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A Geometrical Generalized Model of Failure

Giuseppe Quartieri, Unisrita, ANFeA, CACR, Rome Italy

Reader Aids:
Purpose: Enlarge theoretical state of art
Spec. Math. Needed: Rene Thom Catastrophe Theory and System Theory
Results useful to: Reliability Physicists, Nano-Engineers and Physicists, Condensed Matter Physicists and Engineers, Complex System Engineers.

SUMMARY

This paper develops a new generalized geometrical model of failure by means of a cross fertilization of the classical reliability failure model with the elementary theory of the catastrophe discovered by René Thom. The purpose is to develop a generalized model of the failure physics in terms of evolution and process to approach the structure of the failure and consequent effects till the catastrophe.
The classic fail process starts with some primary malfunction and becomes a defect that propagating transforms to itself to a real failure. Not all the failures are or begin catastrophe in strict sense. Viceversa, all the catastrophes are or begin from critical failures. The application of the RTCT to the above concepts allow to develop (continuous) failure patterns to observe the dynamic of the failure mechanisms.
Usually, it is possible to see and control this comparison along the failure pattern going from smooth degradation failures to "critical unsafe failures".

INTRODUCTION

The physics of failure (POF or failure physics) is a part of the physics and/or engineering covering the:

1. Processes and mechanisms that induce failure.
2. Root cause failure mechanisms based on the specific physics laws.
3. Modelling failure mechanisms based on science and engineering first principles.

The aim of POF is the leveraging of knowledge and understanding of reliability and its prediction for improving product performance. The POF methodology uses modelling and simulations tools to understand and eliminate failures with and in Computer-Aided-Engineering environment. POF supports deterministic and probabilistic approach to reliability and provides a scientific basis for determining and improving performances. POF is part of the process of reducing costs by improving reliability upfront earlier implementing chipper design and product.
The scope of this analysis is to develop a generalized geometrical model of failure by means of a cross fertilization of the classic reliability failure model by using the René Thom Catastrophe Theory (RTCT, Ref. 1, 2). The RT theory was developed in the late 1968 by the eminent mathematician Professor Rene Thom. Many studies and articles have been processed trying to show how RTCT may be related to peculiar mathematical or operations area. In those articles, the thrust of the catastrophe theory is characterized by considering a number of examples. This paper provides similar analysis for the failure model trying to be as much comprehensive as possible.
The most difficult step found in this analysis is that of finding an easy, direct multidimensional space defining the ”failure” to which it could be very easy to apply the known R.T.C.T. geometry. Many computer program of the catastrophe theory have been developed for application to various fields of knowledge.
In this analysis, it is not afforded the peculiar application of the many (or just one) above mentioned catastrophe theory computer programs but only and only the way to find the proper geometrical dimensions and space representing the failure itself. This is the main and unique step of this analysis.
Due to the very generalized nature, the RTCT is born as a mathematical concept that has little if anything to do with ”failure" as it is used in the Reliability theory.
The demonstration of the applicability of the RTCT to failure physics is developed, only and only, in terms of finding the proper and rigorous geometric dimensional space. The demonstration analysis is based upon the following assumptions.

1. A primary malfunction of a material and/or of an elementary part component is a ”catastrophe” in the sense of RTCT.
2. The status (good and/or fault) of a material and/or elementary part component is represented by means of a multidimensional point of the process trajectory in a proper multidimensional space.

3. The most part of material primary malfunction can be «mathematically modeled by means of the solutions of the seven elementary RTCT equations, whilst the remaining cases are dealt with proper multidimensional spaces.
4. No need of applying proper catastrophe theory computer programs, because it is only question of changing variables and number of proper variables in the standard computer programs.

Consequently, the confirmations of the applicability of the proposed geometrical failure model are based only and only upon a survey of very well-known and recognized applications of failure mechanisms in various fields of applied physics. New materials (i.e. lead-free solder, high-K dielectric and so on, software programs using algorithms for prognostic purposes, integrating physics of failure, corrosion patterns and so on) are new topics for geometrical failure model application.
The overall analysis activities involve the modeling of failure mechanisms such as corrosion, relating material degradation as a function of:

- operational and environmental stress on the effect that aging materials including Long-Term Aging of Systems (LAST)
- nature of failure modes and mechanisms in existing designs
- mitigation of failures with proper consideration of environmental severity, condition-based monitoring, and effective maintenance strategies.

An Open Opportunity

At the moment, the analysis is at the very beginning of the study of the potential correlation between the presented geometrical model and the class of models such as stress-strain failure model, Lode angle failure model and its set of tensor invariants for continuous damage and ductile failure plus fracture locus and so on. Other opportunities are given by comparing with micromechanics model use to predict the effective fully coupled time-dependent and non-linear multi-physics responses of smart composites.
At the moment, many different model of failure physics have been developed. Some of them predict the responses of smart composites in terms of effective fully coupled time-dependent and non-linear multi-physics.
Other models are developed on the basis of the variation asymptotic method and implemented by using the finite element method. Taking into account the time-dependent and non-linear characteristics of smart composites, some other procedures have been implemented unifying the instantaneous tangential electromagnetomechanical matrix of composites was established. At the present time, it is not possible to present numerical example to demonstrate the capability of those models.

NOTATIONS AND DEFINITIONS

RT	Renè Thom
RTCT	René Thom Catastrophe Theory
Morphogenesis	Creation or destruction of form.
Catastrophe	Any technical performance of any morphogenesis discontinuity (gap) of medium.
Primary Malfunction (Primal)	Any malfunction occurring during the specified life of a material and/or elementary component which is attributable to itself (material and/or personnel factors, failure of related components or foreign object damage.
Defect	Any primary malfunction in a material, component (and/or equipment, system) which requires a correction by unscheduled maintenance work.
Failure	Any defect of the material and/or elementary part component which creates an inability of a previously acceptable material (etc.) to perform its required function within the limits established in the contractual specification.

Topology	A collection R of sets closed respect to the formation of arbitrary number of unions and of a finite number of intersections which include the null set Φ and the overall space Ω.
Topological space	(Ω,τ) is a space in which a topology τ has been selected.
Ω	Event Space (space of failure and not failure).
E_k	k-th event.
TF	Total failure subset
NF	Not-failure subset
Ω_i	i-th failure space subset
K	Catastrophe set
$W=(\omega_1,\omega_4)$	Failure status = failure subset coordinates
$K= g(x, y, z,)=g(\omega,t)=g(\omega_1,\omega_2,t)$	Analytic function
$K = \Omega_4$	Projection subset of catastrophe set K
F	Attractor
(M,K)	Dynamic system Σ
S(A,B,C,D)	Linear system model Σ (state space)
x(t)	System failure vector status
u(t)	System input vector
y(t)	System output vector = output probability or reliability
A(t)	System status matrix
B(t)	Real matrix
C(t)	Real matrix
D(t)	Real matrix
P(t)	Vector status (transition status) probability at instant t
Z(t)	Transition rate matrix
C	Structure vector
C^T	Transport of a vector C whose components are "1" or "0" according to which it is true that relative status is "good" or "fault".
Γ	Activation energy
Δ	Potential performance
Δ_i	Initial value of performance
Δ_f	Final value of performance
K	Constant
T_e	Temperature
S_{ts}, f_1, f_2	Applied stress function in Arrhenius Law
$n_v(\nu)$	Transformed random variable
V_k	Cut off value of transformed random variable
V_o	Initial value of transformed random variable
a, b, c, d	Control parameter coefficients
T	Arrhenius device life length
J_A	Jonic flow
MTTF	Mean time to failure
dN_v/dt	Hole concentration rate
ΔJ_A	Flow divergence
O.P.	Order Parameter
p.m.	Primary malfunction (primal)
c.l.	Component life
i.c.	Inherent cause of p.m.

