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# Theoretical analysis of unsteady flow between two stretching cylinders with variable physical properties

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#### Abstract

In this paper, we investigate the unsteady flow between two stretching cylinders with variable physical properties. To assuming the two-dimensional flow of a viscous fluid in the zone between two circular cylinders. a(t) is the radius of internal cylinder while the radius of external cylinder is b(t) and both cylinder are allow to stretched. The combine effect of variable thermal conductivity and viscosity are examined in this study. Constitutive equations of motion, energy and mass transport are used as a tool for modeling. For introducing natural parameters of our interest similarity transformation are applied and transformed the governing partial differential equations into the system of ordinary differential equations. For analysis HAM is used and then the results compared with BVP4c package.

Keywords: HAM, stretching cylinder, Unsteady flow, Similarity transformations.

# 1 Introduction

The of study cylinder is used in separate fields including one of the engineering sciences because it keeps a fundamental shape in all structural things. According to its detail the flows about the cylinder have been examined from the performance of fluid mechanics [1-4], for example, through its detailed. The flow structure with separation and consequent, it produces vibration. If we can analyze the data through such a computerized techniques, by use of the CFD (Computational Fluid Dynamics) [5-6] it became possible to point out the structure of three-dimensional unsteady flows in detail and also to apply the computational results to engineering designs, here the computational data depend on the accuracy of numerical methods, grid fineness, and boundary conditions, etc. [7]. Couette [8] was examined all those problems which was associated to the Newtonian fluid which flow between two rotating cylinders. Pillai and Varma [9], Bujurke and Naduvinamani [10], Lin [11] and Ruggiero et al. [12] have studied the flow between annulus region. The temperature and velocity distribution in case of Couette flow analyzed by Schlichting [13]. Channabasappa and colleagues [14] discussed the flow between porous cylinders and analyzed the velocity distribution and wall shear stresses at both walls of the cylinders. The effect of uniform magnetic field in the annulus region were examined by Bathaiah and Venugopal [15]. They analyze that the temperature of the fluid increases in the closeness of the inner cylinder, whereas decreases in proximity of the outer cylinder. Lee [16] analyzed fluid flows in the field of eccentric and concentric cylinders. He investigated the flow effects of the low Prandtl number. The flow between two rotating cylinders in annulus region of Casson fluid were examined by Batra and Das [17]. Beside this, the computational solution of laminar flow between two cylinder Sutterby rheological fluid by using finite difference method were obtained by Batra and Eissa [18]. Leong and Lai [19] were analyzed the heat transfer in between two cylinders. They obtained the solution by using perturbation expansion method. The flow between two concentric cylinders of viscoelastic fluid were analytically investigated by avanchi et al. [20] by using Giesekus model. They consider that the outer cylinder is at rest. More over the fluid flow and heat transfer characteristics between two rotating cylinders in annulus region analyzed by Subotic and Lai [21], where outer cylinder is attached by porous layer. The mixed convection flow between two cylinders were investigate by Barletta et al. [22]. They consider a permeable medium the region of the cylinders. Mahmood et al. [23] investigated velocity profile and shear stress for the flow of viscoelastic fluid in the region between two infinite coaxial cylinders. Recently, Mozayyeni and Rahimi [24] analyzed the viscous fluid in an annulus concentric cylinders, where the cylinders are take place horizontally, and obtained numerical solution for mixed convection flow with different uniform wall temperatures in both unsteady and steady states by using finite



