decimals as we want, provided we are aware of what we are doing. Philosophically speaking, in exact mathematical terms, we can only express geometric figures with whole numbers (Figure 08).

Figure 08: Math view of non-Pythagorean right triangles



When dealing with non-integer numbers, “x”, “y” and “c” (particularly non-terminating decimals), the referred relationship, “$x^2 + y^2 = c^2$ = constant”, is not an exact equality. As a result, we see a distorted circumference (a circle annulus), which circumscribes slightly acute or slightly obtuse triangles, what means that it is not possible to represent non-Pythagorean right triangles in mathematical terms.

To clarify the point, let us mathematically express a practical example of a circumference of diameter “d = 5.00”. We know that said circumference only comprise one Pythagorean right triangle, with sides “3.00” and “4.00”, and hypotenuse “5.00”, which is the circumference diameter. We can represent said Pythagorean right triangle in terms of math by “$3^2 + 4^2 = 5^2$”, as the right triangle “ABC" in Figure 09.

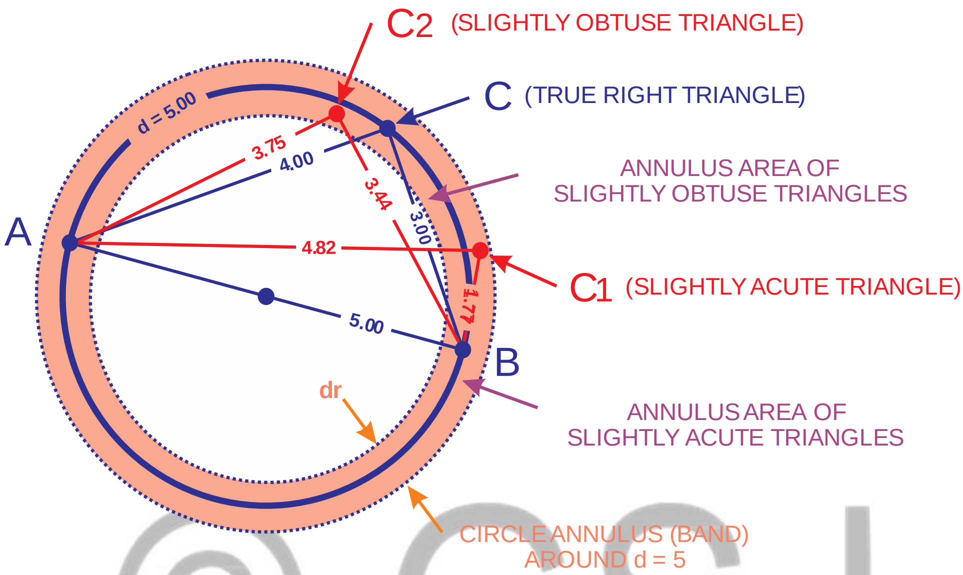
However, geometry (Theorem of Infinite Right Triangles) requires that circumference to comprise an infinite number of right triangles, all of them with variables legs, but with that same hypotenuse “5.00”. Math refers to these other triangles as non-Pythagorean right triangles, since at least one of their sides will not be an integer number. As examples, we see the triangles “ABC_1” and “ABC_2” in that Figure 09.

Mathematically[9], we represent the referred non-Pythagorean right triangles, “ABC_1” and “ABC_2” by “$1.77^2 + 4.82^2 = 5.00^2$” and “$3.44^2 + 3.75^2 = 5.00^2$”, which are not true arithmetic equalities. As previously explained, these two triangles are not true right triangles, but a slightly acute triangle (ABC_1) and a slightly obtuse triangle (ABC_2).

[9] Assuming we will deal with two decimals.

The corners of the angles opposite to the circumference diameter are points that fluctuate within a band (circle annulus around said diameter). These points do not generate a perfect contour line of a circumference, as required by the Theorem of Infinite Right Triangles. Instead, they generate a band, with slightly acute triangles in the external area of that circle annulus or slightly obtuse triangles in the internal area of that circle annulus, when we take the circumference contour line as reference (Figure 09).

Figure 09: Math unconformity regarding non-Pythagorean right triangles



Except when the three sides of a triangle are whole numbers, no other point remains on the circumference contour line. In the example of Figure 09, with the exception of the true right triangle "ABC", arithmetically expressed by "$3^2 + 4^2 = 5^2$", all other triangles comprised by the given circumference are not true right triangles[10].

An integer hypotenuse (the diameter of the circumscribing circumference) is not enough to guarantee the existence of true right triangles (Pythagorean right triangles). It is necessary to occur legs represented by integers too.

That is why, in terms of math, only Pythagorean right triangles are true right triangles. The so-called non-Pythagorean right triangles are in fact slightly acute or slightly obtuse triangles (never perfect right triangles).

This is a limitation of algebra, which does not exist in geometry. Figure 10 shows that a circumference of diameter "5" must circumscribe an infinite number of true right triangles, "$a_i^2 + b_i^2 = 25$" with variable legs (a_i and b_i). However, math is not capable to express them as true equalities, with the sole exception of the right triangle with legs "3" and "4" ($3^2 + 4^2 = 25$).

[10] That right triangle "ABC" can be placed in an infinite number of positions inside the circumference of diameter "d = 5". However, geometry requires an infinite number of right triangles with that same hypotenuse, but variable legs, a requirement math is not capable to fulfil.

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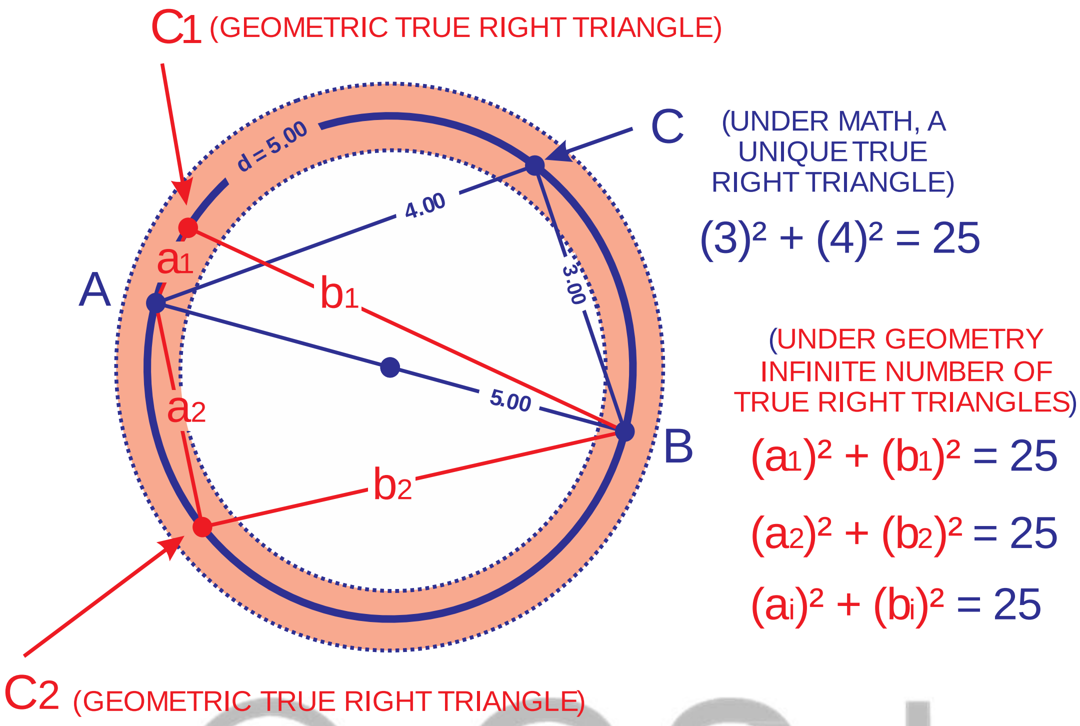
Figure 10: Geometric view of non-Pythagorean right triangles



This math limitation explains why Fermat's Conjecture (Fermat's Last theorem) is a "self-evident statement". Fermat's expression, "$x^n + y^n = z^n$", is an exact equality only when "x", "y" and "z" are integers and the exponent "n" is equal to "1" or "2". In other words, the math expression to represent a perfect contour line of a circumference requires integer numbers and exponent equal to "2" to meet the quadratic relationship, and mathematically express true right triangles. Non-integer numbers accepts a different exponent as an approximate equality (an imperfect circumference), in order to approach the quadratic relationship, either explicitly or implicitly, expressing slightly acute or slightly obtuse triangles (not right triangles).

