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The improved iterative method for the numerical approximation of stiff initial value problems in ordinary differential equations

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Abstract

The new iterative method for the integration of stiff ordinary differential equations has taken a more prominent role in the scientific computing over the past decades. The new iterative scheme offers computational savings for many problems involving large stiff systems of differential equations. Careful design of a practical new iterative scheme is crucial, however, to ensure that the resulting method is efficient for a particular equation, we ensure that the method is efficient, effective and cost effective. In this paper we will state and discuss how important the new iterative scheme is, how to construct high-order and to use the method in the most effective way. In particular, we will address the interplay between the structure of a time integrator and the numerical linear and nonlinear stiff algorithms needed to evaluate the exponential-like functions of large stiff matrices efficiently using the new iterative method. Finally, we will present the results of applications of the new iterative integrator to show how effective the method can be used to portrait the actual representation of the stiff equations' natural or physical interpretation of what they represent in natural phenomena. Comparison with existing numerical methods highlights the advantages of the proposed iterative approach, particularly in terms of stability and convergence properties.

Keywords: Iteration method, System of differential equation, Stiff system,

(2010) Mathematics Subject Classification: 65L05, 65L06, 93C10

1. Introduction

Ordinary differential equations (ODEs) of the stiff variety are particularly difficult to solve numerically. It often occurs in situations when the solution shows abrupt changes or large variability across several scales of the independent variable. Physics, chemistry, biology, engineering, and other scientific and engineering fields all contain stiff ODEs.
In this sense, "stiffness" refers to the difference in the typical time scales of the relevant occurrences. Fast and slow processes are frequently present in stiff equations, with the fast processes taking place over significantly shorter periods than the slow processes. As a result, due to the equation's stiffness, numerical approaches have a difficult time adequately capturing the behavior of the solution. The difficulty with stiff equations is that conventional numerical methods for solving ODEs, such as the explicit Euler method, explicit Runge-Kutta methods, or explicit linear multistep methods, may be very unstable or inefficient when used directly on stiff problem. Since stiff equations require several iterations, these methods depend on small step sizes to ensure stability, which can be computationally expensive. Specialized numerical methods have been developed to deal with stiff differential equations [1]. Implicit methods, such as the backward differentiation formulas (BDF) of Gear methods, are one common type of method. These methods use backward differencing, where each time step's solution is implicitly stated in terms of subsequent time steps. Although implicit approaches require the solution of systems of nonlinear equations, which increases computer complexity, they are more stable for stiff equations. Using stiff solvers found in numerical software libraries is another strategy for dealing with stiff equations. These solvers use sophisticated approaches including automatic switching between explicit and implicit methods and adaptive step-size control dependent on the stiffness of the problem. Therefore, it is crucial to carefully choose appropriate numerical methods and solver algorithms based on the features of the particular situation while working with stiff differential equations. For stiff differential equations to be successfully solved, numerical analysis expertise and knowledge of the underlying physics or processes are essential.

Diverse numerical and analytical techniques are available for studying differential equations. However, the Daftardar-Gejji and Jafari Method (DJM) is one of the most efficient numerical approaches for solving differential equations. The DJM is used in [2] by Batiha et al. to solve the multispecies Lotka-Volterra equation. The differential transformation technique (DTM), variation iteration method (VIM), and Adomian decomposition method (ADM) are compared with the DJM to demonstrate how effective and dependable the DJM is for solving nonlinear equations. This approach was also employed by [3] to resolve the Riccati differential equations (RDEs). According to DJM's findings, the approach is precise and time-effective. The Daftardar-Gejji Jafari approach was taken into consideration by [4] for the approximate solution of some classical Riccati differential equations (RDES). This approach is straightforward in its use, simple to apply, and less stressful in terms of computing. To check the precision and effectiveness of their suggested strategy, three numerical instances were looked into. When compared to certain current methods, the results reached the exact solutions faster. For the purpose of resolving chemical kinetics systems that take the form of nonlinear ordinary differential equations, [3,4] adopted the DJM. The outcome demonstrated that the DJM is a successful method for locating approximate numerical solutions. Three iterative techniques were implemented by [5] to solve a number of second order nonlinear ODEs that appeared in physics. The iterative techniques that have been suggested include the Tamimi-Ansari (TAM), Daftardar-Jafari (DJM), and Banach contraction (BCM) approaches. Each technique may deal with a nonlinear term without making any assumptions. Their results are quantitatively compared to those of other numerical techniques including the

Runge-Kutta of order 4 (RK4) and Euler methods. Additionally, the Banach fixed point theorem was used to demonstrate the convergence of their suggested approaches. The findings of the maximal error of the remaining values demonstrated the efficiency and dependability of the approaches that were described. For calculations, they used Mathematica@10 software. The DJM was used by [6] to resolve the Painlev'e equation I. The results obtained were contrasted with those obtained by applying the ADM, HPM, and VIM. When it comes to solving Painlev's equation I, the solutions obtained by the DJM are in complete accord with one another. Additionally, the computation of the Adomian polynomial or the corrective functional in the case of the VIM is not necessary for DJM. A new numerical approach (NNM) for solving differential equations was presented by [7]. They developed a new way by using the implicit trapezium formula and the Daftardar-Gejji and Jafari techniques. They presented several software packages based on this method and discussed about the method's inaccuracy, stability, and convergence analyses. They use this strategy to solve a variety of equations and demonstrate that the solutions are accurate. The new iterative approach (NIM) developed by Daftardar-Gejji and Jafari is implemented by [8] in 2008. To solve linear and nonlinear partial differential equations of integer and fractional order, an iterative method for solving nonlinear functional equations is used. The outcomes were contrasted with those of other iterative techniques, including variational iteration, homotopy perturbation, and Adomian decomposition.