Figure 1:

volume method. The pioneer work of axisymmetric staganation flow was analyzed by Wang [25]. After looking Wang's idea about axisymmetric flow, many researcher discussed the axisymmetric flows [26, 27, 28, 29, 30, 31 and 32. Gorla [33] analyze the result of viscous fluid of axisymmetric stagnation flow in which the flow is performing harmonic motion in circular cylinder. Hong and Wang [34] discussed annular axisymmetric flow on two cylinders, where one is fixed so as well as the another will be movable. The antisymmetry flow in the region between two stretching/shrinking surface were analyzed by Soid et al. [35]. Awais et al. [36] studied the sisko fluid about a stagnation point over a cylinder. Basic work about squeezing flows was executed by Stefan [37], who presented the fundamental formulation of these flows. According to the importance of flow about stretching cylinders, many researcher have shown their interest in stretching flows. At early time work was done on this topic by Wang [38] when he investigated the flow over a stretching empty cylinder in an ambient fluid and then extended this problem with uniform suction/injection. The effect of magnetic field was done by Ishak et al. [39]. After this, the similarity solution for stretching cylinder with partial slip boundary conditions was obtained by Wang and Ng [40]. The flow and mixed convection due to a vertical stretching cylinder was analyzed by Wang [41]. Fang et al. [42] investigated the flow over an expanding stretching cylinder. Further the unsteady viscous flow on the outside of stretching or shrinking cylinder was done by Fang et al. [43]. After this Mukhopadhyay [44] investigated a flow of boundary layer and transfer of heat over an extending cylinder where the cylinder is attached with a porous medium and obtained the similarity solutions by shooting method. Recently, Wan Zaimi et al. [45] discussed the suction effect on the unsteady flow due to extended cylinder have solved numerically using by shooting method. Study of nano fluid mixed convection flow with viscous dissipation were investigated by Dhanai et al. [46]. The dissipation effect in a nano fluid with constant wall temperature over the boundary layer free convection were analyzed by Mohamed et al. [47]. Most recently, the study of an unsteady nano fluid flow due to a porous shrinking cylinder was done by Khairy Zaimi [48] by using Buongiorno model [49]. In his work, he focuses on the flow induced by a shrinking cylinder in a Nano fluid that was not performed by someone before. He analyzed the unsteady flow of a nano fluid due to a porous shrinking cylinder with constant viscosity and thermal conductivity in one cylinder. It is one of the interesting research area that's why our present work is also on this topic but we take the viscosity and thermal energy as a variable and also take two stretching cylinders.

## 2 Problem formulation

#### 2.1 Governing equations and boundary conditions

Consider an unsteady flow between two stretching cylinders as shown in figure (1). The radius of internal cylinder is a(t) while the internal one is b(t). The cylinders are stretching along z-axis. We take the viscosity and thermal energy as a variables. While  $a(t) = a_o(t)\sqrt{1-\beta t}$  and  $b(t) = b_o(t)\sqrt{1-\beta t}$ . The basic governing equation for incompressible newtonian liquid in the vectorial form are: Continuity Equation:

y Equation.

$$\nabla . \hat{\mathbf{v}} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial \hat{\mathbf{v}}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \hat{\mathbf{v}}$$
<sup>(2)</sup>

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**Energy Equation:** 

$$\frac{\partial T_{emp}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla T_{emp} = \alpha \nabla^2 T_{emp} + \tau \left[ D_B \nabla T_{emp} \cdot \nabla C_{n.f} + \frac{D_T}{T_0} \nabla T_{emp} \cdot \nabla T_{emp} \right]$$
(3)

Mass Transport Equation:

$$\frac{\partial C_{n.f}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla C_{n.f} = D_B \nabla^2 C_{n.f} + \frac{D_T}{T_0} \nabla^2 T_{emp}$$
(4)

where  $\hat{\mathbf{v}} = \langle u_{com}, w_{com} \rangle$  denote the velocity vector, temperature is denoted by  $T_{emp}$ , the volume friction of nano particals is denoted by  $C_{n.f}$ , p represent the pressure, kinematic viscosity is represented by  $\nu$ ,  $\rho$  represented the fluid density, Brownian diffusion coefficient is denoted by  $D_B$  while  $D_T$  is the representation of thermophoretic diffusion coefficient.

