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A Linear Programming Approach for an Industrial Optimal Production

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ABSTRACT

This paper utilized the concept of sengupta(2016) to allocate raw materials to competing variables(20 litres, 4 litres and 2 litres) in a paint industry for the purpose of profit maximization. The analysis was carried out and the result showed that 2.143 units of 20 litres should be produced to make a profit of #128.58. From the analysis, it was observed that 20 litres contribute objectively to the profit. Hence, more of 20 litres are needed to be produced and sold in order to maximize the profit.

keywords- Basic solution, Degeneracy, Linear Programming, Objective functions, Simplex Method, Slack and Surplus variables, Optimal result.

I. INTRODUCTION

A programming problem is a class of problems that determines the optimal allocation of limited resources to meet given objectives. The resources may be men, material, machine and land. (Iheagwara et al 2014). Linear Programming (LP) can be defined as a mathematical technique for determining the best allocation of a firm’s limited resources to achieve optimal goal. (Yahya et al 2012).Linear Programming (LP) is a mathematical technique to achieve profit maximization or cost minimization in a mathematical model whose requirements are represented by linear relationship. A linear programming problem may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. Linear Programming, an operations research technique is widely used in finding solutions to complex managerial decision problems. (Ezema et al, 2012) In a multi-product company it is often a challenging task to find the appropriate quantities of the product to be produced in order to maximize the profit. It is quite a tough management decision to fix the product quantities under the given constraints of the organization. Linear programming comes to much use in finding the optimal product mix.

