

Turbulent Natural Convection using $k - \omega - sst$ Model with varying Aspect Ratios

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KeyWords

$k - \omega - sst$ Model, Heating and cooling, Rectangular enclosure, Turbulent flow, Velocity vectors

ABSTRACT

Natural convection is a means of air mixing inside an enclosure which arises from temperature difference. It's one of the most important modes of heat flow and heat transfer. The aim of this study is to numerically investigate natural turbulent convection in a three dimensional rectangular enclosure using $k-\omega$ - sst model. The research is to investigate the effect of varying aspect ratios ($1.5 \geq A.R. \geq 0.75$) on temperature distribution in a rectangular enclosure with constant Rayleigh number ($Ra = 7.80 \times 10^{11}$). The equations governing the flow (Momentum, Continuity and Energy equations) were first time averaged. The averaging process introduced non-linear terms; Reynolds stress and heat flux which are modelled using $k-\omega$ - sst model. The emerging equations after modelling are non-dimensionalized and then discretized by finite difference method and the results solved using ANSYS Fluent. The results showed that, increasing the height of the enclosure decreases the aspect ratio which then causes vectors of temperature to rise on the left-hand side (hot wall) and sink in the right hand side (cold wall).

Introduction

Most fluids flows occurring in nature and created in engineering applications are turbulent. Turbulent flow is random, diffusive, dissipative, chaotic and irregular. These features have made turbulent flow to be highly and widely used in energy systems in industry. Due to its vast application in day to day activities, buoyancy driven natural convection in an enclosure is receiving more and more research attention. In most turbulent natural convection flows, investigations of velocity and temperatures profiles, heat transfer and turbulent intensities are mostly obtained by means of either experimental or modelling. Mikhail S. and Igor M. (2018) did a model using numerical approaches and also by use of experiments in Sustainable and Renewable energy on Turbulent natural convention heat transfer in rectangular enclosure. Shahid H., in 2021 simulated mixed convective heat transfer in a heated lid-driven right triangular

cavity. Mejri and Mahmoudi (2016) studied natural convection in an inclined triangular cavity filled with water. In 2022 Weppe A., *et al.* did an Experimental investigation of a turbulent natural convection flow in a cubic cavity with an inner obstacle partially heated. This work is aimed at investigating the effects of heating and cooling on temperature in a rectangular enclosure using the $k - \omega - sst$ model.

Mathematical Formulation

We consider a 3 D enclosure shown in fig 1. The vertical walls are isothermal while the horizontal walls are adiabatic. The hot wall is kept at 308K while the cold wall is kept at 288K creating a temperature difference of 18K between them.

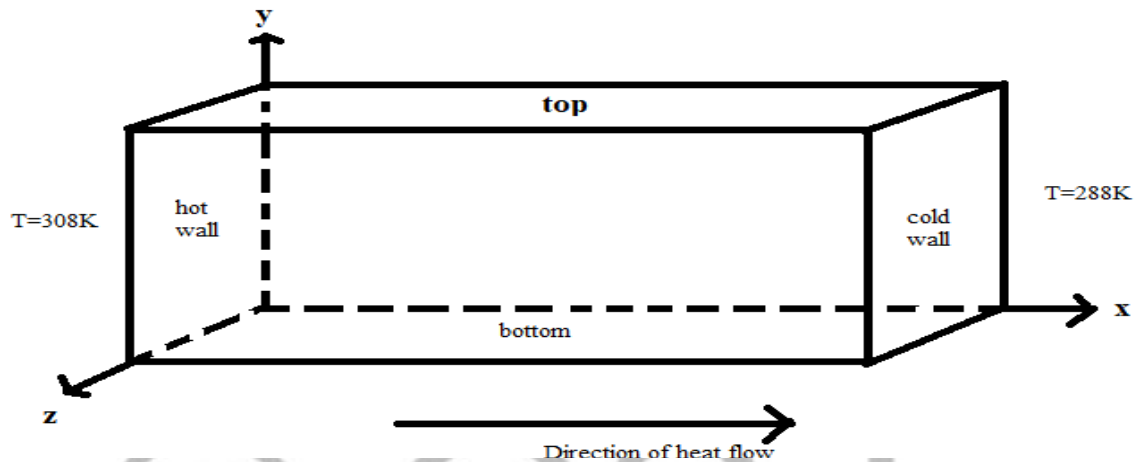


Fig 1

Governing Equations

The set of governing equations in two-dimensional rectangular coordinates which are continuity, momentum and energy equations are derived by Anderson *et al* 1984 [2]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = F_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \Phi \quad (4)$$

$$\text{Where } \Phi = \mu \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right\}$$

The above equations were non dimensionalised to reduce the number of parameters. The resulting equations in general form become;

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (5)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial \tau} = - \frac{\partial P}{\partial X} + Pr \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (6)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + \frac{\partial V}{\partial \tau} = -\frac{\partial P}{\partial Y} + Pr \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra \cdot Pr \cdot \theta_f \quad (7)$$