There is azimuthal velocity component due to the axisymmetry flow, the N-Stokes equations of unsteady incompressible fluid with out body forces in cylindrical coordinate system, (1) - (4) can be represent as, Continuity Equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_{com}) + \frac{\partial w_{com}}{\partial z} = 0$$
(5)

Momentum Equation:

$$\rho \left[ \frac{\partial u_{com}}{\partial t} + u_{com} \frac{\partial u_{com}}{\partial r} + w_{com} \frac{\partial u_{com}}{\partial z} \right] = -\frac{\partial p}{\partial r} + \left[ \mu \frac{\partial^2 u_{com}}{\partial r^2} + \frac{\partial \mu}{\partial r} \frac{\partial u_{com}}{\partial r} + \mu \frac{\partial^2 w_{com}}{\partial z \partial r} + \frac{\partial \mu}{\partial z} \frac{\partial w_{com}}{\partial r} + \mu \frac{\partial^2 u_{com}}{\partial z^2} + \frac{\partial \mu}{\partial z} \frac{\partial u_{com}}{\partial z} \right]$$
(6)

$$\rho(\frac{\partial w_{com}}{\partial t} + u_{com}\frac{\partial w_{com}}{\partial r} + w_{com}\frac{\partial w_{com}}{\partial z}) = -\frac{\partial p}{\partial z} + (\mu \frac{\partial^2 w_{com}}{\partial r^2} + \frac{\partial \mu}{\partial r}\frac{\partial w_{com}}{\partial r} + \mu \frac{\partial^2 u_{com}}{\partial r \partial z} + \frac{\partial \mu}{\partial r}\frac{\partial u_{com}}{\partial z}) + \mu \frac{\partial^2 w_{com}}{\partial z^2} + \frac{\partial \mu}{\partial z}\frac{\partial w_{com}}{\partial z})$$
(7)

Energy Equation:

rgy Equation:  

$$\frac{\partial T_{emp}}{\partial t} + u_{com} \frac{\partial T_{emp}}{\partial r} + w_{com} \frac{\partial T_{emp}}{\partial z} = \alpha \left[ \frac{\partial^2 T_{emp}}{\partial r^2} + \frac{\partial^2 T_{emp}}{\partial z^2} + \frac{1}{r} \frac{\partial T_{emp}}{\partial r} \right] + \frac{1}{\rho c_p} \left[ \frac{\partial k}{\partial r} \frac{\partial T_{emp}}{\partial r} + \frac{\partial k}{\partial z} \frac{\partial T_{emp}}{\partial z} \right] \\
+ \tau \left\{ D_B \frac{\partial C}{\partial r} \frac{\partial T_{emp}}{\partial r} + \frac{\partial C}{\partial z} \frac{\partial T_{emp}}{\partial z} + \frac{D_T}{T_0} \left[ (\frac{\partial T_{emp}}{\partial r})^2 + (\frac{\partial T_{emp}}{\partial z})^2 \right] \right\}$$
(8)

Mass Transport Equation:

$$\frac{\partial C_{n.f}}{\partial t} + u_{com} \frac{\partial C}{\partial r} + w_{com} \frac{\partial C_{n.f}}{\partial z} = D_B \left( \frac{\partial^2 C_{n.f}}{\partial r^2} + \frac{1}{r} \frac{\partial C_{n.f}}{\partial r} + \frac{\partial^2 C_{n.f}}{\partial z^2} \right) + \frac{D_T}{T_0} \left( \frac{\partial^2 T_{emp}}{\partial r^2} + \frac{1}{r} \frac{\partial T_{emp}}{\partial r} + \frac{\partial^2 T_{emp}}{\partial z^2} \right)$$
(9)

There are only two dimensions due to axial symmetry flow i.e r in the radial and z are measured in the axial direction

The boundary condition of these equation are taken to be

$$t < 0: u_{com} = w_{com} = 0, T_{emp} = T_0, C_{n.f} = C_0 \text{ for all } r, z$$
  

$$t \ge 0: u_{com} = \frac{U}{\sqrt{1 - \beta t}}, w_{com} = -\frac{1}{a_0^2} \frac{4\nu z}{1 - \beta t}, T_{emp} = T_i$$
  

$$C_{n.f} = C_0 \text{ at } r = a(t)$$
  

$$C_{n.f} \to 0, T_{emp} \to T_0, C_{n.f} \to C_0 \text{ as } r \to b(t)$$
(10)