The same reasoning about circumferences applies to spheres. We can rewrite the mathematical expressions, "$x^n + y^n = r^n$" and "$x^n + y^n + z^n = R^n$", as "$(x^{n/2})^2 + (y^{n/2})^2 = (r^{n/2})^2$" and ""$(x^{n/2})^2 + (y^{n/2})^2 + (z^{n/2})^2 = (R^{n/2})^2$". Since the sphere surface contains an infinite number of circumferences of constant radius, geometric laws require the exponent "n" to be equal to "2", and arithmetic requires all numbers to be integers. Otherwise, it is not possible to express true equalities (true circumferences and true spheres).

NEAR MISS EQUALITY

The previous reasoning also explains "Fermat's near-misses" examples. There are integer numbers that form "Fermat's near-misses" for "n" greater than "2", as "$x^n + y^n = z^n \pm k$", in which "k" can be a very small number, including whole numbers. These cases are equivalent to the use of non-integer numbers, when we deal with approximate equalities.

As an example, we have Ramanujan's numbers:

$$9^3 + 10^3 = 12^3 + 1^3$$

$$729 + 1{,}000 = 1{,}728 + 1$$

It is possible to find a non-Pythagorean right triangle behind that given numbers, if we rewrite the math expression in a manner to obey the mandatory quadratic relationship, with exponent equal to "2", as below (accepting two decimals):

$$(9^{3/2})^2 + (10^{3/2})^2 = (12^{3/2})^2 + 1.00$$

$$(27.00)^2 + (31.62)^2 = (41.57)^2 + 1.00$$

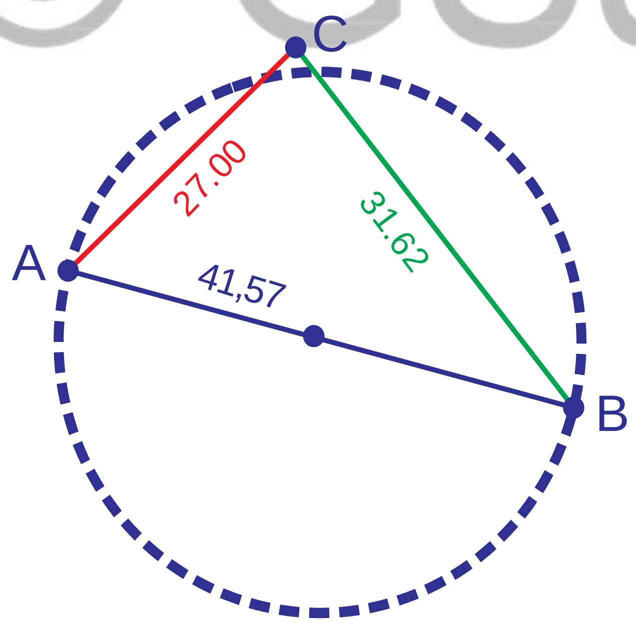
We may avoid the increment (1.00), but the math expression remains as an approximate equality.

$$(27.00)^2 + (31.62)^2 = (41.58)^2$$

Nevertheless, the resulting triangles of near-miss cases, whatever the numbers are, will be slightly acute or slightly obtuse triangles, not true right triangles.

With the rounding criterion adopted in Ramanujan's example above, we see a slightly acute triangle, since the sum of the squares of its minor sides is greater than the square of its bigger side[11] (Figure 11).

Figure 11: Triangle formed by Ramanujan's numbers



[11] If dealing with a different number of decimals, we could find a slightly obtuse triangle.

Similarly, the math expression of a sphere surface, "$x^2 + y^2 + w^2 = z^2$" is an exact equality if, and only if, we deal with integers and exponent equal to "2". Otherwise, we will not have a perfect spherical surface.

FINAL COMMENT

Consider a math expression, which we can rewrite in the form below[12]:

$$(A^{a/2})^2 + (B^{b/2})^2 = (C^{c/2})^2$$

In case we have a true equality, the terms "$(A^{a/2})$", "$(B^{b/2})$" and "$(C^{c/2})$" are whole numbers, and will form a right triangle (Pythagorean right triangle). If any of "$(A^{a/2})$", "$(B^{b/2})$" or "$(C^{c/2})$" is not a whole number, we still may have slightly acute or slightly obtuse triangles (but never a true right triangle), known as non-Pythagorean right triangles.

In other words:

> *As expressed in terms of math, non-Pythagorean right triangles are in fact slightly acute or slightly obtuse triangles (including Fermat's near-miss examples).*
>
> *Math is not capable to write true equalities we need to express all right triangles, which exist in geometry.*
>
> *Clearly, Fermat's Conjecture (Fermat's Last Theorem) is a self-evident statement.*

TOPIC II

RULE OF SIGNS
(An alternative understanding)

INTRODUCTION

In this Topic II, I will present complementary argumentation about:

> *The so-called "rule of signs" used in connection with arithmetic operations involving positive and negative values.*

Among the prevailing math concepts, there is a "rule of signs" under which the result of a multiplication (or a division) of two values of equal signs is a positive value, while the result of a multiplication (or a division) of two values of different signs is a negative value. It implies that the square of a negative value (as a debt) is a positive value (as a greater credit), what is a clear mistake.

---

[12] AMUI, Sandoval, Two famous conjectures (Pierre de Fermat and Andrew Beal), AYA Editora, 2022.

That concept derives from the wrong premise that numbers are positive or negative figures[13]. This misleading understanding about numbers leads to secondary mistakes, as the understanding that positive numbers are equivalent to absolute values (what is not always true) and the existence of complex numbers (an unnecessary theory). In this paper, I say that such rule of signs does not exist. Numbers are neutral indicators of quantities (absolute values), and we simply perform addition and subtraction operations when handling terms of arithmetic or algebraic expressions (also absolute values).

Values commanded by positive signs are not necessarily equivalent to absolute values (it only occurs when dealing with proper equalities, a property that also applies to values commanded by negative signs)[14]. In addition to that, the concept of complex numbers is an unnecessary and useless approach, which only complicates the science of mathematics.

The reader must keep in mind that it is an unquestionable fact that current math coexists with strange results, weird conventions, unproven conjectures and unsolved problems not yet satisfactorily explained. That is why, "thinking outside the box", I assumed that there must be something wrong with the foundation over which we have been building this giant structure we call mathematics. In view of that, I granted myself the right to adopt a reasoning approach completely free from any prevailing math fundamentals.

ARITHMETIC OPERATIONS

There are three main types of arithmetic operations; each type comprises two opposite operations, and one is the inverse of the other, as follows:

Sum and subtraction

Multiplication and division

Power and rooting

MEANINGFUL ARITHMETIC AND ALGEBRAIC EXPRESSIONS

Any meaningful arithmetic or algebraic expression (formulas, equations and others) contains two sides, with terms separated by the equality sign[15]. Each side may show two classes of terms: a class of terms commanded by the plus (+) sign, other class of terms commanded by the minus (–) sign.

A term in an equality may be a single number, a single letter or a combination of numbers, a combination of letters or a combination of numbers and letters. It means that a term may imply any of the arithmetic operations above referred (addition, subtraction, multiplication and the like).

[13] MANNARINO, Remo, *ohomemhorizontal.blogspot.com*.
[14] As "values", I refer to terms (numbers and/or letters) in math expressions.
[15] Except when there exists a meaningful commandment, as "factorial operations" and "limit of math expressions" or dealing with inequalities.

However, at the end of the road, we deal with the balance between a total amount commanded by the plus sign (an absolute value) and a total amount commanded by the minus sign (another absolute value). The final difference between these two classes of terms that form any math expression, no matter which sign commands that difference, also is an absolute value. Obviously, the sign of the resulting balance may require interpretation[16].

It is mandatory to keep in mind that numbers and letters, as well as the terms they form in any math expression represent things of nature, as dollars, books, inhabitants and similar elements, which are neither positive nor negative. When quantifying things of nature with numbers, we deal with absolute values, except by convention.

That is why I state that any arithmetic or algebraic equality only deals with addition and subtraction of absolute values. The plus (+) and the minus (–) signs only mean addition and subtraction of absolute values represented by numbers, by letters or by the combination of numbers and letters. Since numbers are not positive or negative figures, we may infer that we do not need the theory of complex numbers.