Classical failure set classification

Classical reliability theory classification pattern of failure set can be simply and very briefly described as follows according to Fig.1.

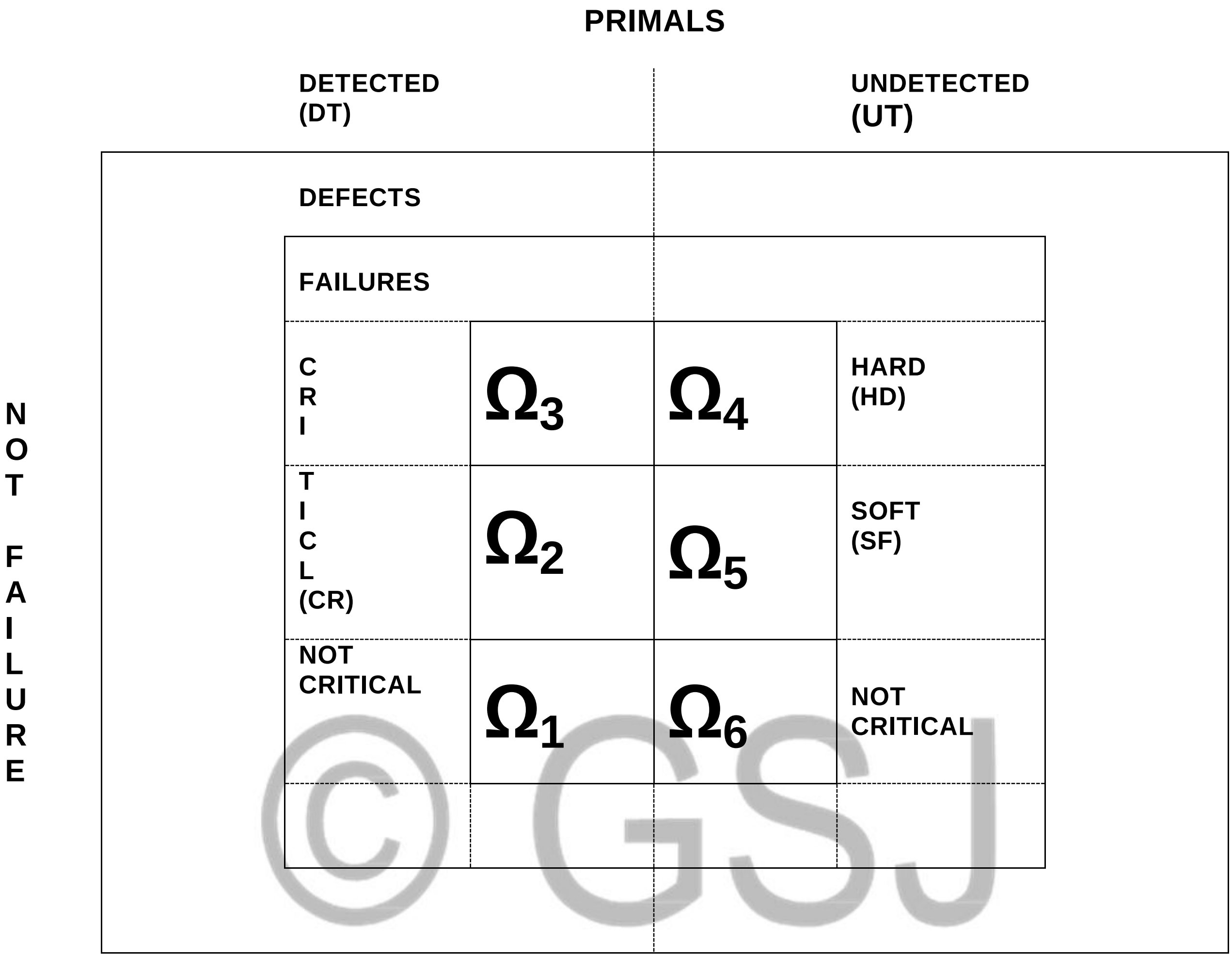

Fig. N° 1 Classical Partition of FAILURE SET

The classical failure set theory partition can be simply represented by the overall space Ω of failures of Fig. 1 that is self-evident. Actually, test result event E_k can belong to any one of the subset Ω_1, Ω_2, Ω_3, Ω_4, Ω_5, Ω_6 that constitute the overall space:

[1] $$\Omega = TF \cup NF$$

The analysis of the definition of primary malfunction (primal), defect and failure are given in the reliability analysis. In the reliability real life, the processing reliability analisys starts from the primal patterns producing defect and failure linked to the above failure set basic classification. In the meantime, failure data base collections are built up. So the analysis of the propagation of the effects of the failure i.e. the FTA, FMEA and FMECA is processed to infer the effect to the higher level of assembly. The hierarchical and indenture level classification of the effect of a failure is also worked up by using the classical the partition of failures reported in Fig. 2 in a very simplified way.

With bottom-up approach an elementary failure, i.e. a hard or soft primal, produces effects to higher indenture level diversifying among unsafe, and safe failures then in safe or unsafe critical failure to summing up the total failures set.

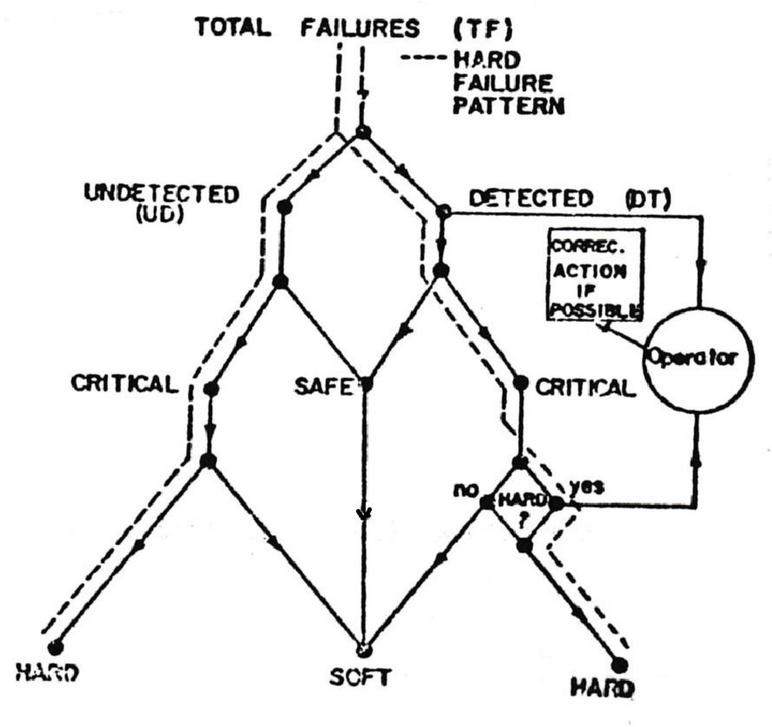

Fig. 2 Failure pattern

Deeper analysis reliability allows to define the following correlations:

1. Primary Malfunction (x_1):
   - a) Component life.
   - b) Inherent cause of primal.
2. Defect (x_2):
   - a) Not-scheduled-maintenance.
   - b) Primary malfunction.
3. Failure (x_3):
   - a) Specification limits.
   - b) Defect.

