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A NEW MATH FOUNDATION

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ABSTRACT

Most students do not achieve in math the same academic results they usually do in respect of other disciplines. Faced at an early school stage, this learning difficulty propagates throughout their entire lives. Math then becomes the least appreciated discipline in almost every school, and people often declare to hate math. A couple of reasons have been listed in a trial to explain such undesirable situation, but except for some few and timid criticisms, nobody clearly states that the reasons may be within math itself. People interested in math may find books and videos on the Internet always teaching the same things in the same way, but I did not see anyone with a different approach, offering a new thesis or at least trying to explain the many strange results yielded by the current math structure. That is the target of this paper. I challenge the current math structure by presenting a new theorem, a new fundamental axiom of algebra, a demonstration of Fermat's Conjecture, and a new math foundation I developed, better aligned with the real world.

MATH ODDITIES AND INCONSISTENCIES

Math teachers have difficulties to transmit their knowledge to the students, and the students feel frustrated and blame math, which is the most under-appreciated discipline in any curriculum framework. Since the students are not able to acquire a solid basis at an early stage of their school lives, this unpleasant situation propagates throughout their academic activities. In addition to the poor learning results, there happens a declared rejection towards math.

As any other area of studies of human knowledge, math relies on postulates, premises, axioms and theories accepted as valid statements, what may not be the case. Undoubtedly, this field of work coexists with inconsistencies and poorly explained matters, indeterminations, unproven conjectures and unsolved problems, what does not make sense in a field of work considered as an “exact science”.

As a very basic question, we may speculate about the nature of numbers:

(a) Current math understands that numbers have signs, but the use of positive and negative numbers are subject to different rules, what implies the acceptance that “$\sqrt{(-1)} = i$”, and of a whole theory about complex numbers that significantly complicates math operations;

(b) Nevertheless, it is possible to say that, in spite of having signs, negative numbers could be subject to the same rules applicable to positive numbers, and approach that, at least, would not need the theory of complex numbers. Accordingly, if $\sqrt{(+4)} = +2$, then $\sqrt{(-4)} = -2$. Although free of the imaginary numbers, this understanding would also complicate math operations;

(c) Finally, we may say that numbers are neither positive nor negative, but neutral elements, a concept which does not require the theory of complex numbers[1], and would make math operations much simpler. This is the approach I will adopt and discuss in this paper.

Numbers are neither positive nor negative math elements.

The theory of complex numbers aims to solve a non-existing problem: to find the roots of a math expression erroneously understood as a cubic equation, and which by another mistaken concept may have up to three roots. Considering the fundamentals of the Cartesian method, it becomes clear that said “cubic equation”, as any other Cartesian representation of geometric figures, is not an equation and has no roots whatsoever.

If we are not able to find a value for the variable “x” when the variable “y” is equal to zero in the math expression of a parabola “$y = 3x^2 - 5x + 6$”, it only means that said parabola, in that specific position in the Cartesian graph, does not touch the axis of the abscissas, as seen in Figure 01. It is not necessary to invent imaginary roots.

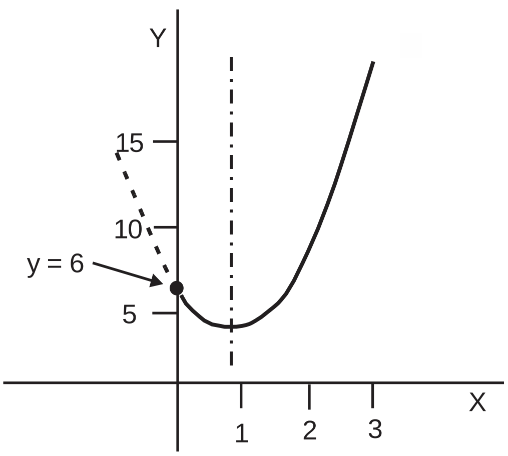


Figure 01: Parabola

[1] http://www.globalscientificjournal.com/researchpaper/THE_MATHEMATICS_OF_NUMERICAL_EXPRESSIONS.pdf

The theory of complex numbers is a huge building constructed over poor quality piles, and remains standing exactly because it is imaginary. Thus:

We may disregard the theory of complex numbers, as well as all further developments relating to the same subject.