ALGEBRAIC NOTATION

To clarify the statement, let us see how to find the balance “B” of a person’s bank account, knowing that, by convention and tradition, said balance is the difference between that person’s credits “C” (positive values) and debts “D” (negative values). Clearly, a positive balance means “credit”, while a negative balance means “debt”. In a real world approach, we should express “B” as:

$$|B| = |C| - |D|$$

In terms of algebra, we use a different notation:

$$+ B = + C - D$$

Assuming that “$|C| > |D|$”, we obtain “$+ B = + |C| - |D| = + |\Delta|$”, what correctly means that said person’s balance is a credit, since we end up with a proper notation (a true equality) that “$+ |B| = + |\Delta|$” or “$B = |\Delta|$”.

However, if “$|C| < |D|$”, we obtain “$+ B = + |C| - |D| = - |\Delta|$”, what correctly means that said person’s balance is a debt, but we end up with an improper notation (a false equality) that “$+ |B| = - |\Delta|$” or “$B \neq |\Delta|$”.

In this example, we should use a notation for “B” in a manner to make clear that said person’s balance could be positive (credit), negative (debt) or zero (null), as “$\pm B = + C - D$”. When dealing with a more general math expression, we need to consider that numbers and letters are absolute values. In case we end up with an improper equality, the result would require interpretation (as in case of equations).

[16] This is particularly true when dealing with equations, a math expression that algebraically equates a previous problem of the real world. The unknown value is the answer to the given problem.

RULE OF SIGNS

As previously said, current math adopts the misleading concept that we deal with positive and negative numbers and/or letters when handling arithmetic and algebraic equalities, what requires the so-called "rule of signs". Numbers and letters in arithmetic and algebraic equalities represent things of nature, as the number of inhabitants, books, dollars and the like, which are neutral elements. That is why numbers and letters in algebraic equalities are neutral elements (absolute values). These numbers and letters form terms, which are parcels of a same nature, each parcel commanded by the plus sign or by the minus sign. The so-called "rule of signs" does not exist.

TERMS OF MATH EXPRESSIONS

The square (or square root) of a positive or a negative value (number, letter or a combination of both) has no meaning if seen isolated[17]. Such element has a meaning only when it is a part of an arithmetic or an algebraic equality, either a term commanded by the plus sign or a term commanded by the minus sign. As a term of arithmetic or algebraic equalities (and complying with the rule applicable to addition and subtraction operations), the operations within said term must obey the commandment of the sign in front of it.

Apparently, the practical results would be the same, and the reader may say that my propositions are mere semantics. That is not the case. Besides, some points I raised are the cause of many strange and inconsistent results yielded by the math model in use.

In fact, when handling terms of arithmetic and algebraic equalities we apply the rule applicable to addition and subtraction operations to other operations, as multiplication and division. That rule applicable to addition and subtraction requires that, in any arithmetic operation, the plus (+) sign in front of a value tells us to keep the sign of the reaming values, while the minus (–) sign in front of a value tells us to change the sign of the remaining values, mere addition and subtraction commandments (algebraic sum), as follows:

$$y = x + 3^2 = |x| + |3^2| = x + 9$$

$$y = x - 3^2 = |x| - |3^2| = x - 9$$

$$y = x + \surd(+ 9) = |x| + \surd(+ |9|) = x + 3$$

$$y = x - \surd(+ 9) = |x| - \surd(+ |9|) = x - 3$$

$$y = x + \surd(- 9) = |x| + \surd(- |9|) = x - 3$$

$$y = x - \surd(- 9) = |x| - \surd(- |9|) = x + 3$$

$$z = + (a - b)^2 = + [(+ |a| - |b|)^2] = + (+ |a^2| - |2ab| + |b^2|) = + a^2 - 2ab + b^2$$

---

[17] That is the reason why current math accepts that "$+2^2 = -2^2 = +4 = 4$", but "$\surd 4 = \surd(+4) = +2$" or "$\surd 4 \neq -2$".

$$z = -(a-b)^2 = -[(+|a| - |b|)^2] = -(+|a^2| - |2ab| + |b^2|) = -a^2 + 2ab - b^2$$

Since numbers are absolute values, complex numbers do not exist. The theory of complex numbers is an unnecessary and useless concept, which only complicates mathematics.

ILLUSTRATING TABLES

The tables below illustrate what stated above, the application of addition and subtraction rules to arithmetic operations with absolute values, with the help of the concepts of credits and debts.

As usual, let us assume that credits are values commanded by the plus sign (positive values) and debts are values commanded by the minus sign (negative values). The reasoning is quite simple. If we add a credit to another credit, we will get a higher credit. If we add a credit to a debt, we may end up with a debt balance or with a surplus credit, depending on the figures. If we subtract a debt from a previous debt, we will have a smaller debt, if not zero (note that if we subtract a debt from a previous debt, which is greater than said previous debt, we will end up with a credit). We apply to arithmetic and algebraic operations the same addition and subtraction rules we see in the table below, nothing else.

ADDITION AND SUBTRACTION OPERATIONS

START		ADD		SUBTRACT		BALANCE	
CREDIT	DEBT	CREDIT	DEBT	CREDIT	DEBT	CREDIT	DEBT
100		50				150	
150			30			120	
120				20		100	
100					30	130	
	80	20					60
	60		50				110
	110			30			140
	140				50		90
60			100				40
	60	60					0
	90				120	30	

The same occurs when performing multiplication and division operations. The positive sign in front of the first element tells us to keep the sign of the second element, while the minus sign in front of the first element tells us to change the sign of the second element.

The plus or the minus sign in front of a resulting product or quotient only indicates to what class of terms said term (product or quotient) belongs in the arithmetic or algebraic equality we deal with, either the class of terms commanded by the plus sign or the class of terms commanded by the minus sign.

MULTIPLICATION AND DIVISION OPERATIONS

START		MEANING		PRODUCT	QUOTIENT
a.b	**a/b**	**ab**	**a/b**	**ab**	**a/b**
(+a)(+b)	(+a)/(+b)	+(+|ab|)	+(+|a/b|)	+ |ab|	+ (|a/b|)
(+a)(–b)	(+a)/(–b)	+(–|ab|)	+(–|a/b|)	– |ab|	– (|a/b|)
(–a)(+b)	(–a)/(+b)	– (+|ab|)	– (+|a/b|)	– |ab|	– (|a/b|)
(–a)(–b)	(–a)/(–b)	– (–|ab|)	– (–|a/b|)	+ |ab|	+ (|a/b|)

FUNDAMENTAL AXIOM OF MATHEMATICS

As a synthesis of all above said, I enunciated in previous works[18] an axiom I named "Fundamental Axiom of Mathematics":

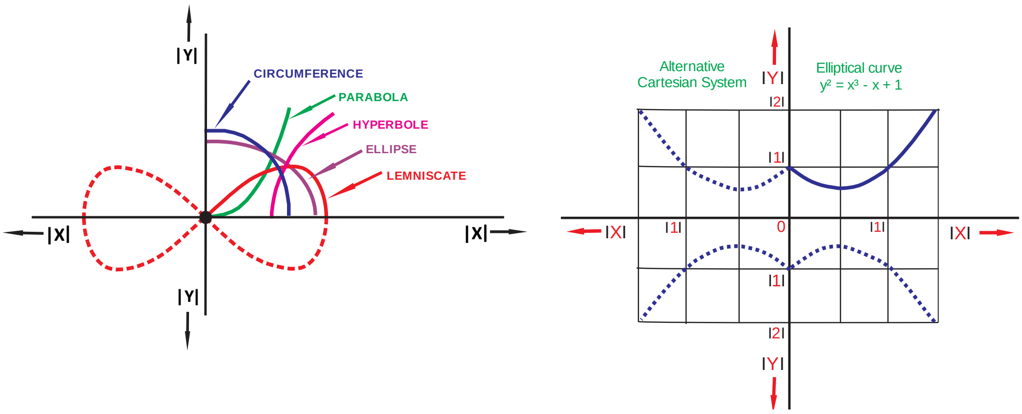
> *Numbers and/or letters, either constants or variables, which form terms of valid arithmetic and algebraic equalities, are absolute values.*

That Axiom implies that the coordinates in the Cartesian system are absolute values (alternative Cartesian system of coordinates), what means we only need one quadrant (often the first one) to represent geometric figures.

The first graph in Figure 01 shows classical geometric figures (circumference, parabola and others), while the second graph shows an abstract geometric figure (elliptic curve), as seen under said alternative Cartesian system of coordinates.

[18] AMUI, Sandoval, You may not enjoy mathematics (but you do not have to hate it), AYA Editora, 2022.