2. The Daftardar-Gejji and Jafari method (DJM)

The nonlinear functional equation can be used to formulate a range of issues in physics, chemistry, biology, engineering, etc., of the form,

$$y = f + N(y) \qquad ...(1)$$

where N is the nonlinear operator and f is a predetermined function. Integral equations, ordinary differential equations (ODEs), partial differential equations (PDEs), systems of ODE/PDE, and other types of equations are all represented by Eq. (1). Solving linear equations has been done using a variety of techniques, including the Laplace transform, Fourier transform, the Green's function method, Runge-Kutta method [9,10], etc. However, numerical iterative techniques are required for solving nonlinear equations. Functional of the form in Eqn. (1) can be solved using the Adomian decomposition method (ADM) [11, 12, 13]. Although the study of Stiff systems of differential equations (SDEs) has been hindered by lack of effective and precise methods, the derivation of approximate solutions to SDEs remains an issue that necessitates the development of some clever and sound strategies that are of interest [14]. For locating the approximate solution of differential equations, Daftardar-Gejji and Jafari devised the iterative technique known as DJM [4, 6]. In contrast to ADM, VIM, and numerical approaches, DJM does not necessitate the arduous calculation of Adomian polynomials in nonlinear terms, the identification of a Lagrange multiplier in its algorithm, or the requirement for discretization make the method very easy and effective. The suggested approach handles both linear and nonlinear equations simply and effectively. For differential equations in the Riccati form, the approach has recently been expanded by [4]. This approach produces answers in the form of infinite series that rapidly converge and can be estimated accurately by merely computing the first few terms. To solve the stiff differential equations, we shall extend and alter the DJM in this study. We will generalize an approach to make SDEs and SSDEs easier to solve.
Consider the following general functional equation for the fundamental concept of the DJM:

$$u = f + L(u) + N(u), \quad ...(2)$$

where f is a function of x, $L(u)$ is linear, and $N(u)$ is a nonlinear term. The series solution of the above functional equation is given by

$$u = \sum_{n=0}^{\infty} u_n \,. \quad ...(3)$$

The nonlinear operator N can be decomposed as

$$N\left(\sum_{n=0}^{\infty} u_n\right) = N(u_0) + \sum_{n=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}. \quad ...(4)$$

From Eqn. (3) and Eqn. (4), Eqn. (2) is equivalent to

$$\sum_{n=0}^{\infty} u_n = f + N(u_0) + \sum_{n=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}. \quad ...(5)$$

We define the recurrence relation:

$$\begin{cases} u_0 = f \\ u_1 = N(u_0) \\ u_{m+1} = N(u_0 + ... + u_m) - N(u_m + ... + u_{m-1}),\ m = 1,2,... \end{cases} \quad ...(6)$$

Then

$$u_1 + ... + u_{m+1}) = N(u_0 + ... + u_m),\ m = 1,2,..., \quad ...(7)$$

and

$$\sum_{n=0}^{\infty} u_i = f + N\left(\sum_{n=0}^{\infty} u_n\right). \quad ...(8)$$

Our focus in this research work will be to improve and extend the DJM to accommodate the system of stiff differential equations.

2.1 Daftardar-Gejji and Jafari first approach

To describe the idea of the first approach of the DJM [15, 16, 17, 18, 19], considered the following general functional equation

$$u(t) = g(t) + N(u(t)) \quad ...(9)$$

where N is the nonlinear operator and g is a known function. We are looking for u which has the series solution in the form

$$u(t) = \sum_{i=0}^{\infty} u_i \,.$$

The operator N can be decomposed into the following

$$N\left(\sum_{i=0}^{\infty}u_i\right)=N(u_0)+\sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i}u_i\right)-N\left(\sum_{j=0}^{i=1}u_i\right)\right\} \quad ...(10)$$

From equations (9) and (10) we have,

$$\sum_{i=0}^{\infty}u_i=g(t)+N(u_0)+\sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i}u_i\right)-N\left(\sum_{j=0}^{i-1}u_i\right)\right\}. \quad ...(11)$$

We define the following recurrence relation

$$u_0=g(t),$$
$$u_1=N(u_0),$$
$$u_{n+1}=N\left(u_o+\ u_1+...+u_n\right)-N\left(u_0+u_1+...+u_{n-1}\right). \quad n=1,2,...$$

The k-term series solution of the general equation (9) takes the following form:

$$u(t)=u_o+u_1+...+u_{k-1}. \quad ...(12)$$

2.2 Daftardar-Gejji and Jafari second approach

This approach is preferred to be used for nonlinear problems. Let us consider the following nonlinear equation:

$$u(t)=g(t)+\varepsilon\left(u(t)\right)+N\left(u(t)\right), \quad ...(13)$$

where ε and N are the linear and nonlinear operators of $u(t)$ and $g(t)$ is a known function. We are looking for u which has the series solution in the form

$$u(t)=\sum_{k=0}^{\infty}u_k(t). \quad ...(14)$$

The linear operator ε can be decomposed into the following

$$\sum_{k=0}^{\infty}\varepsilon(u_k)=\varepsilon\left(\sum_{k=0}^{\infty}u_k\right) \quad ...(15)$$

The nonlinear operator N can be decomposed into the following

$$N\left(\sum_{k=0}^{\infty}u_k\right)=N(u_0)+\sum_{k=1}^{\infty}\left\{N\left(\sum_{j=0}^{k}u_j\right)-N\left(\sum_{j=0}^{k-1}u_j\right)\right\}. \quad ...(16)$$

From equations (13), (14) and (15)

$$\sum_{i=0}^{\infty} u_i = g(t) + \varepsilon\left(\sum_{k=0}^{\infty} u_k\right) + N(u_0) + \sum_{i=0}^{\infty}\left\{ N\left(\sum_{j=0}^{i} u_i\right) - N\left(\sum_{j=0}^{i=1} u_j\right)\right\}.$$

We define the following recurrence relation

$$\begin{aligned} u_0 &= g(t), \\ u_1 &= \varepsilon(u_0) + N(u_0), \\ u_2 &= \varepsilon(u_1) + N(u_0 + u_1) - N(u_0), \\ u_{n+1} &= \varepsilon(u_n) + N(u_0 + u_1 + \dots + u_n) - N(u_0 + u_1 + \dots + u_{n-1}). \quad n = 1, 2, \dots \end{aligned} \quad \dots(17)$$