II. LITERATURE REVIEW

Under this heading, we shall review the existing literatures which are related to the topic. According to miller (2007), linear programming is a generalization of linear algebra use in modelling so many real life problems ranging from scheduling airline routes to shipping oil from refineries to cities for the purpose of finding inexpensive diet capable of meeting daily requirements. Miller argued that the reason for the great versatility of linear programming is due to the ease at which constraints can be incorporated into the linear programming model. Ezema and Amaken (2012) argue that the problem of industries all over the world is a result of shortage of production inputs which result in low capacity utilization and consequently low outputs. They applied linear programming in optimization of profit in golden plastic industry for the purpose of seeking and arriving at the optimal product-mix of the golden plastic industry. This was done or achieve by formulating the linear programming problem for production of plastic and estimated as such, the company was initially producing eight pipes but from the data analysis and estimation it shows that only two size of the total of eight polyvinel Application of Linear Programming for Optimal Use of Raw Materials in Bakery www.ijmsi.org 52 | Page chloride pipes should be produced and they also succeed in establishing that 7,136.564 pieces of 20mm by 5.4m thick pressure pipe and 114317.2 pieces of 25mm by 5.4m conduct pipe should be produced and zero quantities of the remaining size in order to obtain a monthly profit of N1,964,537. Igweet al (2011) reported that linear programming is a relevant technique in achieving efficiency in production planning, particularly in achieving increased agricultural productivity. They observed this when carrying out investigation on maximization of gross return from semi-commercial agriculture in Ohafia zone in Abia state. The general deterministic model is a gross margin maximization model designed to find out the optimum solutions, the decision variables for the model are numbers of hectares the farmer devoted to the production of crop and combination of crop or livestock capacity produced by the farmer. Balogun et al (2012) reported that, the problem in production sectors is the problem of management, that many companies are faced with decision relating to the use of limited resources such as manpower, raw materials, capital etc. In their work titled "use of linear programming for optimal production" in Coca-Cola Company, they were able to applied linear programming in obtaining the optimal production process for Coca-Cola Company. In the course of formulating a linear programming model for the production process, they identified the decision variables to be the following Coke, Fanta, Schweppes, Fanta tonic, Krest soda etc. which some up to nine decision variables and the constraint were identified to be concentration of the drinks, sugar content, water volume and carbon (iv) oxide. The resulting model was solve using the simplex algorithm, after the data analysis they came to a conclusion that out of the nine product the company was producing only two contribute most to their profit maximization, that is Fanta orange 50cl and Coke 50cl with a specified quantity of 462,547 and 415,593 in order to obtain a maximum profit of N263,497,283. They advise the company to concentrate in the production of the two products in order not to run into high cost. Snezanza and Milorad (2009) recognize linear programming as an important tool in energy management despite the non-linearity property of many energy system, they argue that the non-linearity property can be converted to a linear form by applying Taylor series expansion so that the optimization method can be applied to determine the best means of generating energy at a minimum cost. VeliUlucan(2010) reported that a mixed integer linear programming plays an important role in aggregate production planning (i.e a macro production planning) which addresses the problem of deciding how many employees the firm should retain and for manufacturing company, the quantity and mix of products to be produced. Veili argue that the decision variables for an integer programming are required to be an integer in order in order to satisfy both the objective function and the constraints. Fagoyinbo and Ajibode (2010) reported that the success and failure that an individual or organization experience towards business planning depends to a large extent on the ability of making appropriate decision. They argue that a manager cannot make decision based on his/her personal experience, guesswork or intuition because the consequences of wrong decision is very costly, hence an understanding of the applicability of quantitative method to decision making is of fundamental importance to the decision maker. They described linear programming as one of the major quantitative approach to decision-making and hence applied it in effective use of resources for staff training, the decision variables for the model are the junior staff and senior staff and the constraints was the time available for training as the program is in-service training. According to Majeke (2013) commercial farmers are always confronted with the problem of finding the combination of enterprises that will provide them with the highest amount of income through the best use of farm limited resources (constraints), he recognized the over-growing application of linear programming in agricultural sector, particularly in optimization of available farm resources in order to attain an optimal income (profit). He formulated a linear programming model that maximizes the income of farmers in rural area, the decision variables for the model was identified to be hectares allocated for maize production stored for family consumptions, hectares allocated for soya bean production and hectares allocated for tobacco production (i.e five decision variables) and also, six constraints were identified. The resulting model was solved using a computer software (MS excel). Joly (2012) reported that, optimization is a crucial science for high-performance refineries , its main purpose in the oil sector is to push production process or operation towards the maximal profit until it reaches the limit at which any further profitability increase depends on changes in the existing system. Stephanos and Dimitrios (2010), see linear programming as a great revolutionary development which has given mankind the ability to state general goals and to lay out path of detailed decision to take in order to "best" achieve its goals when faced with practical problem of great complexity. They argue that a simple linear programming begins with determination of interrelationship of an objective function as the maximization of profit for one or more products (activities). Nabasirye et al (2011) argue that a linear programming problem is formed when the feasible region is subset of the non-negative portion of Rn , defined by linear equations and inequalities, and the objective function to be minimized or maximized is linear. They also argue that selecting the best alternative out of a large number of Application of Linear Programming for Optimal Use of Raw Materials in Bakery www.ijmsi.org 53 | Page possibilities is called optimization. They successively applied linear programming in minimization of cost of animal feed since animal feed was identified as a major factor in the overall cost of animal production in order to maximize an optimal profits. According Mula et al (2005) production planning problem is one of the most important application of optimization tools using mathematical programming (linear programming). They argue that the idea of incorporating uncertainty in mathematical models is very important in order not to generate inferior planning decisions. This is known as sensitive analysis. According to Waheed et al (2012) linear programming models are frequently used in operation research and management sciences to solve specific problems concerning the use of scare resources. They demonstrated the application of linear programming in profit maximization in a product-mix company, in selecting the best means for selling her medicated soap product which include 1 tablet per pack, 3 tablets per pack, 12 tablets per pack and 120 tablets per pack, which are subject to some constraints. The data analysis was carried out with Rstatistical package, the result of the analysis showed that the company would obtain optimal monthly profitlevel of about N271,296 if she concentrates mainly on the unit sales (one tablet per pack)of her medicated soap product ignoring other types of sales packages. Lenka (2013) argue that global economic crisis makes the business environment unfavorable for industries to survive or manage their resources optimally. Lenka formulated two linear programming models where one of them maximizes the revenue of a company and the other minimizes the cost of operation respectively. Igbinehi et al (2015) applied linear programming model to maximize profit in a local soap production company, the company produces three different type of soap, 5g white soap, 10g white soap and 10g coloredsoap. From the data analysis it was observed that the company spends more on colored soap and they gets more profit from white soap than colored soap. So the company was advised to produce more of white soap (5g and 10g) than the colored soap in order to obtain an optimal profit. Maryam et al (2013) reported that, linear programming plays an important role in improving management decision despite it is still regarded as new science but has proven to be capable in solving problems such as production planning, allocation of resources, inventory control and advertisement. Taha (2003) argue that the idea of differential calculus is required for optimization of objective function subject to constrained continuous functions. For Taha, Langrangean method is the most appropriate method for solving optimization problem with equality constrained continuous functions, if the constraints are continuous and nonlinear then the appropriate method is the Karush-Kuhn-Tucker method for system of non-leaner programming. This is called the classical optimization.