Figure 01: Alternative Cartesian system of coordinates



The above understanding allows us to conclude that there is no "rule of signs" other than the arithmetic rules we use when performing addition and subtraction in handling arithmetic and algebraic operations. We add or subtract absolute values.

EXAMPLES

Some few elementary examples will illustrate the above concepts and demonstrate that my concepts yield the same useful results current applied math does, but is free from strange results and inconsistencies, and does not need any weird convention or poor explanation:

1st) If handling the expression "2ax – 3 = 3by + 5", we have to understand we deal with absolute values as follows:

$$+ |2ax| - |3| = + |3by| + |5|$$

The plus and the minus signs only mean addition and subtraction of absolute values, in a manner that we end up with a balance between a total amount commanded by the plus sign and the total amount commanded by the minus sign (the difference between two absolute values).

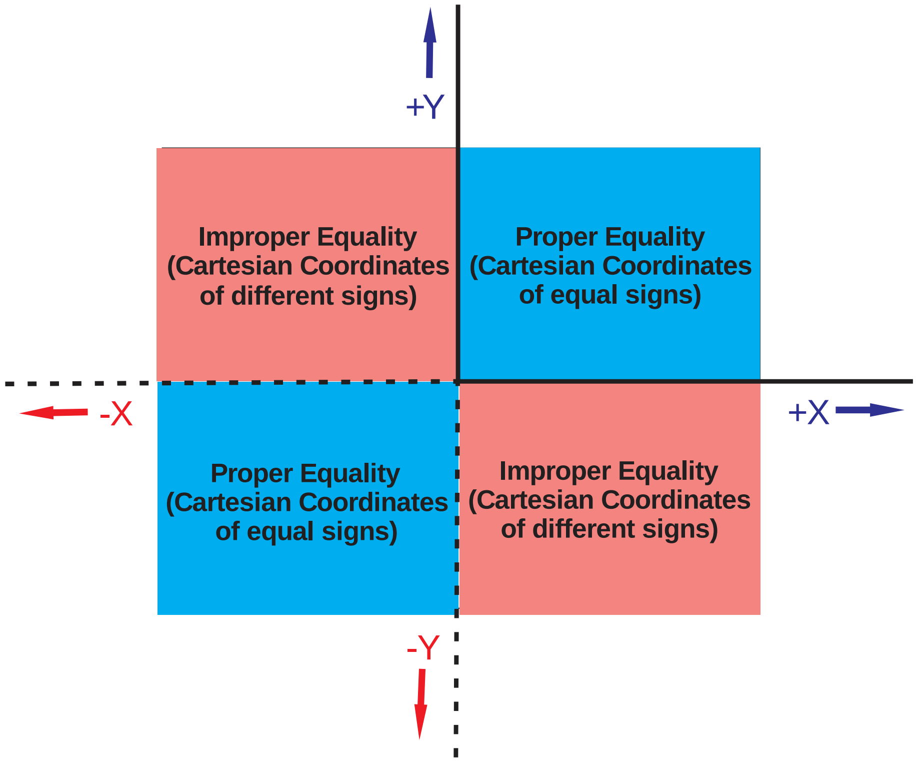
2nd) Consider the algebraic expression "y = x – 5" or algebraically, "+ |y| = + |x| – |5|":

We only have a proper algebraic equality when "|x|" is greater than "|5|". For instance, if "|x| = |8|", "+ |y| = + |3|, a consistent notation, and a case under which a positive value is equivalent to an absolute value (y = |3|).

If "|x|" is equal to or less than "|5|", we face an improper algebraic equality. Assuming "|x| = |1|", "+ |y| = – |4|, an inconsistent notation, and a case under which a positive value is not equivalent to an absolute value (y ≠ |3|).

Figure 02 illustrates the concept of proper (true) and improper (false) algebraic equalities.

Figure 02: Proper and improper algebraic equalities



3rd) Consider the algebraic expression "$-y = \sqrt{(x-5)}$":

If "x" is less than "5", we will have a proper equality. Assuming that "$x = 1$", then "$-y = \sqrt{(1-5)} = \sqrt{(-4)}$" and "$-|y| = -|2|$", also a consistent notation ($y = |2|$). This example leads us to two conclusions: (i) imaginary numbers do not exist; (ii) both positive values and negative values are equivalent to absolute values when we deal with proper (true) equalities.

4th) Consider the algebraic equality "$x^3 = 1$":

Under the prevailing concepts, we face a 3rd-degree equation, which has one real root and two imaginary roots.

According to my principles, we see a ludic algebraic expression[19], and there is only one acceptable value for "x":

$$+|x^3| = +|1| \qquad +|x| = +|1| \qquad x = |1|$$

Under this approach, we have a valid notation and a proper (true) equality. That is a case that a positive value is equivalent to an absolute value.

In case we have "$x^3 = -1$":

[19] The algebraic expression is not an equation, unless it mathematically equates a real problem.

There is no acceptable value for "x" because it contradicts the Fundamental Axiom of Mathematics.

$$+ |x^3| = - |1| \qquad + |x| = - |1| \qquad x \neq |1|$$

We have an invalid notation and an improper (false) equality. That is a case that a positive value is not equivalent to an absolute value.

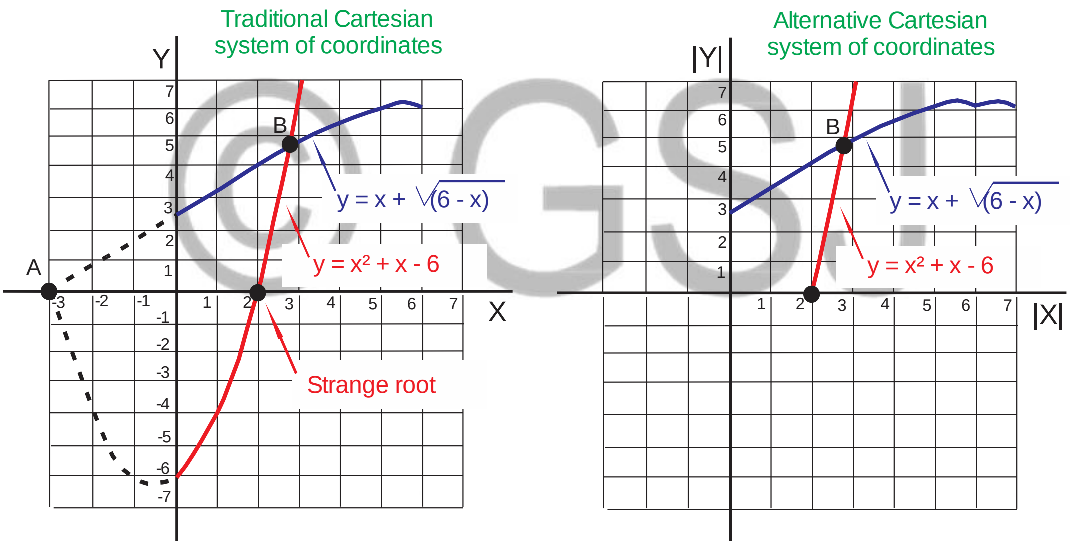
The above examples illustrate why positive values are not necessarily equivalent to absolute values[20], as accepted under the prevailing math concepts.

5th) Let us now consider the "so-called" irrational equation, "$y = x + \sqrt{(6 - x)}$":

We solve this question by mistakenly assuming that the given expression is equivalent to "$y = x^2 + x - 6$". When solving it, we obtain two roots: "$x_1 = -3$" and "$x_2 = +2$". We also see that "$x_1 = -3$" does fit the original math expression, while "$x_2 = +2$" does not.

We say "$x_2 = +2$" is a "strange root", as seen in the first graph of Figure 03.

Figure 03: Irrational equation



Under my approach, we deal with two different math expressions, and neither one is an equation. The given expression is a meaningless random algebraic expression, while the modified expression is one of the infinite ways to represent the formula of a same parabola geometrically defined[21]. It is necessary to understand the given expression as follows:

$$+ |y| = + |x| + \sqrt{(+ |6| - |x|)}$$

[20] For clarity, we used double vertical straight-line segments as to indicate absolute values.
[21] The algebraic expression, "$y = ax^2 + bx + c$", known as a 2nd-degree equation, is nothing else but the generic formula of a parabola, whose symmetry axis is parallel to one of the axis of the Cartesian system of coordinates.

That particular math expression is a valid equality for any absolute value of “x”, since “+ √(+ |6| – |x|)” will always be less than “+ |x|”, even when “+ √(+ |6| – |x|)” is a negative value, in a manner that “+ |y| = + |Δ|”, as illustrates the second graph in Figure 03. To clarify the statement:

If “x = |0|”, “√(+ |6| – |x|) = √+ |6|”, and “+ |y| = + |2.45|”, what shows a proper (true) equality, “+ |y| = + |Δ|”.