The above statement allows me to question the meaning and application of some traditional math truths, as the Fundamental Theorem of Algebra ("complex roots") and the famous Euler's Identity ("$e^{i\pi} + 1 = 0$").

Another difficulty refers to the fact that math is not capable to represent in a sound way the continuity existing in nature, either in geometry or in other fields of work. The concept of real numbers, which encompasses rational and irrational numbers, is not a satisfactory approach, since it is not possible to express and handle a figure with an infinite number of decimals. It means that math equalities are not always true statements, and math results are exact values only when input and output figures are whole numbers.

This is a serious limitation, perhaps the most important and fundamental math problem, which allows us to state that:

Contrarily to the popular belief, math is not a perfect and exact science. In fact, math is the science of approximate results.

The use of the number zero as a common number takes us to some uncomfortable situations, as in the cases we try to divide any number by zero, or find the limit between two math expressions in relation to a common point belonging to their Cartesian representations that rests on the axis of the abscissas.

In this paper, I support the idea that math fundamentals currently in use are inadequate and need improvements. Therefore, all further developments built in accordance with inconsistent premises and concepts often originate mistaken results. That is the reason why I suggest a new math foundation.

However, before introducing that new math foundation and the resulting effects on math applications, it is convenient do review some preliminary subjects. It is also necessary to clarify that, for simplicity, in this paper I will use the wording "math expression" as a general term to refer to any math proposition, as equations, formulas and Cartesian representations of geometric figures.

NEW THEOREM

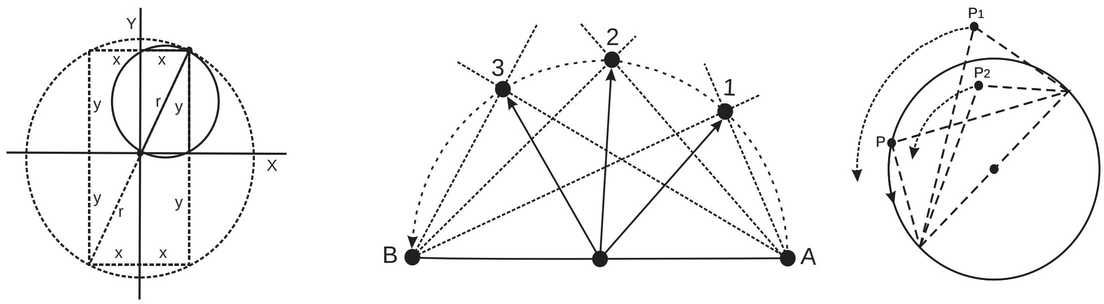
If the Theorem of Pythagoras is not the oldest one, for sure it is the most famous theorem of math. There are references that this theorem is as old as the ancient Mesopotamian civilizations. Its millennial concept informs that the Theorem of Pythagoras is a property of the right triangle under the true quadratic statement that "In any right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse".

After reviewing this famous theorem, I concluded and proved that it is a property of the circumference, and has a much broader meaning.

Figure 02(a) allows us to conclude that mathematically speaking circumference and right triangle are inseparable geometric figures, since they obey the same quadratic relationship, "$x^2 + y^2 = r^2$". Mathematically speaking, circumference and right triangle are a same thing.

Figure 02(b) illustrates that all the triangles formed with points "1", "2" and "3", or with any other of the infinite points belonging to the circumference are right triangles with a circumference diameter (same hypotenuse). We may easily demonstrate that statement by considering that the angle formed by the two chords is always 90 degrees (they encompass a semi-circle arc, which is equal to 180 degrees).

Figure 02(c) confirms that only points as "P", located in the circumference contour line, meet the quadratic relationship and form a right triangle with any diameter. When connected to that diameter, points outside the circumference (as "P_1") form acute triangles, while points inside the circumference (as "P_2") form obtuse triangles. We may then state that all right triangles of hypotenuse "h" are inscribed in a circumference of diameter "d = h".



(a) (b) (c)

Figure 02: Quadratic relationship and new theorem

This new understanding of the quadratic relationship allowed me to suggest a new theorem, the Theorem of Infinite Right Triangles, as a generalization of the Theorem of Pythagoras.