The k-term series solution of the general equation (17) takes the following form

$$u(t) = u_0 + u_1 + \dots + u_{k-1}.$$

2.3 The case of first order stiff system of DEs

Significant scholarly attention has been paid to the study of stiff differential equation systems, both linear and nonlinear. A stiff system of nonlinear differential equations emerges in many models, including the idea of chemical reactions. In order to manage stiff systems of nonlinear differential equations, we express a system in the operation form as,

$$\begin{cases} u'(x) + R_1(u,v,w) + N_1(u,v,w) = g_1, \\ v'(x) + R_2(u,v,w) + N_2(u,v,w) = g_2, \\ w'(x) + R_3(u,v,w) + N_3(u,v,w) = g_3, \end{cases} \quad \dots(18)$$

with initial condition

$$\begin{cases} u_0(x) = f_1(x), \\ v_0(x) = f_2(x), \\ w_0(x) = f_3(x), \end{cases} \quad \dots(19)$$

where $u'(x)$, $v'(x)$ and $w'(x)$ are considered first order differential equation, R_j, $1 \le j \le 3$, are linear and nonlinear operators respectively, and g_1, g_2 and g_3 are forcing terms. In what follows we give the main steps of the improve DJM in handling scientific and engineering problems.
By integrating both sides of Eqn. (17) from zero (0) to x and use the initial conditions, we get

$$\begin{cases} u(x) = f_1(x) + g_1 x - \int_0^x \left(R_1(u,v,w) + N_1(u,v,w)\right) dx, \\ v(x) = f_2(x) + g_2 x - \int_0^x \left(R_2(u,v,w) + N_2(u,v,w)\right) dx, \\ w(x) = f_3(x) + g_3 x - \int_0^x \left(R_3(u,v,w) + N_3(u,v,w)\right) dx, \end{cases} \quad ...(20)$$

To use the IDJM, we let

$$\begin{cases} u(x) = \sum_{n=0}^{\infty} u_n(x), \\ v(x) = \sum_{n=0}^{\infty} v_n(x), \\ w(x) = \sum_{n=0}^{\infty} w_n(x). \end{cases} \quad ...(21)$$

Substituting Eqn. (20) into Eqn. (19) and splitting the resulting equation, we get the following relation owing to the IDJM.

$$\begin{cases} u_0(x) = f_1 x + g_1 x, \\ v_0(x) = f_2 x + g_2 x, \\ w_0(x) = f_3(x) + g_3 x. \end{cases} \quad ...(22)$$

$$\begin{cases} u_1(x) = -\int_0^x \left(R_1(u_0,v_0,w_0) + N_1(u_0,v_0,w_0)\right) dx, \\ v_1(x) = -\int_0^x \left(R_2(u_0,v_0,w_0) + N_2(u_0,v_0,w_0)\right) dx, \\ w_1(x) = -\int_0^x \left(R_3(u_0,v_0,w_0) + N_3(u_0,v_0,w_0)\right) dx, \end{cases} \quad ...(23)$$

$$\begin{cases} u_2(x) = -\int_0^x \left(R_1 \sum_{n=0}^{1} (u_n, v_n, w_n) + N_1 \sum_{n=0}^{1} (u_n, v_n, w_n) \right) dx - u_1(x) \\ v_2(x) = -\int_0^x \left(R_2 \sum_{n=0}^{1} (u_n, v_n, w_n) + N_2 \sum_{n=0}^{1} (u_n, v_n, w_n) \right) dx - v_1(x) \\ w_2(x) = -\int_0^x \left(R_3 \sum_{n=0}^{1} (u_n, v_n, w_n) + N_3 \sum_{n=0}^{1} (u_n, v_n, w_n) \right) dx - w_1(x), \end{cases} \quad ...(24)$$

$$\begin{cases} u_3(x) = -\int_0^x \left(R_1 \sum_{n=0}^{2} (u_n, v_n, w_n) + N_1 \sum_{n=0}^{2} (u_n, v_n, w_n) \right) dx - \sum_{n=1}^{2} u_n(x) \\ v_3(x) = -\int_0^x \left(R_2 \sum_{n=0}^{2} (u_n, v_n, w_n) + N_2 \sum_{n=0}^{2} (u_n, v_n, w_{n1}) \right) dx - \sum_{n=1}^{2} v_n(x) \\ w_3(x) = -\int_0^x \left(R_3 \sum_{n=0}^{2} (u_n, v_n, w_n) + N_3 \sum_{n=0}^{2} (u_n, v_n, w_n) \right) dx - \sum_{n=1}^{2} w_n(x), \end{cases} \quad ...(25)$$

$\vdots$

and so on. Continuing in this manner, the $(n+1)^{th}$ approximation of the exact solutions for the unknown functions $u(x)$, $v(x)$ and $w(x)$ can be obtain.
Therefore, the approximate solutions

$$\begin{cases} u(x) = \sum_{m=0}^{\infty} u_m(x), \\ v(x) = \sum_{m=0}^{\infty} v_m(x), \\ w(x) = \sum_{m=0}^{\infty} w_m(x). \end{cases} \quad ...(26)$$

Three example of stiff systems of differential equations, two linear and one nonlinear, have been chosen to illustrate the effectiveness of the method and provide a clear overview of the analysis as discussed above.

3.0 Experimental rResults

The present section examines the application of the proposed method on different stiff system describing chemical reaction kinetics. Here, we will solve some linear stiff systems of differential equations to demonstrate the efficiency, effectiveness and accuracy of the proposed method.

Example 1: Consider the simple linear stiff differential equation [21],

$$u'(x) = -15u(x), \quad x \geq 0,$$

with initial conditions

$$u(0) = 1$$

The exact solution for the system is

$$u(x) = e^{-15x}$$

The algorithm in the preceding section, yields the following components for example 1

$$u(x) = 1 - 15\int_0^x u(t)dt$$

$$u_0(x) = 1,$$

$$u_1(x) = -15\int_0^x (u_0(t))dt = -15x$$

$$u_2(x) = -15\int_0^x \left(\sum_{n=0}^{1} u_n(t)\right)dt - u_1(x) = \frac{225}{2}x^2$$

$$u_3(x) = -15\int_0^x \left(\sum_{n=0}^{2} u_n(t)\right)dt - \sum_{n=1}^{2} u_n(t) = -\frac{1125}{2}x^3$$

$$u_4(x) = -15\int_0^x \left(\sum_{n=0}^{3} u_n(t)\right)dt - \sum_{n=1}^{3} u_n(t) = \frac{16875}{8}x^4$$

$$u_5(x) = -15\int_0^x \left(\sum_{n=0}^{4} u_n(t)\right)dt - \sum_{n=1}^{4} u_n(t) = -\frac{50625}{8}x^5$$