Here the suction velocity U (<0),  $T_i$  is the representation of temperature the wall of inner cylinder,  $C_i$  is the constant value representation of volume friction of nanopartical at the wall of inner cylinder, while the volume

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friction and the temperature at the outer cylinder is denoted by  $C_0$  and  $T_0$  respectively. In this problem we select the set of similarity variable from (Fang et al. 2012).

$$\mu = \mu_0 (1 + b_1 (T_0 - T_{emp})), k = k_0 (1 + b_2 (T_{emp} - T_0))$$

$$u_{com} = -\frac{1}{a_0} \frac{2\nu}{\sqrt{1 - \beta t}} \frac{f(\eta)}{\sqrt{\eta}}, w_{com} = -\frac{1}{a_0} \frac{4\nu z}{1 - \beta t} f'(\eta)$$

$$\theta = \frac{T_{emp} - T_0}{T_i - T_0}, \phi = \frac{C_{n.f} - C_0}{C_i - C_0}, \eta = (\frac{r}{a_0})^2 \frac{1}{1 - \beta t}$$
(11)

By making use of similarity transformation given in equation (11), the continuity equation (5) satisfied identically and the rest of equation (7-9) takes the following form.

$$f^{''''} - \xi(\theta f^{''''} + 2\theta^{'} f^{'''} \theta^{''} f^{''}) + \frac{1}{\eta} (f f^{'''} - f^{'} f^{''}) + \frac{3}{2\eta} [f^{'''} - \xi(\theta f^{'''} + \theta^{'} f^{''})] - S(f^{'''} + 2f^{''}) = 0$$
(12)

$$\frac{1}{Pr}(\eta\theta^{\prime\prime} + \theta^{\prime} + \varepsilon\eta\theta^{\prime^2}) + f\theta^{\prime} - S\eta\theta^{\prime} + \eta(Nb\theta^{\prime}\phi^{\prime} + Nt\theta^{\prime^2}) = 0$$
(13)

$$\eta \phi^{''} + \theta^{'} + Le(f\phi^{'} - S\eta\phi^{'}) + \frac{Nt}{Nb}(\eta\theta^{''} + \theta^{'}) = 0$$
(14)

The boundary condition (10) now becomes

$$f(1) = 0, \ f'(1) = 1, \ \theta(1) = 1, \ \phi(1) = 1 \ as \ \eta = 1$$
(15)

$$f(2) = 0, \ f'(2) = 1, \ \theta(2) = 0, \ \phi(2) = 0 \ as \ \eta = 2$$
(16)

Prandlt number represented by Pr, the representation of lewis number is Le, S represented the unsteadiness parameter, the Brownian motion is denoted by Nb, Nt shows the thermophoresis parameter,  $\varepsilon$  is the temperature dependent thermal conductivity parameter while  $\xi$  is the temperature dependent viscous parameter, which are the following as

$$Pr = \frac{\nu}{\alpha}, \ Le = \frac{\nu}{D_B}, \ S = \frac{a_0^2 \beta}{4\nu}, \ \xi = b1(T_i - T_0), \ \varepsilon = b2(T_i - T_0), \ Nb = \frac{\tau D_B(C_i - C_0)}{\nu}, \ Nt = \frac{\tau D_T(T_i - T_0)}{\nu T_0}$$

# 3 Solution by Homotopy Analysis Method

By HAM method [], the functions  $f(\eta), \theta(\eta), \phi(\eta)$  are define as

$$f_m(\eta) = \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta)$$
(17)

$$\theta_m(\eta) = \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} c_{m,n}^k \eta^k \exp(-n\eta)$$
(18)

$$\phi_m(\eta) = \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} d_{m,n}^k \eta^k \exp(-n\eta)$$
(19)