By properly overlapping the three ideal curves of primary malfunction, defect and failure in only one space by applying twice the 90 degrees rotation to each single variable, it is obtained the curves of Fig. 3. This allows to demonstrate how primary malfunction, defect and failure do match each other building a very general behavior against performance limits of materials and elementary part components.

The interception of the three tridimensional curves give a bidemensional curve whose projection in the plan of the Ω that can be easily correlated to the RTCT catastrophe set and system failure status process trajectory.

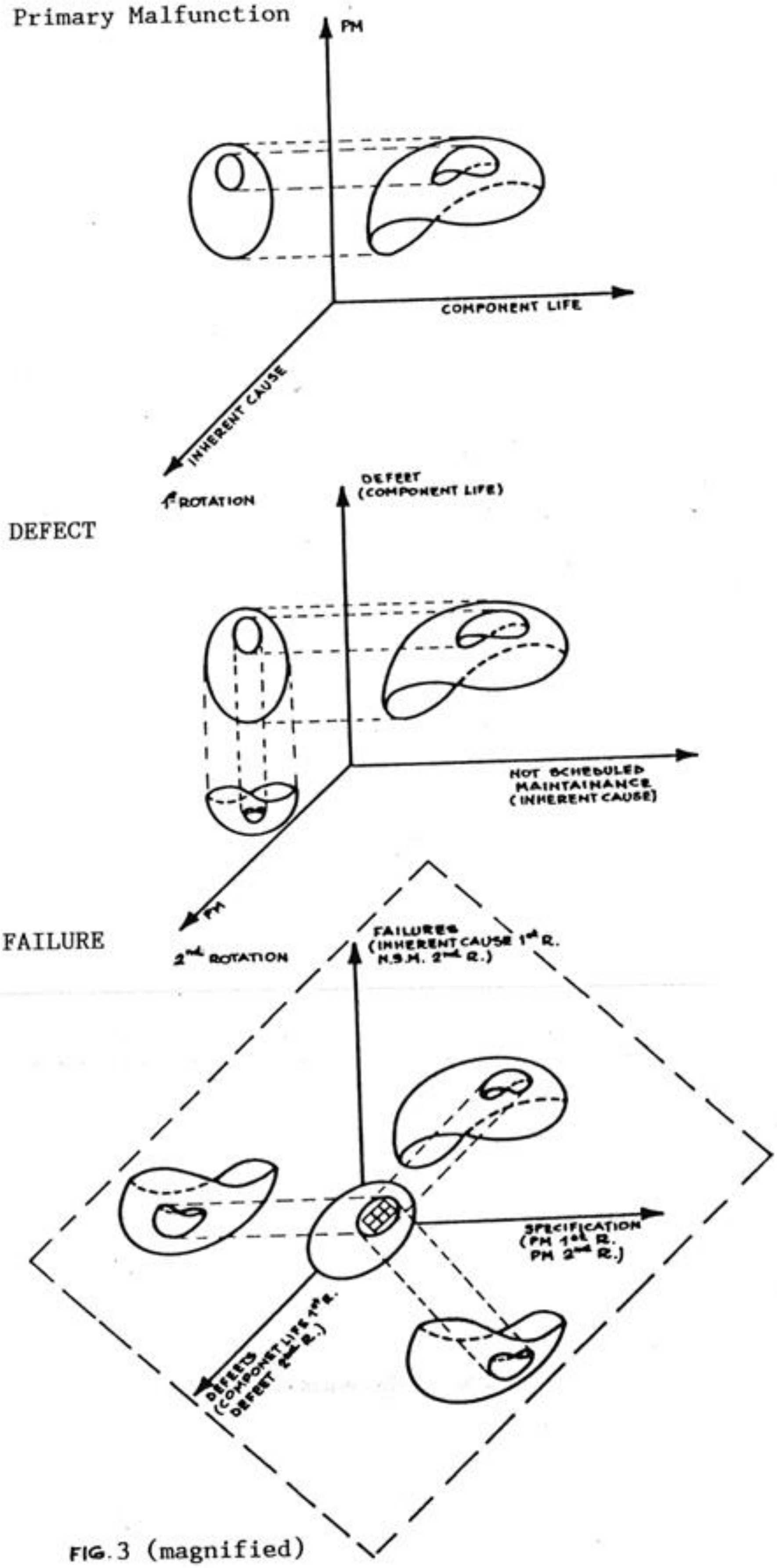

FIG. 3 (magnified)

Fig. N° 3 Relations among Primals (primary malfunctions, defects and failures)

GEOMETRIC FAILURE MODEL

The very general geometrical construction of the proper multidimensional space of failure mechanism is based upon:

1. The revisit in the classical failure classification by means of the application of the set theory;
2. To generalize the above failure set classification to a three dimension space by isolating in the definitions of primary malfunction, defect and failure three main variable for each one definition and then create with these three isolated variables a new failure tridimensional space Ω;
3. To introduce in this Ω failure space the RTCT geometrical approach.

The dynamical failure mechanisms are the following:

1. Failures happen random at various instants t_i;
2. During the infinitesimal period Δt_i, before t_i, the failure trajectory process across one of the seven elementary catastrophes or a more complicate pattern.

The restriction of failure trajectory process duration to the infinitesimal Δt_i, left period before time t_i of failure occurrence is a very limited to the real jump in the process trajectory itself.

In principle, the duration of the overall failure process trajectory could last the entire failure free period to the first failure.

Consequently, the Reliability to the first failure can be written:

[2] $$R(t_1) = R(t_{f1}, -\Delta t_i, t_1)$$

The concept of failure deducible from RTCT

Let now apply the above classical system status failure model to the RTCT model of the system catastrophe. The concept of catastrophe is related to any technical performance of any form or any kind of morphogenesis that presents a discontinuity or a gap in the medium performances. The local states X of a system Σ (material) are parameterized in a space of observables Ω (or ν), in which it exists a closed set $K \in \Omega$, called catastrophe set (Ref. 1, 2, 4).

Let us assume that the system Σ failure status X is defined by an analytical function:

[3] $$X = g(\omega, t) \quad \text{where } \omega = (x, y) \text{ or } \omega = (x_1, x_2, x_3)$$

From the RTCT structural stability point of View it is natural to assume that the function g(x, y, t) is a continuously differentiable function (in practice it is enough a second or third order derivative).

Consequently, it is deducible that a point (x, y, t) in the Ω space is a catastrophic one if the function g(x, y, t) or its first or second derivative has a discontinuity or a gap at this point.