The series solution is then obtained by summing the above iterations as

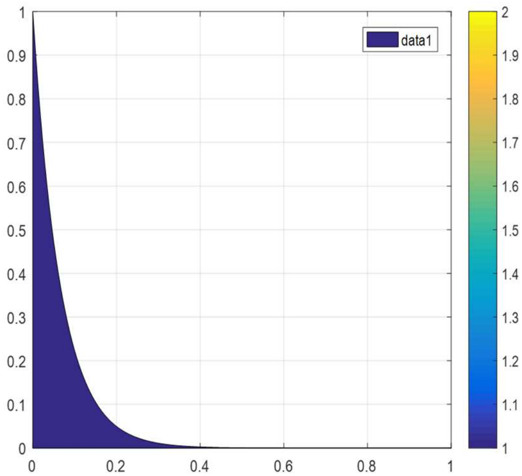
$$u(x) = \sum_{n=0}^{5} u_n(x).$$

This gives the series solution of example 1 as follows

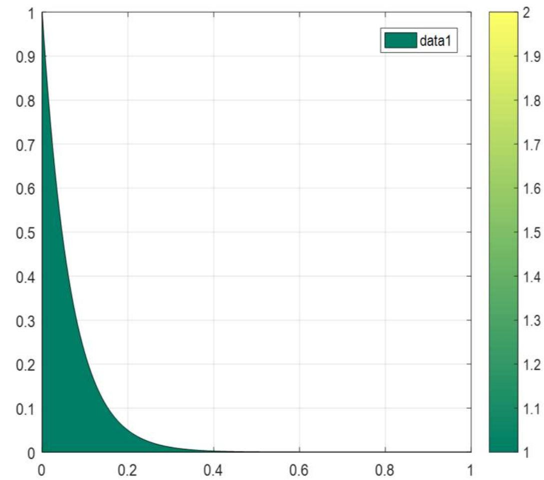
$$u(x) = 1 - 15x + \frac{225}{2}x^2 - \frac{1125}{2}x^3 + \frac{16875}{8}x^4 - \frac{50625}{8}x^5$$

Table 1: Comparison of IDJM solution for example 1 with RK method and the exact solution

x	EXACT	IDJM	RKM
0	1	1	1
0.1	0.210156	0.210156	0.0625
0.2	-0.65	-0.65	-2
0.3	-6.85391	-6.85391	-8.5625
0.4	-33.8	-33.8	-23
0.5	-114.605	-114.605	-48.6875
0.6	-307.7	-307.7	-89
0.7	-704.42	-704.42	-147.313
0.8	-1436.6	-1436.6	-227
0.9	-2684.17	-2684.17	-331.438
1	-4682.75	-4682.75	-464



(a)



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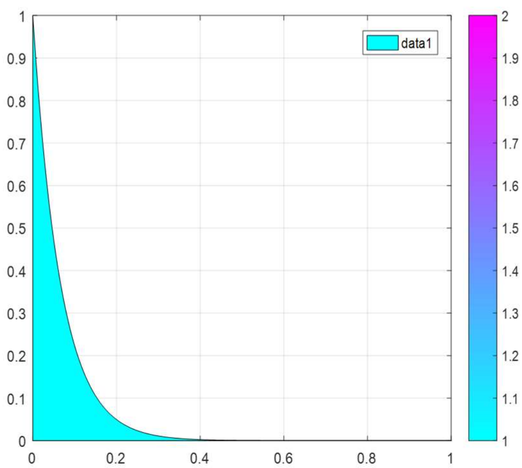
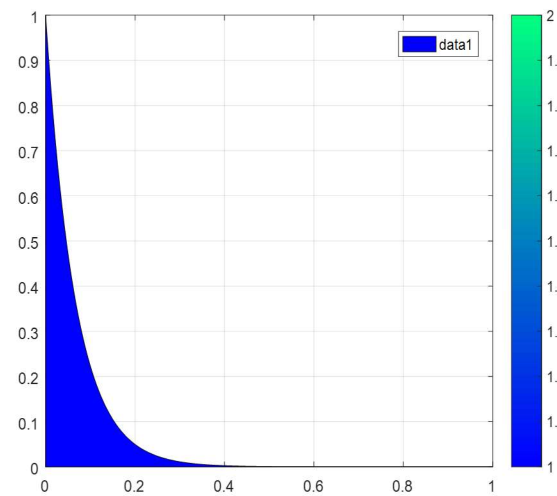
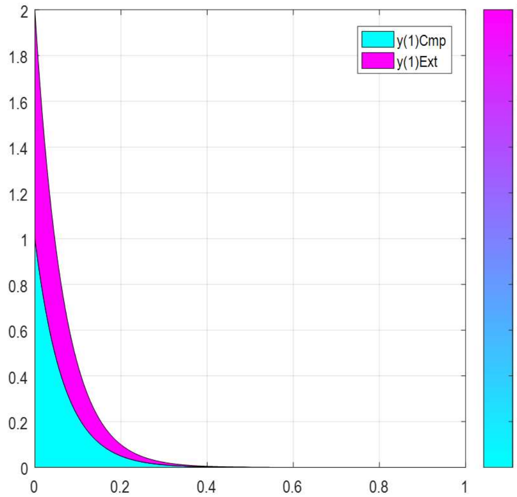


(c)

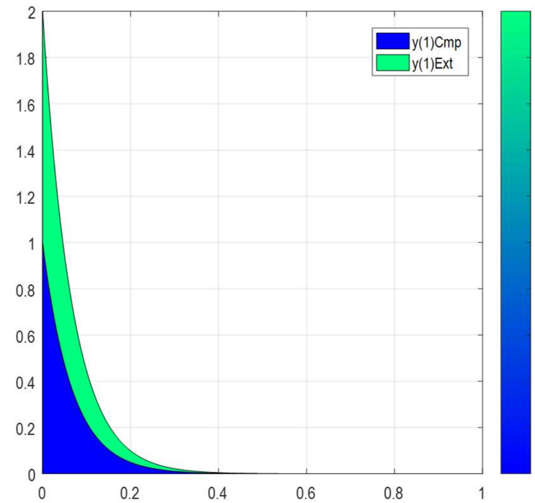


(d)

Computed graphic surface of example 1 without the exact solution



(a)