If “x = |55|”, “√(+ |6| – |x|) = √– |49| = – |7|”, and “+ |y| = + |55| – |7| = + |48|”, what also shows a proper (true) equality, “+ |y| = + |Δ|”.

In case we were dealing with the math expression “+ |y| = – |x| + √(+ |6| – |x|)”, no absolute value of “x” would make it a valid expression, since we would end up with an improper (false) equality “+ |y| = – |Δ|”.

This fifth example shows that we do not face a “strange root”. In fact, we deal with a “strange mathematics”. It illustrates why we do not need the so-called “rule of signs”, as well as the theory of complex numbers. The example also shows it is possible to use a much simpler and more logical math model.

FINAL COMMENT

My approach relies on three principles: simplicity, logic and commonsense. Whenever we need to find a model that fits certain given data, the simpler the better.

The so-called rule of signs is a mistaken concept, which only creates difficulties to math users. In some cases, it introduces inconsistencies, as a (positive) credit that results when we square a (negative) debt. In practice, we only use addition and subtraction rules.

When performing multiplication and division operations, the plus sign in front of a first value, tells us to maintain the sign of the remaining values, while the minus sign in front of a first value, tell us to change the sign of the remaining values, as illustrated below:

A = (– 1)(+ 3)(– 2)(– x)(+ y)

A = (– 1)[(+ 3)(– 2)(– x)(+ y)]

A = (– 3)[(– 2)(– x)(+ y)]

A = (+ 6)[(– x)(+ y)]

A = (– 6x)[(+ y)]

A = – 6xy

TOPIC III

USE OF "ZERO"
"Things of Nature" and "Operating Commandments"
"Counting numbers" and "Operators"

INTRODUCTION

In this Topic III, I will present an alternative approach about:

The use of zero in arithmetic operations.

The implications of using "zero" when performing arithmetic operations certainly is one of the greatest difficulties under current mathematics, possibly even worse than the difficulty of dealing with non-terminating decimals. In performing arithmetic operations using zero we face strange results and odd situations, which implies the need of weird conventions to accept wrong conclusions, as follows:

(a) If we multiply a real number "$m \neq 0$" by "0", the result is "0", but we divide that real number "$m \neq 0$" by "0", or we divide "0" by "0" the result is an indetermination;

(b) The result of any real number (including "0") raised to "0" is "1", as "$0^0 = 1^0 = m^0 = 1$", what could take us to interpret that "0 = 1 = 2 = … m";

(c) Under current concepts, "0! = 1! = 1. This concept allows us to interpret that "0 = 1".

Math has not so far overcome these two math difficulties above referred. We solved the problem relating to non-terminating decimals by accepting approximate results. We have not solved the problem of division by zero. In this paper, I have a suggestion for the latter problem (not yet fully analyzed, I emphasize), which may look complicate, but in fact it is quite simple. Apparently, my suggestion would cause a mess, although that is not the case. Mathematically speaking, it is at a child's level of understanding.

I imagine my proposition will explain the use of "0" as a divisor when performing arithmetic operations. However, the analysis of the suggestion here introduced requires the understanding of some concepts the author has already published, including a distinction between a "math for the real world" (relevant in practice and consistent with field observations and laboratory tests) and an "abstract math" (ludic exercises, with no practical application).

When we perform arithmetic operations, we use specific "operating commandments" to indicate the operation we have to perform. The plus sign ("+") indicates addition and the minus sign ("–") indicates subtraction of "terms" in math equalities. We add or subtract parcels of the same nature (books, students, dollar amounts and the like).

$A = + (2x) - (x/3) + 5 - 1$

We use brackets (or the letter "x") to indicate multiplication and a "slash" (or other graphical sign) to indicate division of a given value "n" by a factor "f".

$B = (n)(f) = (5)(3) = 15$ or $B = n \text{ x } f = 5 \text{ x } 3 = 15$

$C = n/f = 12/3 = 4$ or $C = n : f = 12 : 3 = 4$

Similarly, an exponent "n" indicates the power we shall raise a base number "a", and the "rooting symbol" associated with a number "m" indicates the "m^{th}" root of a given value "b".

$D = a^n = 2^4 = 16$

$E = \sqrt[m]{b} = \sqrt[3]{27} = 3$

There are other operating commandments indicated by specific symbols or math conventions. We indicate a number factorial by the exclamation symbol "!", as in the case of finding the permutations of "s" elements "E_i" (whatever the elements "E_i").

$P_{(s)} = s! = (s)(s-1) \dots (1)$".

We indicate the limit of a function by a specific notation:

$K = \lim_{x \to a} f(x)$

Similarly, there are other symbols for derivatives, partial derivatives, integrals and others.

$d^n y/dx^n$ $\partial z/\partial x$ $\int f(x)dx$

However, when handling math expressions, in addition to the above signs, symbols and conventions indicating operating commandments, we may use numbers and letters in two different ways[22]. When representing things of nature, as dollar amounts, a certain quantity of books, a volume of a water tank, and the like (as terms of math expressions), numbers and letters perform as "counting numbers". When they perform as factors or indicating operating commandments in math expressions (multipliers, divisors, exponents and rooting indexes), numbers and letters function as "operating numbers" ("operators"). In performing two different roles, numbers and letters have distinct attributes and properties, and we must make a clear distinction when they perform one role or the other, what current math does not. This is particularly true in respect of number "zero", the object of this paper.

[22] MANNARINO, Remo, *ohomemhorizontal.blogspot.com*. Formulas used in connection with other sciences (as geometry, physics and others) require a different understanding, but they are not included in the scope of this paper.

For clarity, I will use the green color to indicate numbers and letters representing things of nature (counting numbers) and the red color to indicate numbers and letters as operators. I will also underline numbers and letters when they function as operators.

Since the appearance of the number "0", we should have accepted that "0" became the first number in the sequence of counting numbers, when it quantifies things of nature. Number "1" is no longer the first number in that sequence, as we went on using. In the sequence of counting numbers, number "1" would be the second number, and so on.

On the other hand, "0" does not exist as an operating number. Number "1" remains as the first operating number. We will use operating numbers smaller than "1" (fractional numbers), but not "0". A factor "f" less than "1" in a given operation (let us say, a multiplication) is equivalent to the inverse factor "1/f" in the inverse operations (a division).

This innovative concept is quite important to differentiate the case a number functions as an operating number (operator) from the case it performs as a counting number in any arithmetic operation.

Then, in the sequence of counting numbers representing things of nature (books, cars, dollars, and the like), we should consider that:

> "0" became the "first" counting number in the sequence, as in "There are zero books on the table";
>
> "1" became the "second" counting number in the sequence, as in "There is one car in the garage";
>
> "2" became the "third" counting number in the sequence, as in "He has two dollars in his pocket".
>
> ……….

On the other hand, "zero" and "infinity" are not figures. They are concepts and I will show we cannot use them as operators, as we do with common numbers.

As emphasized in previous books and papers, it is relevant to keep in mind that numbers are mere indicators of quantities, in a manner that the plus and the minus signs only mean addition and subtraction of terms (absolute values) in valid math expressions. The plus and the minus signs, as well as the factorial commandment, limit indication and integral of a math expression, all of them function as operating commandments, but they already have their symbols, meanings and uses.

> *In addition to the innovative concepts presented in previous published material, the adoption of the concepts introduced in this paper about the use of zero would imply a new understanding of math expressions, and a new way to perform math operations. Indeed, a major shift regarding the science of math.*

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OPERATIONS AND OPERATORS

The approach to zero now proposed would imply a new math notation and a compatible way to perform operations, by making a distinction when a number represents a quantity of a certain "thing of nature" (bananas, oranges and books), and when it performs as a mere "operator" (multiplier, divisor, power exponent and rooting index), what current math does not. My suggestion implies an innovative way to understand multiplication, division, power and rooting.

Table 01 shows the sequence of numbers performing as "counting numbers". We see that "0" is the first element in that sequence. We also see "ꝏ" in the sequence.

Table 01

SEQUENCE OF NUMBERS QUANTIFYING "THINGS OF NATURE"

Counting numbers													
0	1	2	3	…	k-1	k	k+1	…	m–1	m	m+1	…	ꝏ

Table 02 shows the sequence of numbers functioning as "operators"[23], and we must notice that "0" and "ꝏ" do not appear, because they are not figures and cannot perform as operators.