Otherwise it could be written:

[4] if $(x, y) \in K \equiv \Omega_4$ the system Σ fails.

[4'] if $(x, y) ¢ K \equiv \Omega_4$ the system Σ operates.

Until the system state $(x, y) \equiv \omega$ does not meet the closed set $K \equiv \Omega_4$ the local system phenomenology does not change. If the state $(x, y) \equiv \omega$ meets the $K \equiv \Omega$ set just moving along a trajectory, then a discontinuity in the system form will happen.

Rene Thom interprets this as a change of the previous form, i.e., a morphogenesis. In other words, the space Ω, associated to the system Σ, is a topological space with the convention. that, if the state (failure state) $(x, y) \equiv \omega$ of the system Σ is out $K \equiv \Omega_4$ then the phenomenological form of the status does not change against a small deformation (perturbation) of the Status itself.

So, any trajectory, any kind of process corresponds to a set of points in Ω-K. Let Ω be a space having differentiable structure (Euclidian R^m space or a differentiable variety), the failure pattern dynamics is identified by means of the vector field X on Ω. The theory of existence and unicity of the solution of a system of differential equations with differentiable equation is the base upon which this deterministic failure model is constructed.

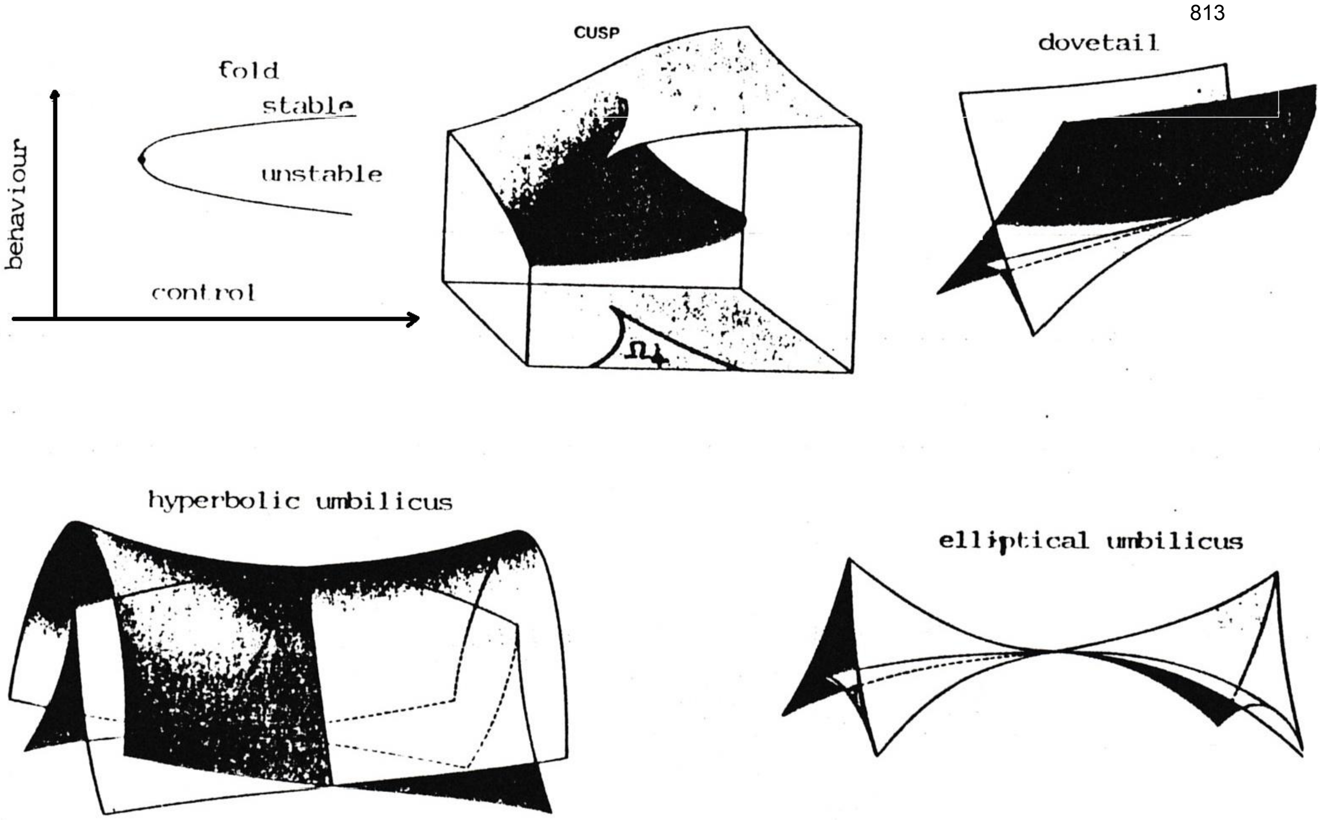

Fig. 4 - Elementary Catastrophe

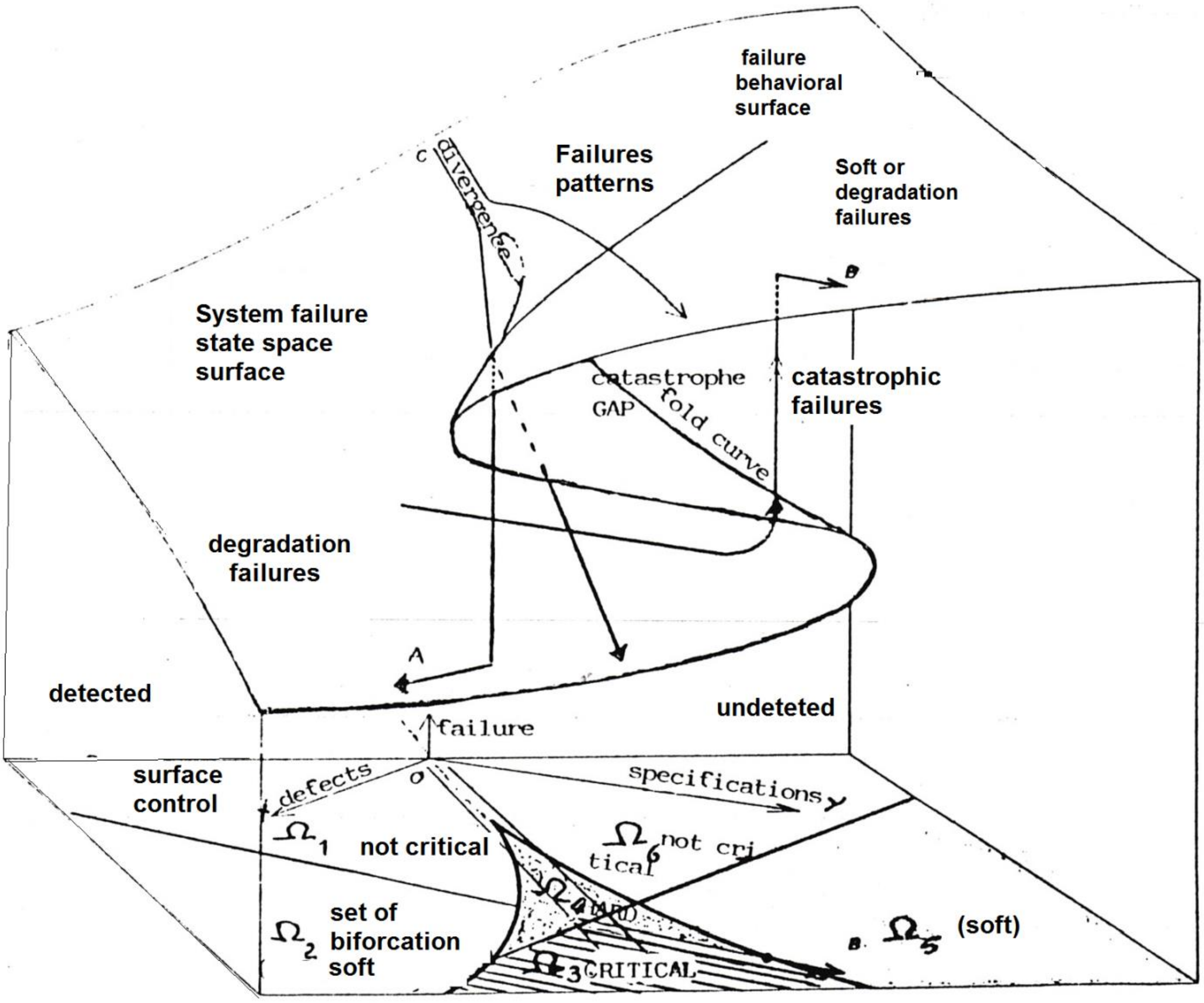

Fig. 4b - Cusp Failure Behavior

The tridimensional failure trajectory goes from a regular point (whose projection is ω in the Ω_4 space) to another regular point in Fig. 4 without meeting "catastrophic set K" (projection Ω). This corresponds to failure pattern going from a degradation point to another degradation point. In the open set Ω-K ($\Omega \equiv \Omega_4$) the overall (projected) process X is a regular process.

By definition, the regular points form an open set in Ω, the closed set complementary to the regular points is the above K set of "catastrophic points". Quite close to each point of the closed set K of catastrophic points, the process presents a "discontinuity" (a gap). In each elementary neighborhood of any int c of the closed set K "it happens something". So, to know the closed set K≡Ω, describing each singularity, it is just the morphology of the process. It is possible to define catastrophic any failure if the failure detection and analysis usually is made up by means of very sophisticated and very sensible and accurate measurement equipments. Furthermore, as a little digression, let remember that the lack of a comprehensive understanding of the failure process in structural materials has resulted in catastrophic failure of a variety of engineering structures. Many kinds of structural mechanical failures (fracture, cracks, flaws, brittle fracture, cleavage, extrusions, forgings, stress concentration such as cut-outs, bolt, rivet holes, evidence of fitting from flaws and from imperfections in welding and so on) could be and are usually interpreted in term of the R.T.C.T. application.

In the real life application, many equipments and systems have been and are developed for detecting mechanical structural failure, i.e. ultrasonic equipments, industrial (infrared tomography, gamma tomography or positron tomography and so on) tomography (Ref. 11) for detailed inspection complex and critical parts as jet engine, turbine parts, blades and disks, fuel rods for nuclear reactors.

So really, the distinction between catastrophic and just degradation failure is only a matter of the level of precision in the mathematical model. In other words, it is just the hierarchical and indenture level in the system analysis that defines precisely the level of severity and criticality of the "catastrophicity" of the failure. So, a part from the fault tree effect and propagation law the failure pattern is the best quantum of information.

The qualitative dynamic notion (system failure status space)