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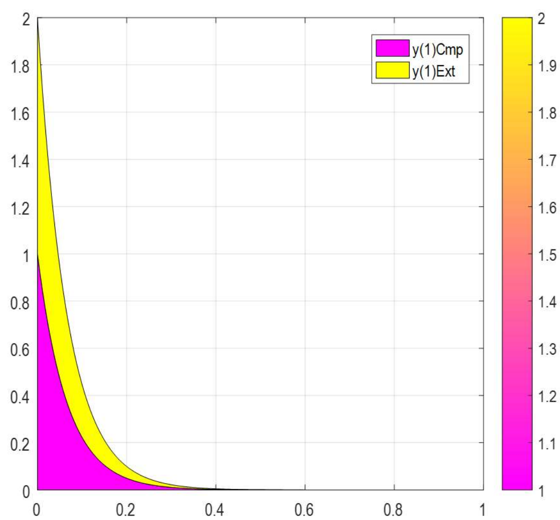
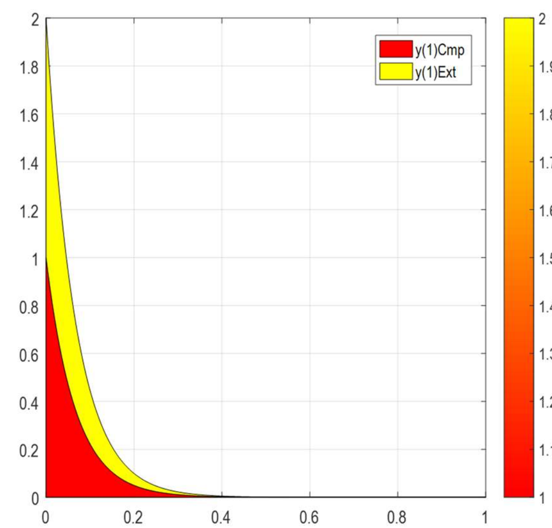


(c)



(d)

Computed graphic surface of example 1 together with the exact solution

Figure 1: *Graphical surface curves for example 1 using the new iterative method with nfe =500*

Example 2: Consider a linear stiff system of differential equations [21]

$$\begin{cases} u'(x) = -u(x) + 95v(x), \\ v'(x) = -u(x) - 97v(x), \end{cases}$$

with initial conditions

$$\begin{cases} u(0)=1, \\ v(0)=1. \end{cases}$$

The exact solution for the system is

$$\begin{cases} u(x)=\dfrac{1}{47}\left(95e^{-2x}-48e^{-96x}\right), \\ v(x)=\dfrac{1}{47}\left(48e^{-96x}-e^{-2x}\right). \end{cases}$$

The algorithm in the section above, yields the following components for example 2 above

$$\begin{cases} u(x)=1+\int_0^x \left(95v(t)-u(t)\right)dt\,, \\ v(x)=1-\int_0^x \left(u(t)+97v(t)\right)dt \end{cases}$$

$$\begin{cases} u_0(x)=1, \\ v_0(x)=1 \end{cases}$$

$$\begin{cases} u_1(x)=\int_0^x \left(95v_0(t)-u_0(t)\right)dt=94x \\ v_1(x)=-\int_0^x \left(u_0(t)-97v_0(t)\right)dt=-98x \end{cases}$$

$$\begin{cases} u_2(x)=\int_0^x \left(95\sum_{n=0}^{1} v_n(t)-\sum_{n=0}^{1} u_n(t)\right)dt-u_1(x)=-4702x^2 \\ v_2(x)=-\int_0^x \left(\sum_{n=0}^{1} u_n(t)+97\sum_{n=0}^{1} v_n(t)\right)dt-v_1(x)=4706x^2 \end{cases}$$

$$\begin{cases} u_3(x)=\int_0^x \left(95\sum_{n=0}^{2} v_n(t)-\sum_{n=0}^{2} u_n(t)\right)dt-\sum_{n=1}^{2} u_n(t)=\dfrac{45177}{3}x^3 \\ v_3(x)=-\int_0^x \left(\sum_{n=0}^{2} u_n(t)+97\sum_{n=0}^{2} v_n(t)\right)dt-\sum_{n=1}^{2} v_n(t)=-\dfrac{451780}{3}x^3 \end{cases}$$

$$\begin{cases} u_4(x)=\int_0^x \left(95\sum_{n=0}^{3} v_n(t)-\sum_{n=0}^{3} u_n(t)\right)dt-\sum_{n=1}^{3} u_n(t)=-\dfrac{11181547}{3}x^4 \\ v_4(x)=-\int_0^x \left(\sum_{n=0}^{3} u_n(t)+97\sum_{n=0}^{3} v_n(t)\right)dt-\sum_{n=1}^{3} v_n(t)=\dfrac{10503893}{3}x^4 \end{cases}$$

$$\vdots$$

The series solution is then obtained by summing up the above iterations

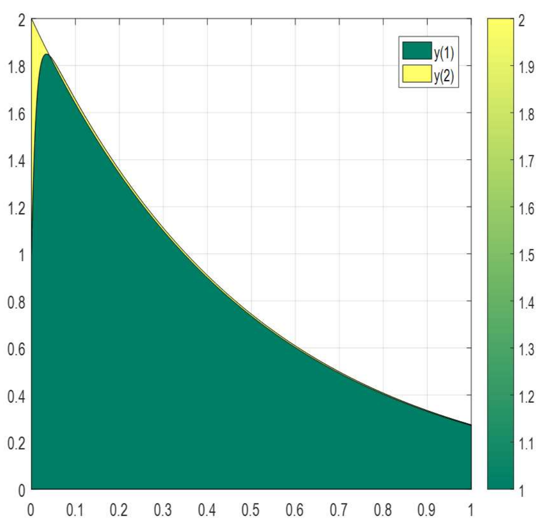
$$\begin{cases} u(x) = \sum_{n=0}^{4} u_n(x) \\ v(x) = \sum_{n=0}^{4} v_n(x) \end{cases}$$

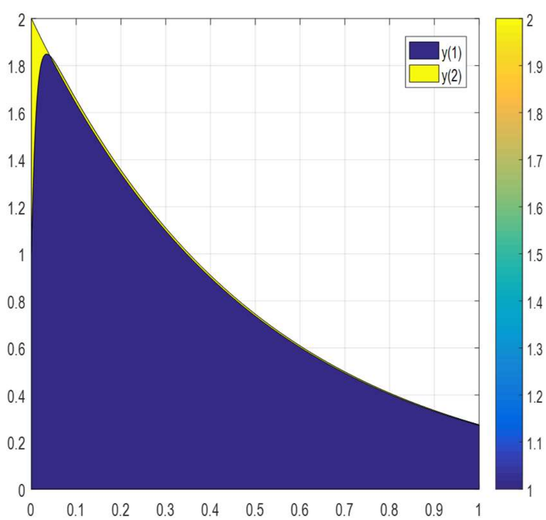
This gives the series solution of the example in 2 as follows

$$u(x) = 1 + 94x - 4702x^2 + \frac{451772}{3}x^3 - \frac{11181547}{3}x^4$$
$$v(x) = 1 - 98x + 4706x^2 - \frac{451780}{3}x^3 + \frac{10503893}{3}x^4$$