Table 02

SEQUENCE OF NUMBERS FUNCTIONING AS "OPERATORS"

Operators													
…	…	0.5	…	1	2	…	k	…	…	m-1	m	…	…

The sequence in Table 01 contemplates "0" and "ꝏ" as counting numbers, because they have meaning when used as that. We may say "There are zero ("0") books on the table" (meaning the absence of books on the table) as well as "Math deals with a set of an infinite quantity ("ꝏ") of odd numbers" (meaning there are countless odd numbers in the universe of numbers).

However, the sequence in Table 02 does not include "0" or "ꝏ" because they are not figures, they do not exist as operators. They are concepts, since "ꝏ" means a countless quantity of a certain thing of nature, while "0" means the absence of that thing in any universe of things considered. Therefore, an operator can be a very small or a very large number, including any real number, but not "0" or "ꝏ".

[23] Tables 01 and 02 show whole numbers, but the sequences comprise other real numbers.

Since we cannot use zero (“$\underline{0}$”) and infinity (“$\underline{\infty}$”) as operators, the following notation in Table 03 are meaningless math expressions (“n” being any real number in the sequence of counting numbers, including “0”):

Table 03

Invalid use of (“$\underline{0}$”) and (“$\underline{\infty}$”) as operators									
With “$\underline{0}$”	$\underline{0}n$	$n/\underline{0}$	$n^{\underline{0}}$	$(\underline{0})(\underline{0})$	$\underline{0}/\underline{0}$	$0/\underline{0}$	$\underline{0}!$	$0^{\underline{0}}$	
With “$\underline{\infty}$”	$\underline{\infty}n$	$n/\underline{\infty}$	$n^{\underline{\infty}}$	$(\underline{\infty})(\underline{\infty})$	$\underline{\infty}/\underline{\infty}$	$\infty/\underline{\infty}$	$\underline{\infty}!$	$\infty^{\underline{\infty}}$	

However, we may use “0” as a counting number. Then, we can say that for any real number “$\underline{m} \neq \underline{0}$” and “$\underline{m} \neq \underline{\infty}$” functioning as an operator:

$\underline{m}0 = 0$ $\quad$ $0^{\underline{m}} = 0$ $\quad$ $0/\underline{m} = 0$ $\quad$ $\sqrt[\underline{m}]{0} = 0$

We may accept “$(\underline{1})(0) = 0$, but not “$(\underline{0})(1)$”. In addition, we may accept “$0^{\underline{1}} = 0$” but not “$\underline{0}^1$”.

Within some restrictions, it is valid to multiply or to divide two or more than two operators, “$(\underline{m_1})(\underline{m_2})$” or “$\underline{m_1}/\underline{m_2}$”, provided they are different from “$\underline{0}$” and “$\underline{\infty}$”. Examples:

$(\underline{3})(\underline{4})(2) = 24$ $\quad$ $[(\underline{4})/(\underline{3})](12) = 16$

If we write “(5)(2)(3)(6) oranges”, it is necessary to make clear what figure quantifies the numbers of oranges (counting number), and what numbers function as operators, as follows:

“$(\underline{5})(\underline{2})(\underline{3})(6)$ oranges = 180 oranges

As an example, we can divide “zero” oranges among “five” kids (“zero” oranges for each kid), but we cannot divide “five” oranges among “zero” kids. Although subtle, there is a difference when a given number quantifies a “thing of nature” (including “zero”) and when that number (excluding “zero”) functions as an “operator”.

Based upon the concepts above, and since “$\underline{0}$” and “$\underline{\infty}$” are not figures, we conclude that:

The employment of “$\underline{0}$” and “$\underline{\infty}$” as operators does not create an indetermination. It creates an invalid operation.

When we deal with counting numbers (quantifying things of nature), we need to have a different understanding:

$\underline{m}a = b$, $\quad b/a = \underline{m}$,

If “a” = “b”, “$\underline{m} = \underline{1}$”, and

$12/2 = \underline{6}$ $\quad$ since $\quad$ $(\underline{6})(2) = 12$

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$0/0 = \ldots\ 12/12\ \ldots = m/m = \underline{1}$ since $(\underline{1})(0) = 0$, $(\underline{1})(12) = 12$ $(\underline{1})(m) = m$

If Mary has "m" books, and John has "m" books (whatever the value of "m", including "zero"), the number of Mary's books is equal to the number of John's books ($m/m = 0/0 = \underline{1}$). We may say that "$0/0 = \underline{1}$" because "$(\underline{1})(0) = 0$".

However, "$\underline{0}/\underline{0}$", "$0/\underline{0}$", and "$\underline{0}/0$" are all invalid operations, because "$\underline{0}$" is not a counting number and "0" is not an operator.

On the other hand, for "$\underline{r} \neq \underline{0}$", "$\underline{r} \neq \underline{\infty}$", "$\underline{s} \neq \underline{0}$" and "$\underline{s} \neq \underline{\infty}$", we can say that "$\underline{rs}$" or "$\underline{r}/\underline{s}$" is a meaningful operating commandment, provided we are aware of what we understand by these operations, as follows:

> *Norman missed one math class this week. John missed three times more math classes than Norman, while Henry missed two times more math classes than John. "One" is a counting number (the number of Norman's absences). "Three" and "two" are operators. Then, Henry missed "$\underline{3}x\underline{2}x1 = 6$" math classes ("six" is a counting number, the number of Henry's absences).*
>
> *Carol missed eight classes of geometry, twice more than Nancy's absences, since Nancy only missed four classes. Debora's absences was half of Nancy's. "Twice" and "half" are operators; "eight" and "four" are counting numbers. The total of Debora's absences amounts to "$(8/\underline{2})/\underline{2} = 2$". "two" is a counting number.*

Examples of valid operations:

$(\underline{3})(0) = 0$	$0/\underline{3} = 0$	$(\underline{5})(2) = 10$	$(\underline{0.7})(4) = 2.8$	$4/(\underline{0.5}) = 8$
$(3)(\underline{2}) = 6$	$6/\underline{2} = 3$	$(\underline{2})(\underline{3})(\underline{10})(3) = \underline{60}(3) = 180$		

Examples of invalid operations:

$(\underline{0})(0)$	$\underline{5}/4$	$(\underline{0})(1)$	$(\underline{0})(0.7)$	$\underline{8}/(0.5)$
$(5)(2)(4)(\underline{3})$				

> *It is mandatory to keep in mind that the improper use of an operator different from "zero" and "infinity" (as "$\underline{6}/2$") is not a major problem, since the practical result would be the same. However, when using "zero" or "infinity" as an operator, we may face problems (as $3/\underline{0}$"). That is why we should not use "zero" or "infinity" as an operator.*

As previously said, the adoption of the concepts above introduced would imply the acceptance of alternative math fundamentals, a new understanding about math expressions, and a new way to perform arithmetic operations. The approach would need verification and, if feasible, would require the implementation of a new math model, a task not contemplated in this paper.

> *Indeed, and unfortunately, at a high "overall cost".*

NUMBERS AND THEIR USE

As explained, I believe there is an alternative approach to overcome the problem of using the number zero as a common number. The practical results would be the same, but we would employ a different concept (we will not use "$\underline{0}$" or "$\underline{\infty}$" as an operator). Operating numbers will be any real number (either integer, fractional or irrational numbers, but not "zero"). Operators will always be equal to or greater than "$\underline{1}$" in any arithmetic operations (I will revert to this matter at a later moment).

$(36)(\underline{0.5}) = (36)/(\underline{1}/\underline{0.5}) = (36)(2) = 72$

$(25)/(\underline{0.5}) = (25)(\underline{1}/\underline{0.5}) = (25)(2) = 50$

OPERATIONS

As previously stated, there are three basic operations, and each one comprises two opposite operations, one being the inverse of the other, as follows:

Addition and subtraction

Multiplication and division

Power and rooting

Addition and subtraction

Addition and subtraction mean an algebraic sum of parcels of a same nature. Said parcels are absolute values, which may contain numbers, letters and the combination of numbers and letters. Those numbers and letters and their combinations may be counting numbers (quantifying things of nature) or perform as operating numbers (operators). In any math equality, there are two types of parcels, one type commanded by the plus sign, the other commanded by the minus sign, and we look for the resulting balance.