Without recalling all the R.T.C.T., let us summarize some very basic notions such as qualitative dynamic, attractor, and ordinary and essential catastrophic point definition, and later the notion of generalized catastrophe.

Let (M, X) be a dynamical system Σ defined by the vector field X upon the variety M (≡Ω). In the usual system theory, the dynamical system Σ is represented by means of a couple equations (R. Somma, V. Amoia Ref. 10):

5) $$\frac{dx}{dt}(t) = A(t)\,x(t) + B(t)\,u(t)$$

6) $$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

assumed that the system Σ is linear time variable or not stationary (coefficient matrix depend upon time) and not homogenous ($u(t) \neq 0$). In the reliability theory, these equations are specialized as follows (Ref.10):

7) $$\frac{dp}{dt}(t) = Z(t)\,p(t)$$

8) $$y(t) = c^t\,p(t)$$

They represent the model in differentiable form of a dynamic system Σ linear and homogenous with state probability $p_s(t)$ and output probability $p_o(t)$.

Cross fertilization of the two models gives that the vector field X upon the variety M is just the status vector of the system theory. In the R.T.C.T. the status X (x, y) in study is the system failed status itself which is analyzed from the structural stability point of view. So, to the dynamic system defined as (M, X) (where M=A(t)) is associated an "attractor F" that is a closed set invariant against X, having peculiar properties (Ref. 1).

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The attractor F of a field X constitutes the basic entity defining the structural stability. In the failure behavior surface usually the attractors can be considered isolated points or just ordinary closed trajectories, for which the structural stability is just straight forward. Losing structural stability brings failure in the system status. Let remember that the ordinary catastrophic points in X, projected in $K \equiv \Omega_4$, form an open set in the K set of catastrophic failure points. Moreover, a not-ordinary catastrophic (failure) points are said an essential catastrophic points. They form a closed subset of the set K of the catastrophes. By definition, an essential catastrophic point cannot be isolated. If the number of elementary and very small catastrophes is very high and quite closed each other, then a field reduction is applied usually by the observer to process an average of all those little catastrophes building up a static model instead of a metabolic (dynamic) model. So, essential catastrophic points can be of two kinds: the ones that are all inside the round of ordinary catastrophic points and the others having a round that includes only essential catastrophic points. Then, there is a demarcation line between points that are inside ordinary catastrophes and points belonging to essential catastrophe round.
For analogy, being the R.T.C.T. the most important step improvement to the analogy theory from the time of Aristotle, the R.T. essential catastrophe set of joints could be made to correspond to the usual "cut set" or "tie set" of the fault tree theory.

Improves, verifications and validations
Any experimental fact reproving the R.T.C.T. approach of this generalized failure physics can be reported in support of this generalized failure model. As above mentioned, many kinds of structural mechanical failures could be theoretically interpreted by means of this generalized failure model and concept. The proposed approach is basically a phenomenological description of the generalized failure physics model. Let us limit the analysis to the most important topics:

1. Arrhenius law extensible to other known models (Eyring, Peck, Reick-Hokm and so on).
2. Electromigration in R.F. devices, and semiconductors [later enlargeable to failure mechanisms like Negative Bias Temperature Instability (NBTI), Hot Carrier Injection (HCI) and Time Dependent Dielectric Breakdown (TDDB) etc.].
3. Order-disorder phenomenon including idealized endurance curve during vibration tests.