Table 2: Comparison of IDJM solution of Example 2 with RK method and the exact solution

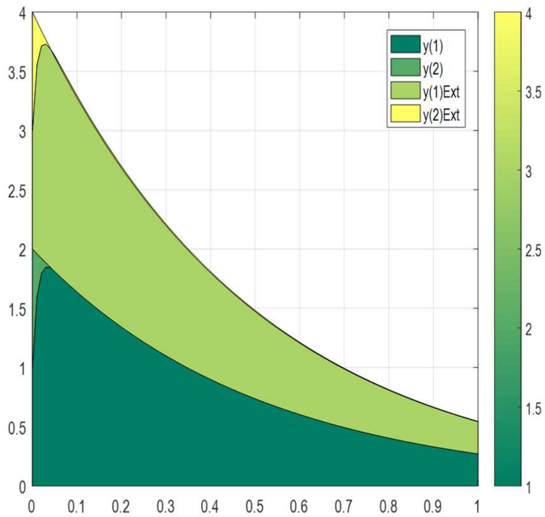
x	$^{u}EXACT$	$^{u}IDJM$	^{u}RKM
0	1	1	1
0.1	-968.352	-979.646	-663.8138
0.2	-326465	-326646	-53599.31
0.3	-9324702	-9325617	-666382.2
0.4	-9.9E+07	-9.9E+07	-3920779
0.5	-6.1E+08	-6.1E+08	-15388004
0.6	-2.7E+09	-2.7E+09	-46843392
0.7	-9.4E+09	-9.4E+09	-1.2E+08
0.8	-2.8E+10	-2.8E+10	-2.7E+08
0.9	-7.2E+10	-7.2E+10	-5.51E+08
1	-1.7E+11	-1.7E+11	-1.04E+09



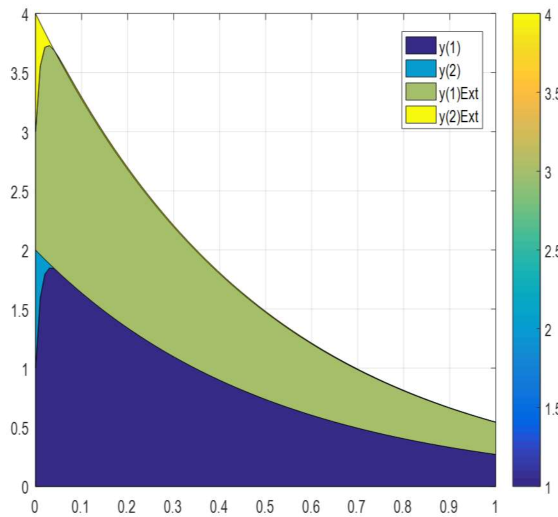


(a) (b)

Computed graphic surface of example 2 without the exact solution



(a)



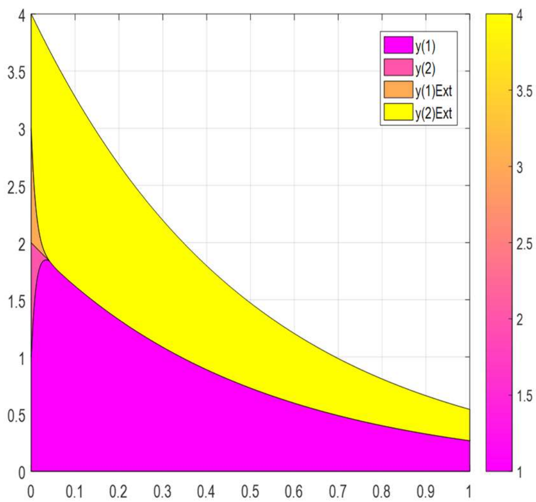
(b)

Computed graphic surface of example 2 together with the exact solution

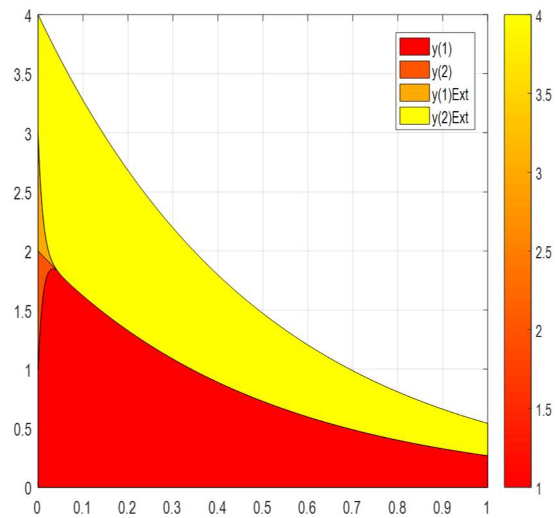
Figure 2: *Graphical surface of the stiff problem of example 2 using the new iterative method*

Table 3: Comparison of IDJM solution of example 2 with RK method and the exact solution

x	$^{v}EXACT$	$^{v}IDJM$	^{v}RKM
0	1	1	1
0.1	969.9892	958.6949	665.4513
0.2	326466.7	326286	53600.65
0.3	9324703	9323788	666383.3
0.4	98722237	98719345	3920780
0.5	6.1E+08	6.1E+08	15388005
0.6	2.69E+09	2.69E+09	46843393
0.7	9.4E+09	9.4E+09	1.2E+08
0.8	2.77E+10	2.77E+10	2.7E+08
0.9	7.19E+10	7.19E+10	5.51E+08
1	1.69E+11	1.69E+11	1.04E+09



(a)



(b)

Figure 3: *Graphical surface curves for example 2 using the new iterative method with nfe =500*

Example 3: Consider a non-linear stiff system of differential equations of one of the chemical kinetic Problems [22, 23]

$$\begin{cases} u'(x) = \lambda u(x) + v^2(x), \\ v'(x) = -v(x), \end{cases}$$

where $\lambda = 10{,}000$ with initial condition

$$\begin{cases} u(0) = 1, \\ v(0) = 1. \end{cases}$$

The exact solution for the system is

$$\begin{cases} u(x) = -\dfrac{e^{-2x}}{(\lambda + 2)}, \\ v(x) = e^{-x}. \end{cases}$$