> *There is an infinite number of right triangles with a hypotenuse "h" inscribed in a circumference of diameter "d = h" and, in all of them, the legs "a" and "b" may freely vary, provided the sum of their squares remains constant and equal to "$h^2 = d^2$".*

Or, in other words,

> *Any straight line segment of length "l" is the hypotenuse "h = l" of an infinite number of right triangles and, in all of them, the legs "a" and "b" may freely vary, provided the sum of their squares is constant and equal to "$h^2 = l^2$".*

Then, we may mathematically express the Theorem of Infinite Right Triangles, a property of the circumference, as follows:

$(a^2 + b^2) = \text{constant}$.

It is worthwhile to point out that we use the Theorem of Pythagoras and the Theorem of Infinite Right Triangles in connection with whole and fractional numbers. However, mathematically speaking only the Pythagorean right triangles are true right triangles, as we see in Figure 03(a). All others, as shown in Figure 03(b), are approximations.

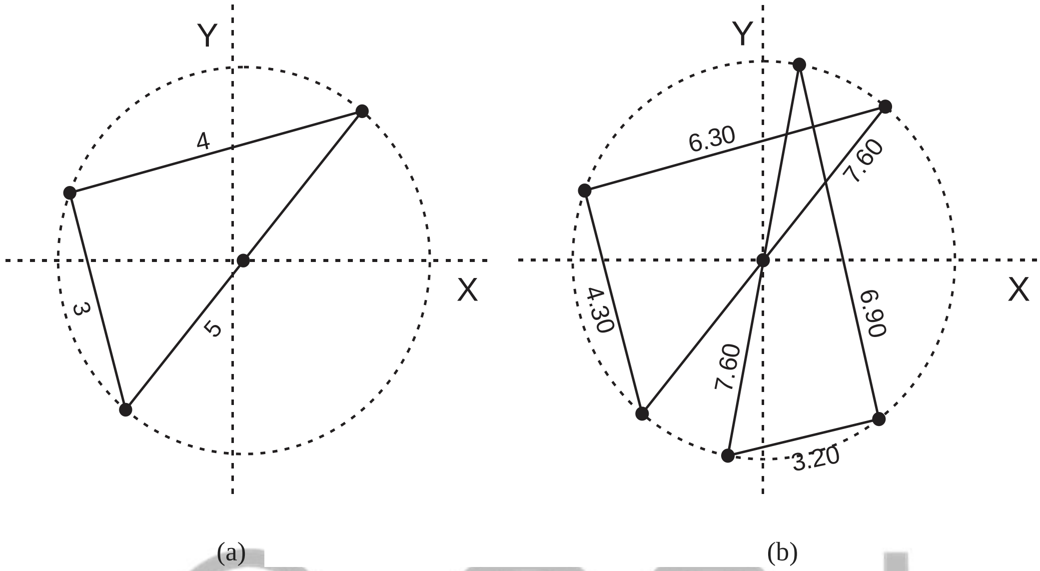


Figure 03: Pythagorean and non-Pythagorean right triangles

MATH AT THE SERVICE OF OTHER FIELDS OF WORK

As a language at the service of other fields of human activities, math uses the same framework under different rules and for distinct purposes, but the employment of a same foundation (which by itself coexists with inconsistencies) under different rules and distinct purposes originates some conceptual confusion.

In some cases, we deal with formulas and a consistent unit system to define the behavior of natural phenomena and events, as we do in physics. As an example, as established by the English physicist and mathematician Isaac Newton, the law of universal gravitation says "the attraction force "f_G" between two masses "m_1" and "m_2" is directly proportional to the product of the masses and indirectly proportional to the square of the distance "d" between the centers of the masses". Then, as a math expression, "$f_G = [k(m_1)(m_2)]/d^2$".

Math serves geometry with formulas to quantify the properties of the geometric figures (length, areas and volumes). The volume of a sphere, expressed as a function of its radius "r", is given by "$V = (4/3)\pi r^3$". Defined the sphere radius, its volume will be the same, no matter if the sphere is in my mind, as a drawing in a piece of paper or as a real soccer ball over a table. The sphere volume is a geometric property, and its value is solely dependent on the sphere radius.

Math also serves geometry with the help of a Cartesian system of axes to represent geometric figures in the math language. A different application. The representation of a sphere in a Cartesian system is given by the math expression "$x^2 + y^2 + z^2 = r^2$", and contrarily to a geometric property of the sphere, the math expression will be different if the sphere is placed in a different position in the Cartesian graph.