Arrhenius law applications
The usual failure model employed in reliability physics assumes that a failure occurs after that the performance value of a device varies from an initial value V_0 to a critical value V_F. At this instant t_F a gap threshold is reached and the device fails. The length of this period is the device life. The relation between the rate of degradation of V and the applied stress is given by the empirical Arrhenius mode1, borrowed from the chemical physics and statistical dynamics (Ref. 6).
The classical Arrhenius failure model is:

$$\frac{dV}{dt} = \text{K} \exp \left[\frac{\varepsilon - f(x)}{kTe} + \text{f}(\text{x})\right]$$

Integrated with respect to t, it is:

$$\text{t}(\text{T}_e) = \frac{V_F - V_O}{K} \text{K} \exp \left[\frac{\varepsilon - f(x)}{kTe} + \text{f}(\text{x})\right]$$

where: V = Vo when t = 0. Defined $\frac{V_F - V_O}{K} = \bar{U}$, it is:

$$\text{t} \quad (\text{T}_e) \quad = \quad \bar{U} \quad \text{-exp} \quad (\text{-f}) \quad = \quad \bar{U}_F \qquad \bar{U} > 0$$

$$\ln F = S \qquad F > 0$$

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Let now suppose that the applied stress function f (s) = Sts is not a random variable, but it has one of the standard seven elementary catastrophic forms developed by Rene Thom for his Theory of the Catastrophe. So, from reliability point of view the system Σ is described by the failure behavior surface, corresponding to each of the seven very well-known elementary catastrophes, given in Fig. A (classification theorem).
These seven elementary catastrophes describe all the possible gaps that can be verified in the phenomena controlled by four-factor-parameters. To each elementary catastrophe it is associated a function that includes the control parameters represented by means of four coefficients (a, b, c, d).
Those are the failure controlling properties. The failure behavioral surface is such that each point the first derivative is zero. If the first two derivatives exist, then the behavioral surface is the point "1oci" in which the couple of derivatives is nullified. The table N. 1 gives the Arrhenius failure behavior and life length T in correspondence to each of the seven elementary catastrophes.

Tab. N° l

elementary catastrophe type		control dimension	behaviour dimension	failure behaviour	Arrhenius life length T
cusps	fold	1	1	$\frac{1}{3}x^3 - ax$	$\nu \exp(\frac{1}{3}x^3 - ax)$
	cusp	2	1	$\frac{1}{4}x^4 - ax - \frac{1}{2}bx^2$	$\nu \exp(\frac{1}{4}x^4 - ax - \frac{1}{2}bx^2)$
	dovetail	3	1	$\frac{1}{5}x^5 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3$	$\nu \exp(\frac{1}{5}x^5 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3)$
	throttle (butterfly)	4	1	$\frac{1}{6}x^6 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3 - \frac{1}{4}dx^4$	$\nu \exp(\frac{1}{8}x^6 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3 - \frac{1}{4}dx)$
umbilicus	hyperbolic	3	2	$x^3 + y^3 + ax + by + cxy$	$\nu \exp(x^3 + y^3 + ax + by + cxy)$
	elliptical	3	2	$x^3 - xy^2 + ax + by + cx^2 + cy^2$	$\nu \exp(x^3 - xy^2 + ax + by + cx^2 + cy^2)$
	parabolic	4	2	$x^2y + y^4 + ax + by + cx^2 + dy^2$	$\nu \exp(x^2y + y^4 + ax + by + cx^2 + dy^2$

Electromigration in RF Devices (In Thick and So On)
By using the Huntington and Grone theory (Ref. 21) it is possible to link the jonic flow J_A to the electric field E (or current density):

$$J = \frac{ND}{KD} z e$$

So the mean time to failure against the stress is‘

$$MTTF = c \exp\left(\frac{\varepsilon}{KT}\right) \left(\sum_{0}^{\infty} M_n J^n\right)$$

The process of electro-migration degradation fault is usually described by means of:

$$\frac{dNv}{dt} = - \Delta J_\Lambda$$

i.e. the hole concentration rate (in time) depends upon the flow divergence. In any point of the film where there is a flow divergence (thermal gradient, structural not-homogeneities, defects and so on) it is also possible to find a starting point of failure due to electromigration. The overcoming the activation energy ε figure a failure mechanisms can initiate (A1: $\varepsilon = 0.6 \pm 0.2$ V). By reasoning in similar way of Arrhenius theory it is possible to build up a new table corresponding to the seven elementary catastrophes.

Order-disorder
White and Ceballe (Ref. 3) had treated very deeply the disorder to order phenomena, classifying the so called "dirty physics” in:

a) ordering with no transition;

b) ordering via first-order transition, i.e. a first derivative of the Gibbs free energy is discontinuous;
c) ordering via a second order transition, a discontinuity in a second derivative of the Gibbs free energy G.

Physically, the most phase transitions are characterized by the appearance of some not zero quantity in the ordered state (lowering the Σ symmetry).
Such a quantity is called the "order parameter". Two orders of transitions are possible:
1) first: the order parameter vanishes discontinuously;
2) second: the order parameter vanishes continuously.

Table 2 gives the most important order parameters (O.P.) in solid state. The proposed generalized "order parameter" behavior, reported in Fig. 5, is the generalized phenomenological description of the physics model of O.P.

PARAMETERS CLASSIFICATION (dirty physics)

<table>
<tr><td colspan="3"></td></tr>
<tr><td>TIPE OF ORDER</td><td colspan="2">ORDER PARAMETER</td></tr>
<tr><td>Magnetic Order</td><td colspan="2">Generalized susceptibility (ferromagnetic)
(spontaneous magnetization)
Staggered magnetization (antiferromagnetic)
Magnetization $M_x+M_y = M (\exp \iota\theta)$
$q = |<\overline{\Sigma}>|^2$ (spin glass)</td></tr>
<tr><td>Crystallization</td><td colspan="2">Electronic charge density ρ(r) (diffraction patterns)
Scattering amplitude or periodic potential V (G)</td></tr>
<tr><td>Superconductivity</td><td colspan="2">The fraction of electrons condensed below T_c in a superconducting state with a fraction $(1- n_s/n)$ remaining normal is:
$|\varepsilon|^4 = n_s/n = \left(\frac{T_c - T}{T_c}\right)^4$
Pair wave function (BCS), a quantity that in special case, is the energy gap:
<table>
<tr><th>Quantity</th><th>Formula</th></tr>
<tr><td>(BCS) Coherence length</td><td>$\xi = \hbar v_F/\pi\Delta$</td></tr>
<tr><td>BCS gap</td><td>$\Delta/T_c = 3.5$</td></tr>
<tr><td>Critical temperature</td><td>$T_c = (...)\hbar\omega_0 \exp(-1/N(0)V_0)$</td></tr>
<tr><td>Quasiparticle dispersion</td><td>$E_k = \sqrt{|\epsilon(k) - \mu|^2 + |\Delta(\mathbf{k})|^2}$</td></tr>
<tr><td>(BCS coefficients)</td><td>$|u_\mathbf{k}|^2, |v_\mathbf{k}|^2 = \frac{1}{2}(1 \pm (\epsilon(\mathbf{k}) - \mu)/E_\mathbf{k})$
$u_\mathbf{k} v_\mathbf{k} = \Delta(\mathbf{k})/2E_\mathbf{k}$</td></tr>
</table>
The gap equation:
$\Delta_\mathbf{k} = -\frac{1}{2}\sum_{\mathbf{k}'} V^{(\mathbf{0})}_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{|\epsilon_{\mathbf{k}'}|^2 + |\Delta_{\mathbf{k}'}|^2}}$</td></tr>
<tr><td rowspan="2">Structural transitions</td><td colspan="2">DOF (Degree OF Freedom): splitting the electronic state for cooperative Jahn-Teller transitions.
ELECTRONIC INSTABILITIES: Spin Peierls distortion amplitude of charge density Δ_k</td></tr>
<tr><td>VIBRATIONAL INSTABILITIES</td><td>Dissipative transition amplitude of distortion
Twist ϕ (soft made)
Amplitude of distortion flow rate:
• interstitial and substitution impurity atoms
• dislocation lines
• color defects and flow defects
• grand edge
• $\rho_{liq} - \rho_{glas}$ = density difference.</td></tr>
<tr><td>Liquid cristals
(molecular ordining)</td><td colspan="2">θ = angle between long axis of molecule and preferred axis
$s = <3/2 \cos^2\theta - 1/2> = \frac{Xz - Xx}{N(Xii - Xi)}$ in diamagnetic parameter density in the direction of the preferred axis.</td></tr>
<tr><td>Superfluid Helium</td><td colspan="2">In 4He→OP = $\sqrt{x_0}\, e^{j\phi}$
$\phi(o) = \xi_0 + 1/\sqrt{\nu}\, \Sigma/kt_0\, e^{ik-r}$ ak wave function
OP→ 2 x 2 matrix in spin space
$\Delta_{\alpha\beta}(k) \cong <C_{kd}\, C_{kb}>$</td></tr>
</table>