III. LINEAR PROGRAMMING

The general linear programming model with n decision variables and m constraints can be state in the following form.

Optimize(max or min) $z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$

s.t

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_1 \,(\leq, =, \geq)\, b_1$

$a_{12}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_1 \,(\leq, =, \geq)\, b_2$

. . .

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$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_1 \,(\leq, =, \geq)\, b_1$

$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_1 \,(\leq, =, \geq)\, b_m$

Standard form of Linear Programming Model

The use of the simplex method as proposed sengupta to solve a linear programming problem requires that the problem be converted into its standard form. The standard form of a linear programming problem has the following properties.

i. All the constraints should be expressed as equations by adding slack or surplus variables.
ii. The right - hand side of each constraint should be made of non - negative(if not). This is done by multiplying both sides of the resulting constraints by -1.
iii. the objective function should be of a maximization type.

For n decision variables and m constraints, the standard form of the linear programming model can be formulated as follows.

Optimize(max or min) $z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$

s.t

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_1 \,(\leq, =, \geq)\, b_1$

$a_{12}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_1 \,(\leq, =, \geq)\, b_2$

. . .

. . .

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_{1\,(}\leq, =, \geq_{)}b_1$

$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_{1\,(}\leq, =, \geq_{)}b_m$

Assumptions

1. It assumed that the raw materials required for production paint are scarce.
2. It is assumed that an effective allocation of raw materials to the variables(20litres, 4litres and 2litres) will aid optimal production mix and at the same time maximizing the profit of the paint industry.
3. It is assumed that the qualities of raw materials used in paint production are standard(not inferior)

Data Presentation and Analysis

The data for this research project was collected from Miller Augustine Paint Limited, Nigeria. The data consist of total amount of raw materials (calcium carbonate, water, titan colour, preservatives, ammonia, defoamer and acrylic) available for daily production of three different sizes of emulsion paints (20litres, 4litres and 2litres) and profit maximization per unit size of emulsion paint produced. the content of each raw material per each product of emulsion paint produced is shown below.

Calcium Carbonate

Total amount of calcium carbonate available = 30kg

Each unit of 20litres requires 14kg of calcium carbonate

Each unit of 4litres requires 2.8kg of calcium carbonate

Each unit of 2litres requires 1.4kg of calcium carbonate

Water

Total amount of water available = 25kg

Each unit of 20litres requires 9kg of water

Each unit of 4litres requires 1.8kg of water

Each unit of 2litres requires 0.9kg of water

Titan Colour

Total amount of titan colour available = 35kg

Each unit of 20litres requires 0.15kg of titan colour

Each unit of 4litres requires 0.03kg of titan colour

Each unit of 2litres requires 0.015kg of titan colour

Preservatives

Total amount of preservatives available = 40kg

Each unit of 20litres requires 0.1kg of preservatives

Each unit of 4litres requires 0.02kg of preservatives

Each unit of 2litres requires 0.01kg of preservatives

Ammonia

Total amount of ammonia available = 20kg

Each unit of 20litres requires 0.05kg of ammonia

Each unit of 4litres requires 0.01kg of ammonia

Each unit of 2litres requires 0.005kg of ammonia

Defoamer

Total amount of defoamer available = 10kg

Each unit of 20litres requires 0.1kg of defoamer

Each unit of 4litres requires 0.02kg of defoamer

Each unit of 2litres requires 0.01kg of defoamer

Acrylic

Total amount of acrylic available = 10kg

Each unit of 20litres requires 0.1kg of acrylic

Each unit of 4litres requires 0.02kg of acrylic

Each unit of 2litres requires 0.01kg of acrylic

Profit contribution per unit product(size) of emulsion paint produced

Each unit of 20litres of emulsion paint = #60

Each unit of 4litres of emulsion paint = #25

Each unit of 2litres of emulsion paint = #40

The above data can be summarized in a tabular form.

Raw materials	Product			Total available raw materials
	20litres	4litres	2litres	
Calcium carbonate(kg)	14	2.8	1.4	30
Water(kg)	9	1.8	0.9	25
Titan colour(kg)	0.15	0.03	0.015	35
Preservatives(kg)	0.1	0.02	0.01	40
Ammonia(kg)	0.05	0.01	0.005	20
Defoamer(kg)	0.1	0.02	0.01	10
Acrylic(kg)	1	0.2	0.1	15
Profit(#)	60	25	40	

Model Formulation

Let the quantity of 20litres to be produced = X_1

Let the quantity of 4litres to be produced = X_2

Let the quantity of 2litres to be produced = X_3

Let Z denote the profit to be maximize.