The reasoning is quite simple. If we add a credit to another credit, we will get a higher credit. If we add a credit to a debt, we may end up with a debt balance or with a surplus credit, depending on the figures. If we subtract a debt from a previous debt, we will have a smaller debt, if not zero (note that if we subtract a debt from a previous debt, which is greater than said previous debt, we will end up with a credit). Figure 01 clarifies that reasoning about addition and subtraction:

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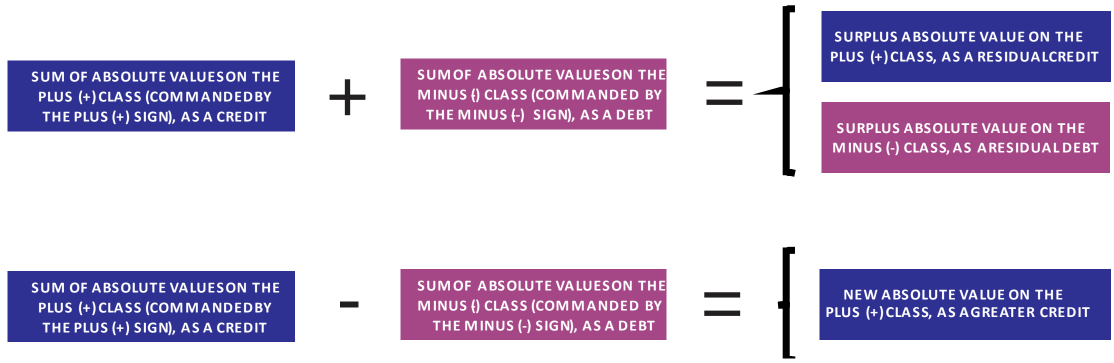
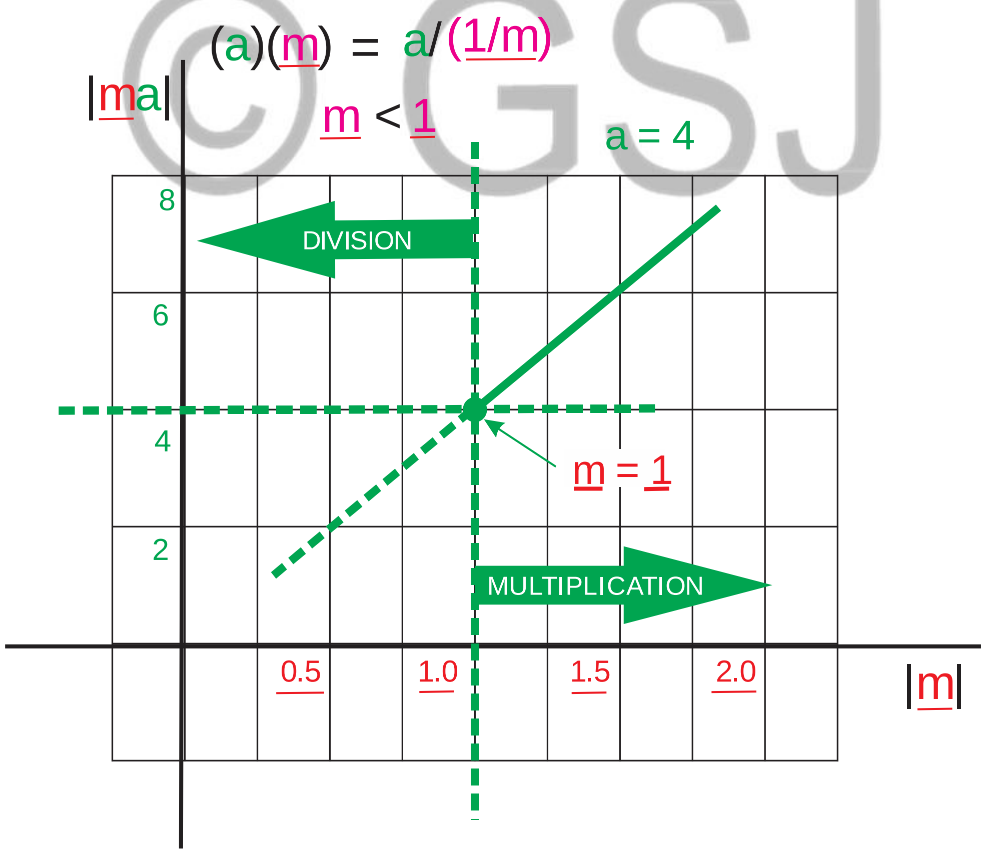
Figure 01: Addition and subtraction



Multiplication and division

Multiplication and division are a same arithmetic operation. If we multiply a number expressing a thing of nature “n > 1” by an operator “m > 1”, the result increases. On the other hand, if we multiply that number “n > 1” by an operator “m < 1”, the result decreases, because in fact we performed a division of “n > 1” by the operator “1/m” (the dashed line at left side of “m = 1”). Figure 02 illustrates that[24]:

Figure 02: Multiplication



[24] Figures for illustrative purposes.

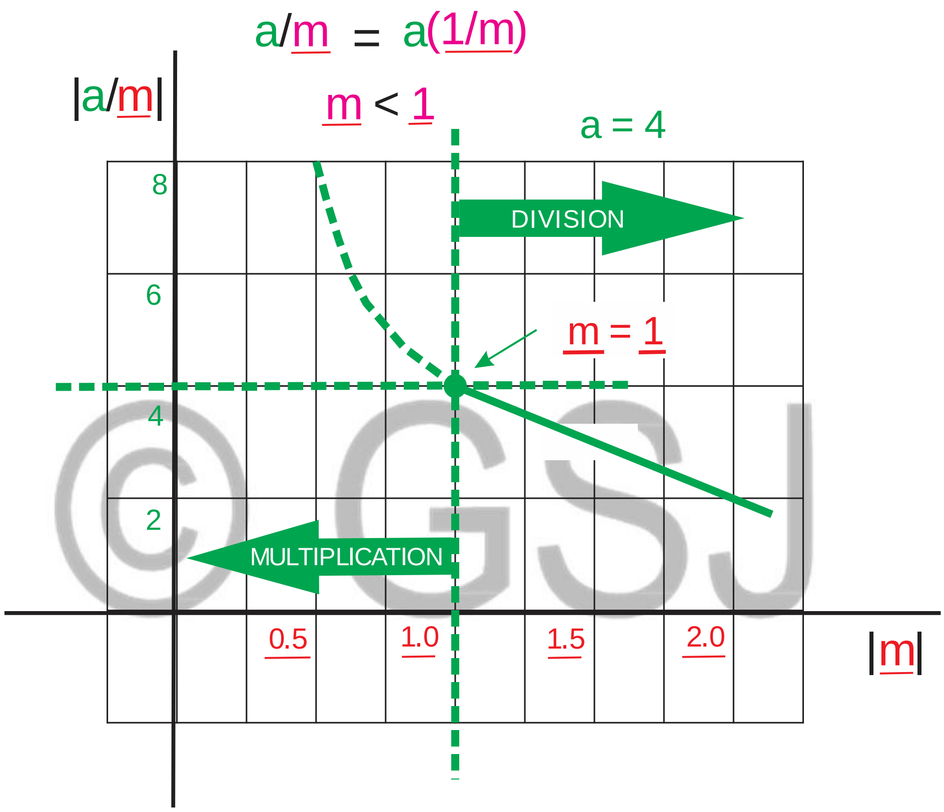
Example:

m.n = 1.5n Multiplication

m.n = 0.5n = n/(1/0.5) = n/2 Division

If we divide a number expressing a thing of nature “n > 1” by an operator “m > 1”, the result decreases. On the other hand, if we divide that number “n > 1” by an operator “m < 1”, the result increases, because in fact we performed a multiplication of “n > 1” by the operator “1/m” (the dashed line at left side of “m = 1”). Figure 03 illustrates that:

Figure 03: Division



Example:

n/m = n/1.5 Division

n/m = n/0.5 = n(1/0.5) = 2n Multiplication

Given any real number performing as a counting number “n > 1”, the greater the multiplier “m > 1”, the greater the result “mn”, but we will never reach “mn = ꝏ”. Similarly, we will never reach “n/m = 0” when dividing a certain counting number “n > 1” by “m > 1”, no matter how big “m” can be.

Power and rooting

The same effect occurs when handling power and rooting, as in Figures 04 and 05.