Where  $b_{m,n}^k \eta^k, c_{m,n}^k \eta^k, d_{m,n}^k \eta^k$  are the constant coefficient to be determined and the following are the initial guesses for the above equations

$$f_0(\eta) = \frac{-b - b^2 + \eta + 4b\eta + b^2\eta - 3\eta^2 - 3b\eta^2 + 2\eta^3}{(-1+b)^2}, \theta_0(\eta) = \frac{b - \eta}{-1+b}, \phi_0(\eta) = \frac{b - \eta}{-1+b}$$
(20)



Figure 2:  $S = 2, Pr = 0.2, Nb = 0.2, Nt = 0.05, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 



Figure 3:  $S = 2, Pr = 0.2, Nb = 0.2, Nt = 0.05, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 



Figure 4:  $S = 2, Pr = 0.2, Nb = 0.2, Nt = 0.05, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 

### 3.1 Error Analysis

In this section we analyze the error for the solution of the set of equation (12-14) by HAM method. For error analysis we plotted Fig (2-4), while table 1 and 2 are made. As we increase the approximation order, then the total average squared error and the total residual error have decreased for different values of natural parameters.

Numerical results of average residual error (ASRE) for velocity field, temperature and concentration for fixed values of natural parameters i.e S = 2, Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02,  $\xi = 0.05$  and  $\varepsilon = 0.002$  is given in table 1. While the convergent control parameters verses order of approximation by selecting S = 2, Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02,  $\xi = 0.05$  and  $\varepsilon = 0.002$  is given in table 2.

In table 1 we noticed that the ASRE reduces and become  $10^{-10}$ ,  $10^{-11}$  and  $10^{-12}$  for f,  $\theta$  and  $\phi$  respectively for m = 12. In table (3-4) we compared the values obtained from Homotopy Analysis Method with the numerical results obtained from BVP4c Package.

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m	$f(\eta)$	$ heta(\eta)$	$\phi(\eta)$						
2	4.92884	0.000276256	0.000679512						
4	0.00578015	$4.98305 \times 10^{-6}$	$9.87536 \times 10^{-6}$						
6	0.0000795539	$1.73602 \times 10^{-7}$	$1.6344 \times 10^{-7}$						
8	$7.53319 \times 10^{-7}$	$7.67869 \times 10^{-9}$	$3.20489 \times 10^{-9}$						
10	$1.07725 \times 10^{-8}$	$4.16759 \times 10^{-10}$	$8.14959 \times 10^{-11}$						
12	$2.82476 \times 10^{-10}$	$2.75373 \times 10^{-11}$	$3.06297 \times 10^{-12}$						

Table 1: Numerical data of ASRE.

Table 2: Data of convergence control parameters for various values of approximation.

Order of approximation	$h_f$	$h_{ heta}$	$h_{\phi}$	Error
3	-0.638804	-0.774898	-3.33702	0.354242
4	-0.670805	-0.792896	-3.22879	0.02318
5	-0.695343	-0.701385	-3.03664	$2.26199 \times 10^{-3}$
6	-0.672706	-0.839571	-2.82589	$3.0227 \times 10^{-4}$
7	-0.67851	-0.80836	-2.91789	$2.56283 \times 10^{-5}$

Table 3: Comparison of the values of  $f(\eta)$ ,  $f'(\eta)$ ,  $\theta(\eta)$  and  $\phi(\eta)$  with S = 2, Pr = 0.2, Nb = 0.2, Nt = 0.05, Le = 0.02,  $\xi = 0.05$ ,  $\varepsilon = 0.002$  and various values of  $\eta$  using (HAM) via *Mathematica* package and **BVPh4c**.