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Inside the order-disorder phenomena, it is peculiarly interesting to recall the idealized endurance curve due to its clear analogy to a dovetail failure behavior (Fig. 6). This further verification of R.T.C.T. application to the failure concept is very important because it could be the foundations for a new approach of the overall vibration failure behavior.
From all the above mentioned R.T.C.T. application examples it could be infer red that instabilities are the malfunctions, defects and failures. In this contest, men is excluded as component and as system. Many other very important fields could be mentioned (i.e. Tunnel Diode performance, quarks and so on), but it convenient to limit this discussion to a very peculiar and important field: the MHD instabilities and beams (dispersive relations) instabilities inside the R.T. C.T.

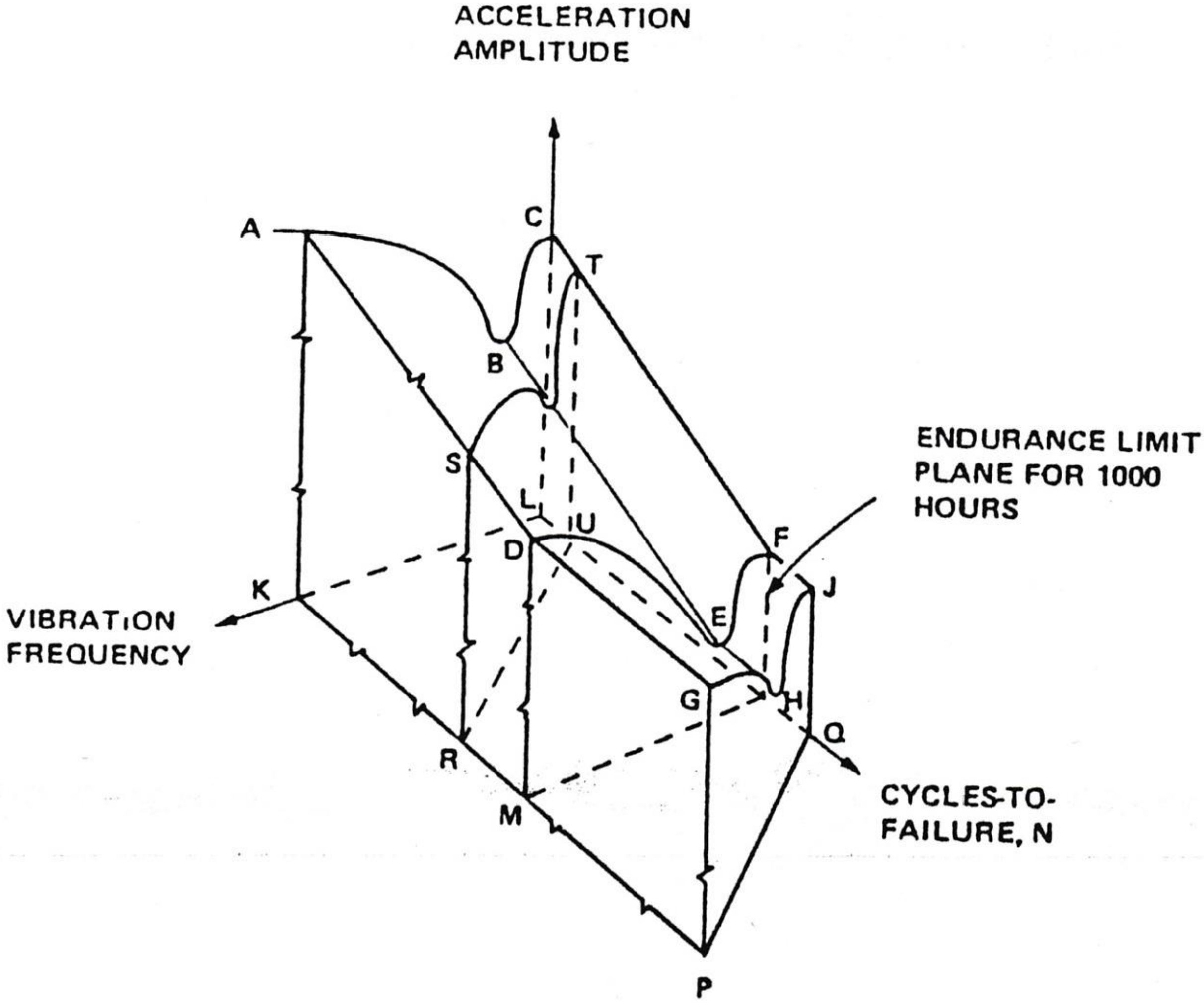

Fig.5 Order Parameter failure behavior

Those topics are very big to the extent they could cover a book, so they are' not really entered here neither tentatively. However intuitively, eruditely and boldly (see Ref. 8 , 9), it can be said that the well know "sausage instability", the "toroid instability" cab be interpreted and descripted by means of the generalized R.T.C.T. With a big effort the interpretation and description of "non-linear in stabilities" could be interpreted and descripted too.
Deeper C.T. analysis should be able to describe also the beams instabilities (dispersion relations) and beams coupling physical behaviors (like beating, evanescence, convective instabilities, and absolute instabilities and so on). Actually, the tridimensional topologies of dispersion curves are still not available (at least to me in the open literature) but many projections give sound basis for deducing that they can belong to the standard seven elementary catastrophes or to the more generalized sketch. All those aspects need a very big and sound analysis for reducing them inside the generalized R.T.C.T. This would request almost a book effort.

Let remember the R.T. proposition (Ref. 1) that the *every kind of catastrophe or morphogenesis could be treated and explained by means of his theory*. Frequently new ideas are first introduced informally and, later, more rigorous and complete treatment is performed and presented.