The algorithm in the above section, yields the following components for the Equation,

$$\begin{cases} u(x) = 1 + \lambda \int_0^x \left(u(t) + v^2(t)\right)dt, \\ v(x) = 1 - \int_0^x \left(v(t)\right)dt \end{cases}$$

$$\begin{cases} u_0(x) = 1, \\ v_0(x) = 1 \end{cases}$$

$$u_1(x) = \lambda \int_0^x \left(u_0(t) + v_0(t)\right)dt = \lambda x + x$$

$$v_1(x) = -\int_0^x \left(v_0(t)\right)dt = -x$$

$$u_2(x) = \lambda \int_0^x \left(\sum_{n=0}^{1} u_n(t) + \sum_{n=0}^{1} v_n(t)\right)dt - u_1(x) = \frac{1}{2}x^2\lambda^2 + \frac{1}{2}x^2\lambda + \frac{1}{3}x^3 - x^2$$

$$v_2(x) = -\int_0^x \left(\sum_{n=0}^{1} v_n(t)\right)dt - v_1(x) = \frac{1}{2}x^2$$

$$u_3(x) = \lambda \int_0^x \left(\sum_{n=0}^{2} u_n(t) + \sum_{n=0}^{2} v_n(t)\right)dt - \sum_{n=1}^{2} u_n(t) =$$

$$\frac{1}{12}\lambda x^4 + \frac{1}{6}x^3\lambda^3 + \frac{1}{6}x^3\lambda^2 - \frac{1}{3}x^3\lambda + \frac{1}{20}x^5 - \frac{1}{4}x^4 + \frac{1}{3}x^3$$

$$v_3(x) = -\int_0^x \left(\sum_{n=0}^{2} v_n(t)\right)dt - \sum_{n=1}^{2} v_n(t) = -\frac{1}{6}x^3$$

$$u_4(x) = \lambda \int_0^x \left(\sum_{n=0}^{3} u_n(t) + \sum_{n=0}^{3} v_n(t)\right)dt - \sum_{n=1}^{3} u_n(t) =$$

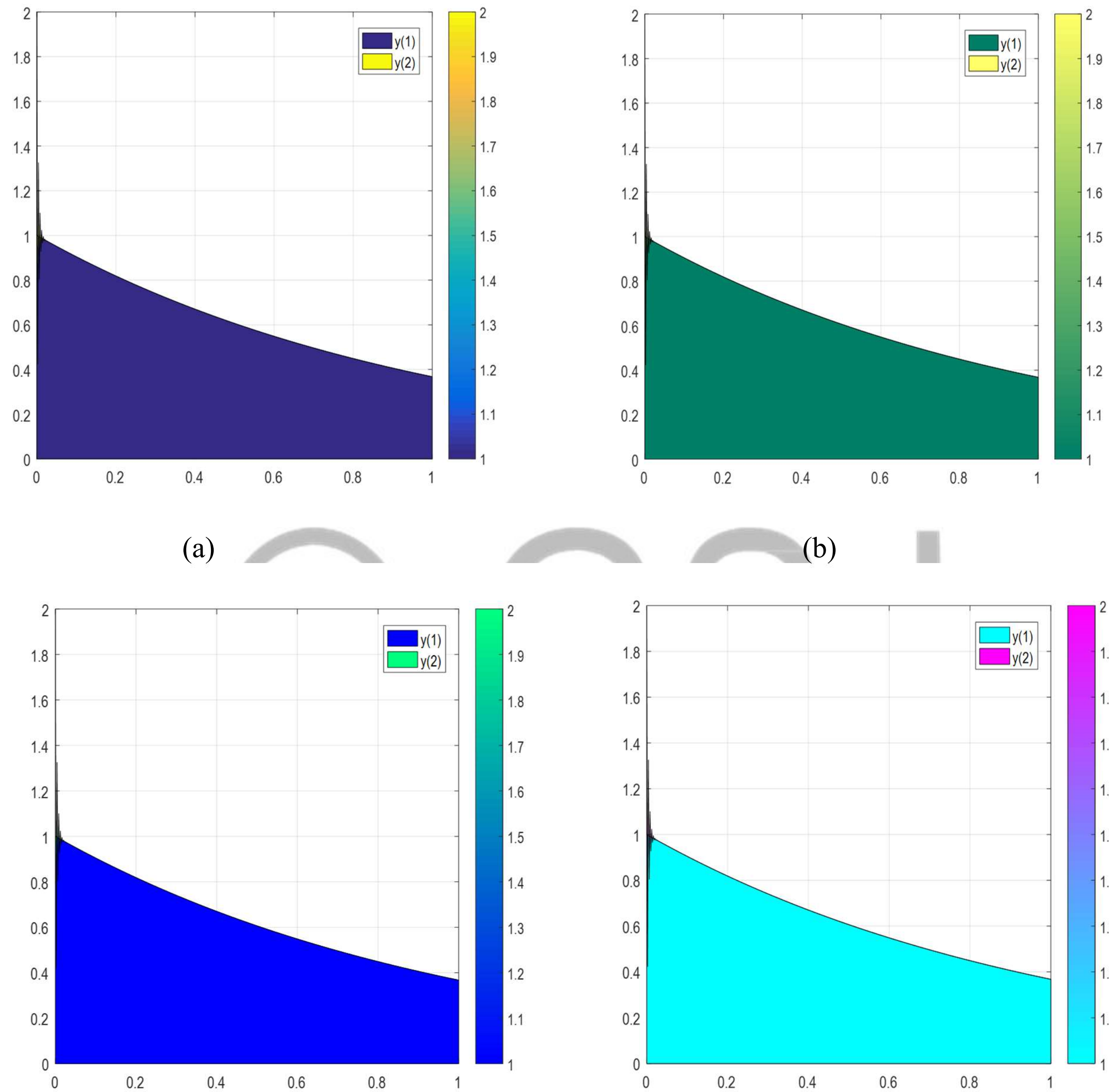
$$\frac{1}{120}\lambda x^6 + \frac{1}{60}x^5\lambda^2 - \frac{1}{20}x^5\lambda + \frac{1}{12}\lambda x^4 + \frac{1}{24}x^4\lambda^4 + \frac{1}{24}x^4\lambda^3 - \frac{1}{12}x^4\lambda^2 + \frac{1}{252}x^{7}$$
$$- \frac{1}{36}x^6 + \frac{1}{15}x^5 - \frac{1}{12}x^4$$

$$v_4(x) = -\int_0^x \left(\sum_{n=0}^{3} v_n(t)\right)dt - \sum_{n=1}^{3} v_n(t) = \frac{1}{24}x^4$$