η	HAM			BVP4C				
	$f(\eta)$	$f'(\eta)$	$ heta(\eta)$	$\phi(\eta)$	$f(\eta)$	$f'(\eta)$	$\theta(\eta)$	$\phi(\eta)$
1.0000	$6.4670 \times 10^{-15}$	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000
1.1000	0.0736	0.4928	0.8856	0.8588	0.0736	0.4929	0.8857	0.8588
1.2000	0.1023	0.1006	0.7764	0.7306	0.1024	0.1007	0.7764	0.7307
1.3000	0.0972	-0.1875	0.6713	0.6134	0.0972	-0.1875	0.6714	0.6135
1.4000	0.0681	-0.3767	0.5696	0.5056	0.0682	-0.3768	0.5697	0.5057
1.5000	0.0251	-0.4666	0.4709	0.4059	0.0252	-0.4667	0.4707	0.4060
1.6000	-0.0216	-0.4516	0.3739	0.3133	-0.0217	-0.4516	0.3739	0.3134
1.7000	-0.0612	-0.3202	0.2787	0.2271	-0.0613	-0.3202	0.2788	0.2271
1.8000	-0.0812	-0.0543	0.1849	0.1465	-0.0812	-0.0543	0.1850	0.1465
1.9000	-0.0667	0.3729	0.0921	0.0709	-0.0668	0.3730	0.0921	0.0710
2.0000	$-3.7252 \times 10^{-9}$	1.0000	$-1.7462 \times 10^{-10}$	$-5.2750 \times 10^{-11}$	0.0000	1.0000	0.0000	0.0000

#### 3.2 Result and discussion

Flow between two concentric stretchable cylinders with variable physical properties along with heat transfer and concentration are modeled in the system of equations (3.12-3.14), corresponding to boundary conditions (3.15-3.16). For the solution we use HAM and for numerical simulation we use BVP4c. The tables (3.3-3.4)

	TTAN T							
$\eta$	HAM				BVP4C			
	$f(\eta)$	$f'(\eta)$	$ heta(\eta)$	$\phi(\eta)$	$f(\eta)$	$f'(\eta)$	$\theta(\eta)$	$\phi(\eta)$
1.0000	$-5.1451 \times 10^{-15}$	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000
1.1000	0.0775	0.5649	0.9425	0.8602	0.0775	0.5650	0.9425	0.8601
1.2000	0.1157	0.2123	0.8790	0.7324	0.1157	0.2123	0.8790	0.7324
1.3000	0.1223	-0.0676	0.8086	0.6149	0.1224	-0.0676	0.8086	0.6149
1.4000	0.1044	-0.2785	0.7301	0.5064	0.1045	-0.2786	0.7302	0.5064
1.5000	0.0690	-0.4175	0.6423	0.4058	0.0691	-0.4175	0.6424	0.4058
1.6000	0.0237	-0.4727	0.5436	0.3124	0.0238	-0.4727	0.5437	0.3124
1.7000	-0.0218	-0.4196	0.4323	0.2254	-0.0219	-0.4196	0.4324	0.2254
1.8000	-0.0551	-0.2143	0.3062	0.1446	-0.0551	-0.2144	0.3062	0.1446
1.9000	-0.0572	0.2176	0.1629	0.0695	-0.0573	0.2176	0.1630	0.0695
2.0000	0.0000	1.0000	$-3.4924 \times 10^{-9}$	$4.3655 \times 10^{-11}$	0.0000	1.0000	0.0000	0.0000

Table 4: Comparison of the values of  $f(\eta)$ ,  $f'(\eta)$ ,  $\theta(\eta)$  and  $\phi(\eta)$  with S = 4, Pr = 0.4, Nb = 0.6, Nt = 0.05, Le = 0.02,  $\xi = 0.05$ ,  $\varepsilon = 0.002$  and various values of  $\eta$  using (HAM) via *Mathematica* package and **BVPh4c**.



Figure 5: The effect of S on radial velocity with Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02,  $\xi = 0.05$ ,  $\varepsilon = 0.002$ 



Figure 6: The effect of  $\xi$  on radial velocity with S = 2, Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02,  $\varepsilon = 0.002$ 



Figure 7: The effect of S on axial velocity with  $Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 



Figure 8: The effect of  $\xi$  on axial velocity with  $S = 2, Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02, \varepsilon = 0.002$ 



Figure 9: The effect of  $\varepsilon$  on temperature with  $S = 2, Pr = 0.2, Nt = 0.05, Nb = 0.2, Le = 0.02, \xi = 0.05$ 