The linear programming model for the above production data is given by

Max $Z = 60X_1 + 25X_2 + 40X_3$

subject to;

$14X_1 + 2.8X_2 + 1.4X_3 \leq 30$

$9X_1 + 1.8X_2 + 0.9X_3 \leq 25$

$0.15X_1 + 0.03X_2 + 0.015X_3 \leq 35$

$0.1X_1 + 0.02X_2 + 0.01X_3 \leq 40$

$0.05X_1 + 0.01X_2 + 0.005X_3 \leq 20$

$0.1X_1 + 0.02X_2 + 0.01X_3 \leq 10$

$X_1 + 0.2X_2 + 0.1X_3 \leq 15$

$X_1, X_2, X_3 \geq 0$

Converting the model into its corresponding standard form;

Max $Z = 60X_1 + 25X_2 + 40X_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + 0s_7$

subject to;

$14X_1 + 2.8X_2 + 1.4X_3 + 0s_1 = 30$

$9X_1 + 1.8X_2 + 0.9X_3 + 0s_2 = 25$

$0.15X_1 + 0.03X_2 + 0.015X_3 + 0s_3 = 35$

$0.1X_1 + 0.02X_2 + 0.01X_3 + 0s_4 = 40$

$0.05X_1 + 0.01X_2 + 0.005X_3 + 0s_5 = 20$

$0.1X_1 + 0.02X_2 + 0.01X_3 + 0s_6 = 10$

$X_1 + 0.2X_2 + 0.1X_3 + 0s_7 = 15$

$X_1, X_2, X_3, 0s_1, 0s_2, 0s_3, 0s_4, 0s_5, 0s_6, 0s_7 \geq 0$

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Basic variable	X1	X2	X3	S1	S2	S3	S4	S5	S6	S7	Z
Z	-60	-25	-10	0	0	0	0	0	0	0	1
S1	**14**	2.8	1.4	1	0	0	0	0	0	0	0
S2	9	1.8	0.9	0	1	0	0	0	0	0	0
S3	0.15	0.03	0.015	0	0	1	0	0	0	0	0
S4	0.1	0.02	0.01	0	0	0	1	0	0	0	0
S5	0.05	0.01	0.005	0	0	0	0	1	0	0	0
S6	0.1	0.02	0.01	0	0	0	0	0	1	0	0
S7	1	0.2	0.1	0	0	0	0	0	0	1	0

Iteration One

Basic variable	X 1	X2	X3	S1	S2	S3	S4	S5	S6	S7	Z	Value
Z	0	9	5	4.26	0	0	0	0	0	0	0	128.58
X1	1	0.2	0.1	0.071	0	0	0	0	0	0	0	2.143
S2	0	-1.44	-0.81	-0.639	1	0	0	0	0	0	0	34.288
S3	0	-0.024	-0.135	-0.01065	0	1	0	0	0	0	0	74.6835
S4	0	-0.016	-0.009	-0.0071	0	0	1	0	0	0	0	85.5057
S5	0	-0.008	-0.0045	-0.00355	0	0	0	1	0	0	0	42.7528
S6	0	-0.016	-0.009	-0.0071	0	0	0	0	1	0	0	21.2157
S7	0	-0.16	-0.09	-0.071	0	0	0	0	0	1	0	30.002

Interpretation of Result

Based on the data collected, the optimum result derived from the model indicates that one size of emulsion paint should be produced; 20litres of emulsion paint. The production quantity should be 2.134units. This will produce a maximum profit of #128.58.

IV. SUMMARY

The objective of this research work was to apply sengupta method for optimal product mix in profit maximization in emulsion paint industry. Miller Augustine Paint limited was used as my case study. The decision variables in this research work are the three different sizes of paint (20litres, 4litres and 2litres) produced by Miller Augustine Paint Limited. The researcher focused mainly on seven raw materials (calcium carbonate, water, titan colour, preservatives, ammonia, defoamer and acrylic) used in the production and the amount of raw material required of each variable (paint size). The result shows that 2.143 unit of 20litres of paint, 0unit of 4litres of paint and 0unit of 2litres of paint should be produced which will give a maximum profit of #128.58.

V. CONCLUSION

Based on the analysis carried out in this research work and the result shown, Miller Augustine Paint Limited should produce the three litres of paint(20litres, 4litres and 2litres) in order to satisfy his customers. Also, more of 20litres of paint should be produced in order to attain maximum profit because they contribute mostly to the profit earned by the company.

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