This gives the series solution of example 3 as

$$u(x) = 1 + \frac{2}{3}x^3 - x^2 - \frac{1}{20}x^5\lambda + \frac{1}{24}x^4\lambda^3 - \frac{1}{12}x^4\lambda^2 + \frac{1}{120}\lambda x^6 + \frac{1}{60}x^5\lambda^2 +$$
$$\frac{1}{24}x^4\lambda^4 + x + \frac{7}{60}x^5 - \frac{1}{3}x^4 + \frac{1}{6}x^3\lambda^2 - \frac{1}{3}x^3\lambda + \frac{1}{6}\lambda x^4 + \frac{1}{6}x^3\lambda^3 +$$
$$\frac{1}{252}x^7 - \frac{1}{36}x^6 + \lambda x + \frac{1}{2}x^2\lambda + \frac{1}{2}x^2\lambda^2$$

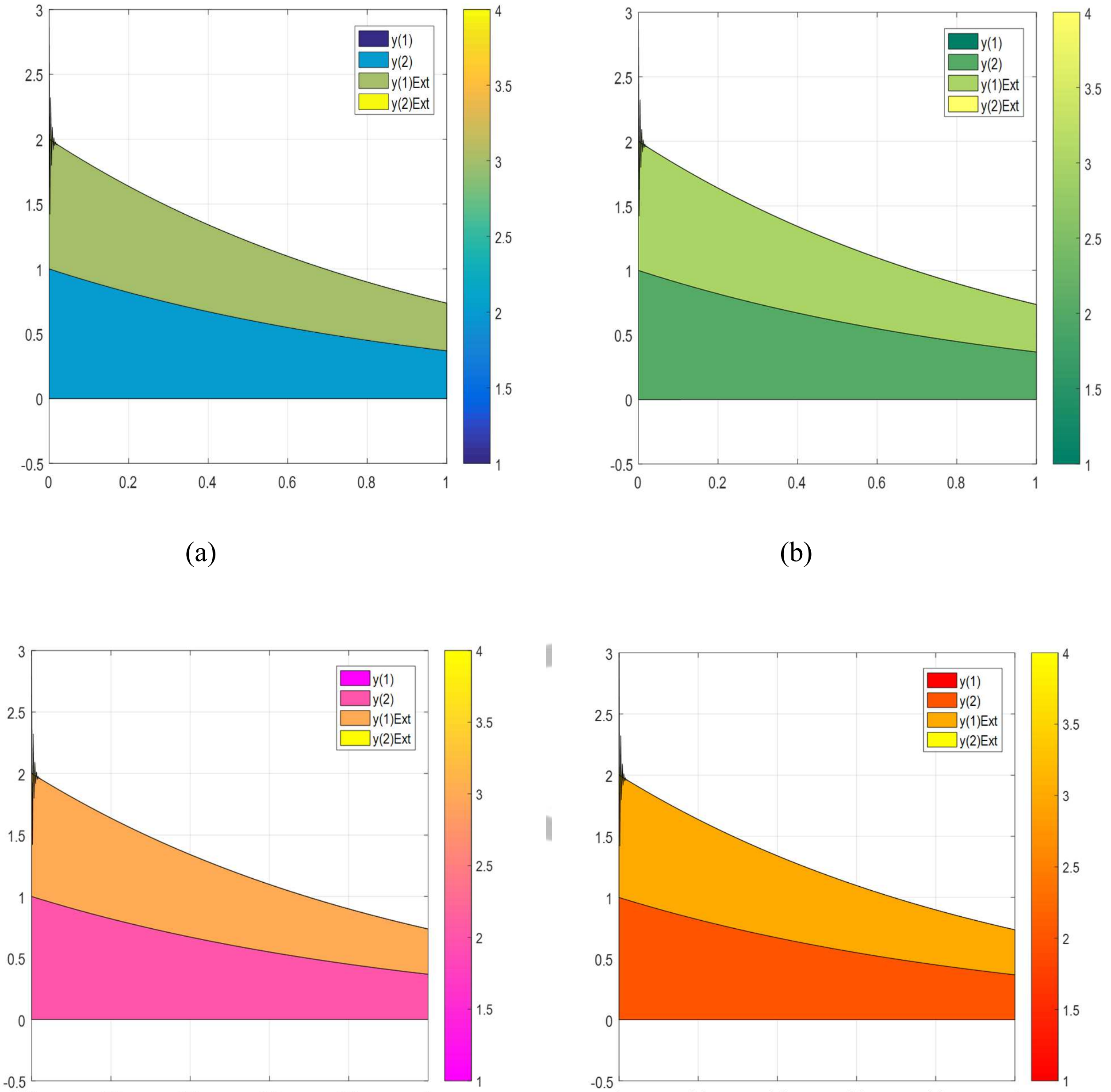
$$v(x)=1-x+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{24}x^4$$



(a) (b)

(c) (d)

Computed graphic surface of example 3 without the exact solution



(c) (d)

Computed graphic surface of example 2 together with the exact solution

Figure 4: *Graphical phase plots for example 3 using the new iterative method with nfe =500*

4. Discussion of Results

The main objective of this work has been achieved by solving some linear and nonlinear system of stiff differential equations by the proposed method. This has been obtained by implementing the improved DJM. In addition, the comparison of the results obtained by the proposed method, the exact solution and results obtained by other methods were presented in tables. The IDJM is efficient and reliable to find the approximate solution of linear and nonlinear systems of stiff differential equation (see Tables 1-3). The proposed methods did not require any assumption to deal with the nonlinear terms unlike other methods. When comparing the results of the IDJM with those of the DJM the numerical solutions obtained by IDJM are more accurate (see, tables). The approximate error decrease when there is more iteration which are clarified in the computations (see, tables). In comparing the results obtained by the IDJM with those of the existing methods, it is observed in general that the approximate solutions obtained by the IDJM converge faster without any restricted assumptions and possesses high-order of accuracy (see, Tables 1-3).

5. Conclusion

In this work, a semi-analytical method based on the DJM was introduced, and which was used to solve stiff system of differential equations. To support the analysis, two linear stiff systems of differential equations and one nonlinear stiff system of differential equations were solved. The obtained results revealed that this method is simpler and effective in its computational procedures than the other methods. Therefore, this method is more suitable and convenient for solving stiff systems. Finally, as demonstrated, the applications of the new iterative integrator can be effectively used to portrait even the actual representation of the stiff equations' natural or physical interpretation of what they represent in natural phenomena, depicted as phase plots as shown in the figures.

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