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Figure 10: The effect of Pr on temperature with  $S = 2, Nt = 0.05, Nb = 0.2, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 



Figure 11: The effect of Nb on temperature with  $S = 2, Pr = 0.2, Nt = 0.05, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 



Figure 12: The effect of Nt on mass transfer with  $S = 2, Pr = 0.2, Nb = 0.2, Le = 0.02, \xi = 0.05, \varepsilon = 0.002$ 



Figure 13: The effect of Le on mass transfer with S = 2, Pr = 0.2, Nb = 0.2, Nt = 0.05,  $\xi = 0.05$ ,  $\varepsilon = 0.002$ 

shows the comparison of non-dimensional axial velocity radial velocity, temperature and mass transport for different values of involved parameters. It is clear from the tables (3.3-3.4) that the results of HAM and the results obtained by numerical BVP4c package are same up to three decimal places. For analysis of the effect of natural parameters arises during modeling is graphed in figures (3.5-3.13).

In Fig (3.5) radial velocity  $f(\eta)$  verses  $\eta$  is plotted for S = 2, 4, 6 and 8 by other parameters as fixed shown in Fig (3.5). From this figure it is observed that by increasing unsteadiness parameter S,  $f(\eta)$  also increases because the stretching of cylinders is directly proportional to unsteadiness parameter, therefore it causes to increases velocity in this direction. In Fig (3.6) it is clear that if the temperature dependent viscous parameter decrease the radial velocity increases, because viscosity is inversely proportional to velocity. In Fig (3.7-3.8), if the parameters S and  $\xi$  are increases the axial velocity increases up to a specific limit of radius, because there is also some opposing force from outer cylinder, therefore the axial velocity increases up to specific radius and then decreases shown in Fig (3.7-3.8). In Fig (3.9), the temperature profile directly depend on temperature dependent thermal conductivity parameter. If the  $\varepsilon$  decrease, then temperature profile may decrease. In Fig(3.10), it is observed that if the Prandlt number increase then temperature profile increases. Prandlt number depend on kinematic viscosity and thermal conductivity. If the thermal conductivity increase then Pr decrease because thermal conductivity and temperature are directly depend on each other so it mean temperature inversely depend on Prandlt number, but in energy equation Prandlt number exist in the denominator shown in equation (3.13), therefore the effect is directly on temperature. In Fig (3.11) it is clear that if the Nb parameter increases, then temperature of the liquid is also increases, because Brownian parameter arise due to the collision of molecules, when molecules collides to each other than the temperature of liquid is increases. Fig (3.12) shows the effect of thermophoresis parameter. It is clear that if the thermophoresis parameter increase then the mass transport decreases, because thermophoresis parameter directly depend on temperature difference. If the temperature difference increase, it mean thermal conductivity decreases and the mass transport increasing with the increasing thermal conductivity, so the mass transport decreasing with the increasing temperature difference, therefore mass transport inversely depend on thermophoresis parameter. In Fig (3.13) mass transport equation  $\phi(\eta)$  verses  $\eta$  is plotted for Le = 0.05, 1.05, 2.05 and 3.05 by other parameters as fixed shown in Fig (3.13). It is observed that by increasing Lewis number parameter Le,  $\phi(\eta)$  also increases because Lewis number is the ratio of thermal diffusivity to mass diffusivity. If the thermal diffusivity increases then the Lewis number is increase, and mass transport is increases by increasing the thermal diffusion, therefore by increasing the Lewis number the mass transport also increases.

# 4 Concluding remarks

Consider the flow between two concentric stretching cylinder of non-newtonian fluid. The non linear system of differential equation into the system of ordinary differential equation by using some transformation. HAM (which is an analytical method ) is used for the solution and the results are compared with the numerical method BVP4c package. The effect of viscosity and thermal conductivity on the flow and heat transfer are

presented graphically and discussed. Results are compared in table 3.3 and 3.4 which are correct up to three decimal places. The effect of different parameters on radial velocity, axial velocity, temperature and mass transfer in r and z direction are shown by different graphs.

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