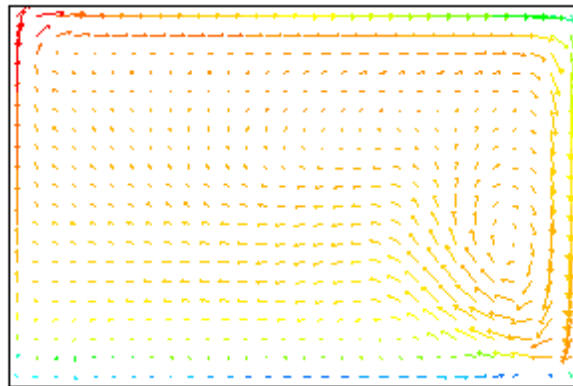
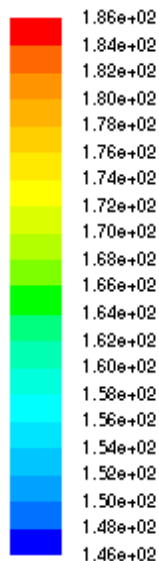
$$\frac{\partial \theta_f}{\partial \tau} + u \frac{\partial \theta_f}{\partial X} + v \frac{\partial \theta_f}{\partial Y} = k \left(\frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2} \right) + \Phi$$

Results and Discussion

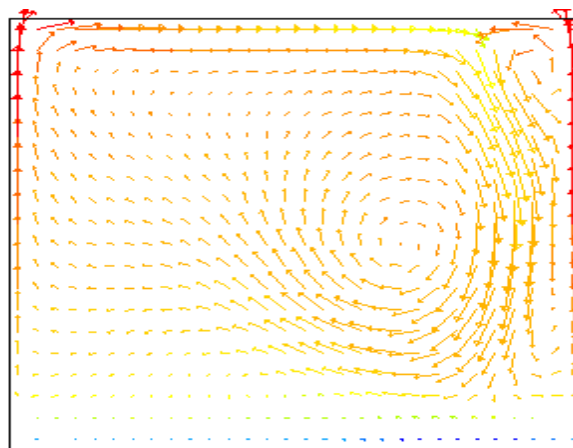
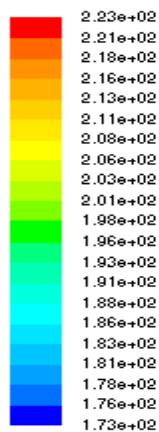
The results presented here were obtained by use of finite difference method to solve the governing equations numerically and together with the boundary conditions give the numerical solutions for variables in SST $k - \omega$ model. The Rayleigh numbers is kept constant by keeping the length constant.

Velocity vectors colored by temperature $Ra = 7.80 \times 10^{11}$

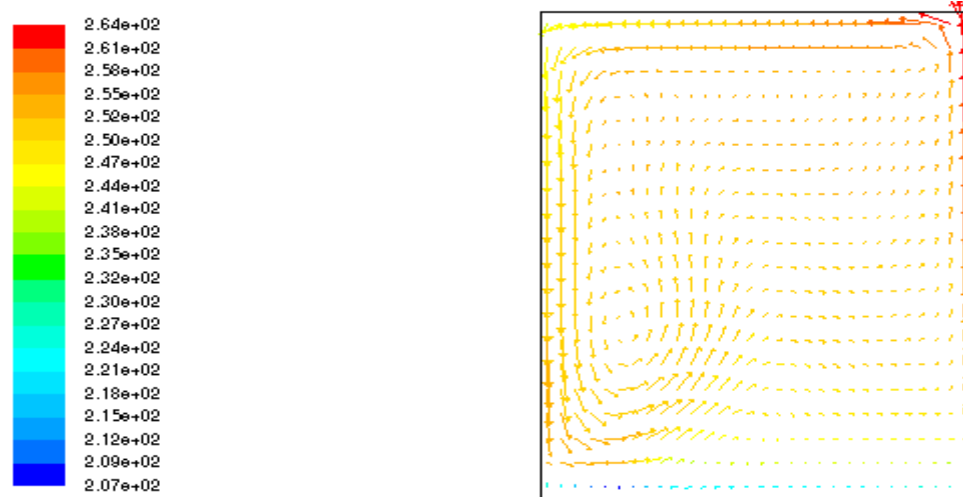
a) A.R=1.5



b) A.R=1



c) A.R=0.75



Looking at the temperature distribution in the enclosures, we consider the isotherms which are lines connecting points of equal temperature. The heat from the hot wall is transferred to the remaining parts of the enclosure through the working fluid. As the aspect ratio decrease the vectors of temperature tend to rise on the left-hand side (hot wall) and sink in the right hand side (cold wall).

Conclusion

This work was aimed at investigating the effect of varying aspect on temperature distribution in a rectangular enclosure with heating and cooling. The Rayleigh number was kept constant but the aspect ratio was varied by varying the height of the enclosure. The results showed that, increasing the height of the enclosure decreases the aspect ratio which then causes vectors of temperature to rise on the left-hand side (hot wall) and sink in the right hand side (cold wall).

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