Figure 04: Power

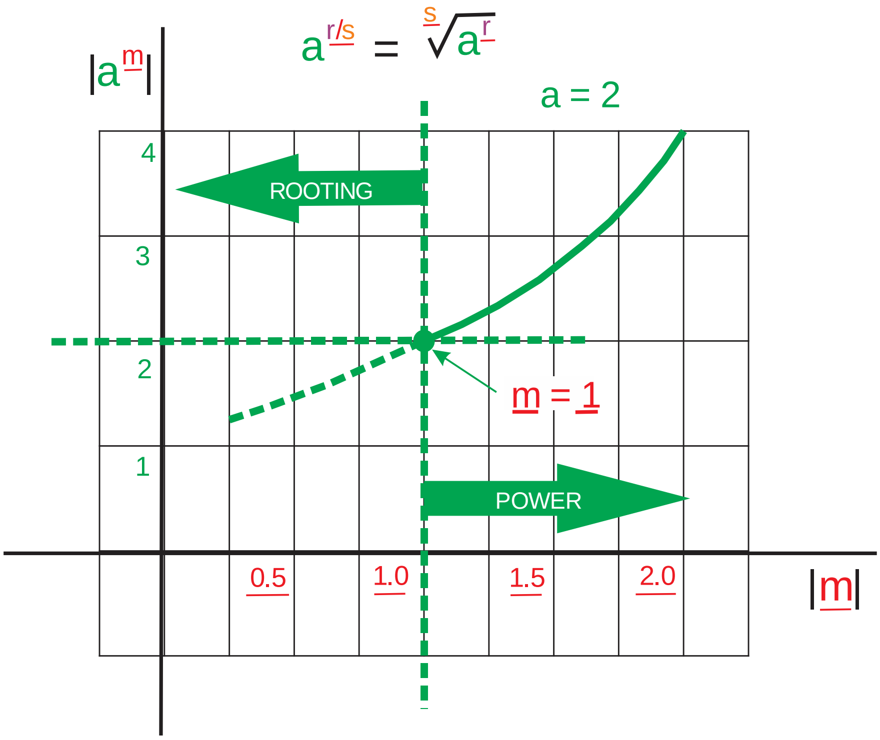
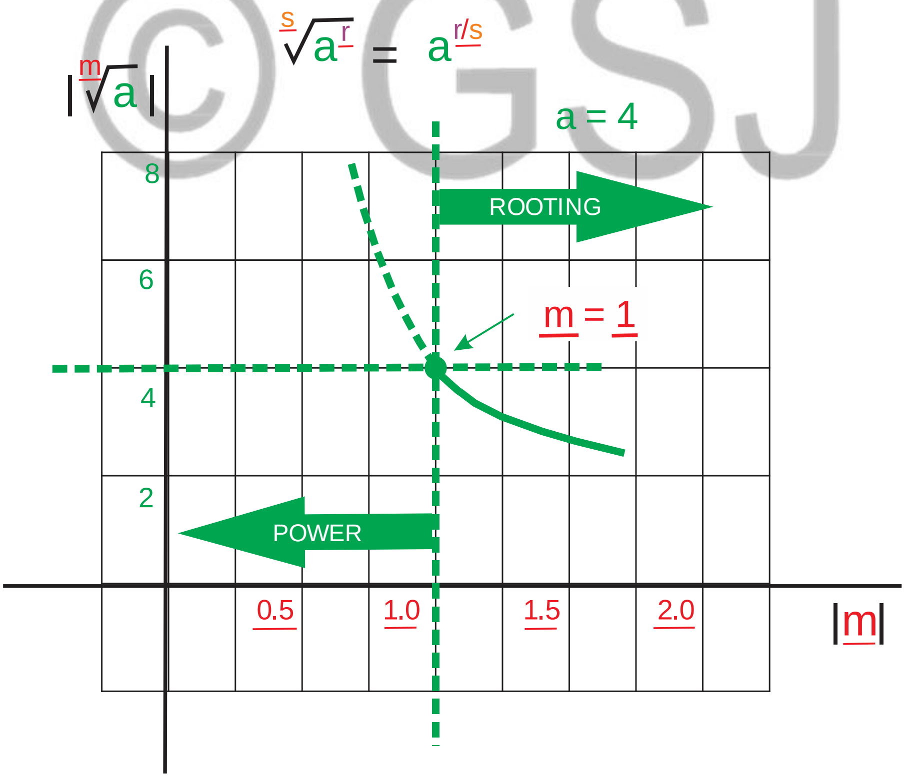


Figure 05: Rooting



In case of “m < 1”, the reasoning would be the other way around. We can conclude that multiplication and division are a same operation, as well as power and rooting. What distinguishes one from another is the operator (if greater than or smaller than “1”).

It is necessary to keep in mind that, contrarily to the present understanding, we have to recognize there are restrictions to accept when dealing with arithmetic and algebra, except when we handle aleatory math expressions with no practical relevance, mere abstractions or ludic exercises, which do not equate any problem in the real world.

> *We have to accept an innovative meaning of multiplication and division, in a manner that a multiplication only occur when the result of "mn" or "n/m" is greater than "n", while a division only occurs when the result of "mn" or "n/m" is lower than "n", whatever the values of "m" and "n". A similar reasoning apply to power and rooting operations.*
>
> *In other words, directly or indirectly, in arithmetic operations, an operator will never be a number less than "1".*

Table 04 illustrates the sequence of operators, and the application of operators to "n = 8" things of nature (books, students, dollar amounts and the like).

Table 04
SEQUENCE OF OPERATIONS
(MULTIPLICATION)

n = 8 is a counting number (quantifies things of nature)										
---	0.25	---	0.5	---	1	2	3	---	---	Operators
---	0.25n	---	0.5n	---	1n	2n	3n	---	---	Operations
---	n/4	---	n/2	---	1n	2n	3n	---	---	Operations
---	2	---	4	---	8	16	24	---	---	Results

|........................ ($m < 1$) ....|($m = 1$)|.................. ($m > 1$) ....|

As from "1" up, the sequence of operators grows and the result of "m.n" tends to infinitely big (multiplication). As from "1" down, the sequence of operators decreases and the result of "m.n" tends to infinitely small (division). "0" is not an element in that sequence of operators, and we will never reach "m.n = ∞" or "m.n = 0".

FINAL COMMENT

In summary, we can use zero as a counting number, quantifying things of nature, because it makes sense. However, we cannot use zero as an operating number (functioning as an operator), because we will face an invalid and meaningless operation. Zero only exists as a counting number.

Notwithstanding the above statement, we can use fractional numbers as counting numbers and as operators. In performing an operation with a fractional number, it is equivalent to the inverse operation, when we consider the inverse of that fractional number (which becomes a number greater than one).

Differently from the present understanding, in a math model for the real world, a math expression must have meaning, and the results it yields must be consistent with field observations and laboratory measurements.

The present proposition on how to consider the number zero (not including it as an operator) need further analysis to verify if it is practically feasible and, in case it is feasible, further implementation to adjust the new concept to other related math matters, accordingly. A major shift, indeed.

This matter remains as an open question.

FINAL WORDS

In previous books and papers, I stated we have been practicing and teaching our students a bad mathematics, which coexists with strange results, weird conventions and other unclear matters. Current math model is a poor model because it relies on a poor foundation. In spite of that, the math community keeps adding concepts and theories to the present math model without any serious initiative to question its poor foundation.

I also stated it is feasible to have an improved math model by adopting different math fundamentals, particularly the concept of numbers, the main pile in the referred foundation, as well as by complying with the unbreakable interdependence of geometry and some branches of mathematics, as number theory, algebra and trigonometry.

If, as declared by some math experts, applied mathematics is a bad mathematics, pure mathematics is a bad mathematics too, because it relies on the same poor foundation. In its applied version, and in spite of its many limitations, mathematics allows us to handle real world matters. On the other hand, pure mathematics, as an abstract approach, not necessarily committed to solve any real world problem, has no practical use.

I also say that the math model in use originates certain unproven conjectures and unsolved problems (including some of the Millennium Problems), which are solely invalid statements, resulting from misleading math fundamentals.

I have not so far received any comment, either in favor or against my innovative and polemic propositions, and I am convinced that in near future nobody will pay attention to my ideas. I also know that academic authorities will not take any action to update math school programs and teaching methods to improve students' grades and achievements in their academic lives.

Nobody wants to step outside his or her comfort zone to face challenges and contradict the *status quo*. It means we will insist on using a poor teaching approach to teach our students a bad mathematics. It also means that students will go on with low grades and poor achievements, and repeating that mathematics is the most difficult discipline they face. In the same line, people in general will keep saying they hate mathematics.

This unpleasant situation will only change if we recognize the need to implement a major shift in respect of the science of mathematics, starting with its primeval concept, the concept of numbers. Indeed, a costly challenging decision to take.

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