Two Models Coexistence

Let the system Σ have an exponential failure distribution, i.e., defined only by the failure rate λ . Let λ be a not constant function of type:

$$\lambda = g \quad (w_1 \quad w_2, \quad t)$$

As usual, this function defines the failure (rate) behavior by means of the usual four coefficients (a, b, c, d). Let suppose that this function has first derivative zero in each behavior point. Again, if two derivatives exist then the failure rate surface behavior is the "loci" of points in which the couple of derivatives is nullified. Physically, in each of the seven standard elementary catastrophe surfaces failure rate gaps are identifiable. According to the different failure rate trajectories, it can happen a smooth change always inside not-critical or detected critical areas. Otherwise if the failure rate trajectory goes through a gap, the projection across the “hard critical subset” ($\Omega_k = k$), producing a critical failure.
So, the failure model based upon R.T.C.T. can be combined with the failure rate model based upon the R.T.C.T. in order to have a deeper knowledge of the overall failure behavior. This rediscovering of the deterministic approach should be consistent with the random one. A way to indicate the matching is to suppose that all the deterministic R.T.C.T. approach happens inside a small period Δt before the instant t in which the critical failure happens.

CONCLUSIONS

The conclusions of this research are open to many opportunities of continuing to develop this way of thinking for trying to rationalize better and deeper the proposed model. Since it is a not sponsored study, it should be intended as a tentative research having in mind the purpose of defining the bases for a more detailed analysis. This basic analysis is based on the R.T.R.T. but the effort is focused on the failure model in a very general way. So the applicability is very large and
Reproves, verification and validations are and can be easily and mostly deduced from the open literature in the field of physics of Failure (POF). IN the future a lot of effort should be devoted to theexecution of specific tests and some peculiar experiment.

REFERENCES

l. René Thom: *Structural Stability and Morphogenesis*, W.A. Benjamin Inc. 1975
2. R. Thom and C.C. Zeeman. *Catastrophe Theory: its present state and Future Perspective*, Preceeding of "Dynamic System Warwicg 1974, University of Warwich 1973-74 Springler-Verlog 1975
3. R.M. White and T.M. Geballe: "*Long Range order in solid*", Academic Press Inc. 1979.
4. A. Woodcoock M. Davis, "*Catastrophe Theory*" Garzanti Ed.1982.
5. Yoshiro Kato and Hideyasu "*Some Approach to reliability physics*", IEEE Proceeding on Reliability Vol. R-l7 N1 March. 1968.
6. Roberto Somma, Vito Amoia "*Proprietà e soluzione analitica dell'equazione fondamentale”*, Rivista Tecnica Selenia Vol. 4 N°3 1977.
7. Gianfranco Piacentini, *Affidabilità dei dispositivi per microonde*, Castro Marina- Lecce 22-27 Sept. 1980.
8. Clem Bateman, ”*MHD Instabilities*" The MIT Press Cambridge.
9. J.D. Lawson, ”*The Physics of Charged particles beams*" Claredon Press Oxford.
10. Giorgio Careri, *Ordine e Disordine nella Materia*, Laterza 1982.
11. B. W. Rosen, *Tensile failure of fibrous composites,* G.E. Company – King of Prussia, AIAA Journal, Vol. 2, No. 11, (1964), pp. 1985-1991.
12. George H. Miley, *Study of a power source based on Low Energy Nuclear reactions (LENRs) using Hydrigen Puressurized nanoparticles,*
13. Stefano Lencia, Francesco Clementi, Giuseppe Rega, *Nonlinear free vibrations of Timoshenko beams with mechanical or geometric curvature definition*, 24th International Congress of Theoretical and Applied Mechanics Comparing, Procedia IUTAM 20 (2017) 34 – 41; https://www.sciencedirect.com/

14. Ebrahim Lamkanfi, WimVan Paepegem, Joris Degrieck, *Shape optimization of a cruciform geometry for biaxial testing of polymers,* Polymer Testing Volume 41, February 2015, Pages 7-16; https://doi.org/10.1016/j.polymertesting.2014.09.016
15. Borja Erice, María Jesús Pérez-Martín, Francisco Gálvez, *An experimental and numerical study of ductile failure under quasi-static and impact loadings of Inconel 718 nickel-base superalloy*, International Journal of Impact Engineering, Volume 69, July 2014, Pages 11-24.
16. https://en.wikipedia.org/wiki/Physics_of_failure
17. Charles Kittel, *Elementary Solid State Physics*, Wiley.
18. Douglas S. Billington, James H. Crawford Jr. *Radiation Damage in Solid*, Oxford, 1961.
19. Borja Erice, Francisco Gálvez, *A coupled elastoplastic-damage constitutive model with Lode angle*, International Journal of Solids and Structures 51 (2014) 93–110.
20. Mitul Sisodiya Arghya Das: *Influence of Lode angle-dependent failure criteria on shear localization analysis in sand, International Journal of Numerical and analysis method of Geomechanics*, Wiley online Library, 19 February 2018 https://doi.org/10.1002/nag.2778
21. Mitul Sisodiya Arghya Das: *Influence of Lode angle-dependent failure criteria on shear localization analysis in sand*, International Journal of Numerical and Analysis Method of Geomechanics, Wiley online Library, 19 February 2018 https://doi.org/10.1002/nag.2778
22. Tian Tang n, Sergio D. Felicelli: *Numerical characterization of effective fully coupled thermo-electro-magneto-viscoelastic-plastic response of smart composites,* International Journal of Non-Linear Mechanics, Volume 71, May 2015, Pages 52-62.

ADDITONAL REFERENCES

1. Lubomir Vlcek , Pavol Jozef Safarik, *New Trends in Physics: Extraordinary proofs*, Ed. *International* Journal of Nanotechnology in Medicine & Engineering, November 15, 2016, Lubomir Vlcek, IJNME 2016, 1:2
2. *The New Coordinate Systems in Physics and Magic Numbers*, viXra. http://vixra.org/author/lubomir_vlcek; http://vixra.org/author/lubomir_vlcek
3. Gi-Chul Yang, Sio-Iong Ao, Xu Huang - *Mathematics,* Nanotechnology 20:65307 2. – 2013.
4. Noa Lachman, Haiping XuYue Zhou, Brian L Wardle, *Carbon Nanotube Electrodes for Energy Storage*, Oct 2014Advanced Materials Interfaces
5. Curtin William, *Mechanics of composites*, ME-430
6. Philippe Colomban1, *Understanding the nano- and macromechanical behaviour, the failure and fatigue mechanisms of advanced and natural polymer fibres by Raman/IR microspectrometry Advances in Natural Sciences*: Nanoscience and Nanotechnology, Volume 4, Number 1, , Published 19 December 2012 , © 2013 Vietnam Academy of Science & Technology
7. Roberto Guzman de Villoria, *Multi-physics damage sensing in nano-engineered structural composites*
8. Nanotechnology, 22(18):185502, May 2011, DOI, 10.1088/0957-4484/22/18/185502
9. *Multi-Physics Nano-Engineered Structural Damage Detection and De-Icing*
10. *Multi-physics damage sensing in nano-engineered structural composites*, Nanotechnology 22(18):185502, May 2011.