Math is a mean to give an answer to a pre-formulated question when it builds and solves certain math expressions of a single unknown value, defined as "equations", in which said unknown value is the answer to the question. An equation occurs when we impose an equality to two algebraic sums of a same variable, as:

$$\pm a_1x \pm b_1 = \pm a_2x \pm b_2$$

In Figure 04, we see what math understands as a system of two equations:

$$\begin{cases} y = a_1x + b_1 \\ y = a_2x^2 + b_2x + c_2 \end{cases}$$

These expressions are not equations and do not form a system of equations. They are the Cartesian representations of a straight line and a parabola. The two figures may have common points or not. The straight line may intercept the parabola in two ways: in two points "A_1" and "A_2", or be tangent to it in a single point. The two figures may also not touch each other. Any common point that may occur will depend on the positions of the two figures in the Cartesian system. Except in case of a previous assumption, any common point, as well as the points where the two figures may intercept the axis of the abscissas, mean nothing.

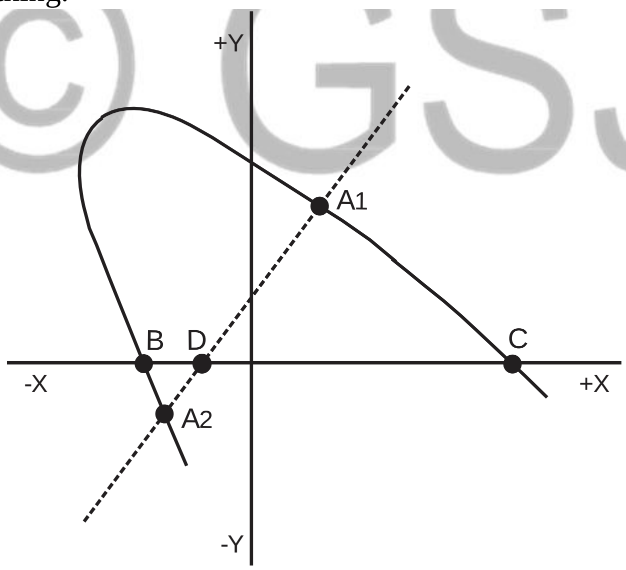


Figure 04: Intersection of a parabola by a straight line

Finally, at a higher level, as in research, the math segment called "pure mathematics" takes care of abstract and imaginary matters, which, in many cases, do not aim at any immediate practical application.

Each one of these different applications of math relies on its own fundamentals not clearly explained to the students. They are not aware that formulas, equations and Cartesian representations of geometric figures are three different uses of math. Just to illustrate the point, Cartesian representations of geometric figures do not create equations nor systems of equations. A polynomial made equal to zero is not an equation[2]. There is no equation with imaginary or strange roots. In brief:

> *Algebra does not have the widespread field of work and applications as currently accepted by math.*
>
> *When at the service of other area of human knowledge, as geometry, math must comply with the laws ruling the object of studies of such other area.*

Current math courses treat geometry as different specialties of math, as algebraic geometry, differential geometry, projective geometry, and the like. We have a unique geometry, an independent area of human knowledge with a proper object of studies (the geometric figures), which uses math as any other science does.

TYPICAL AND MODIFIED GEOMETRIC FIGURES

I developed a new approach in full disagreement with the traditional understanding on how to interpret and treat math expressions in what concerns the concept and meaning of numbers and letters that represent constants and variables of Cartesian representations. There are two aspects to consider.

As the first aspect of said approach, the number of variables in any math expression only indicates if we are dealing with a flat geometric figure (two variables, “x” and “y”) or a spatial geometric figure (three variables, “x”, “y” and “z”). Obviously, the Cartesian system does not accept more than three variables. The exponents of the variables inform if we deal with a typical geometric figure (follows a geometric law and a math expression) or with a modified geometric figure (only follows a math expression).

We see below some examples of math expressions of typical open and closed geometric figures, in their simplest Cartesian versions:

Straight line: $y = ax + b$

Parabola: $y = ax^2 + b$

Circumference: $x^2 + y^2 = r^2$

Sphere: $x^2 + y^2 + z^2 = r^2$

I will show that modified geometric figures, either open or closed, will occur if we alter the exponents of the variables in the math expression of a typical geometric figure.

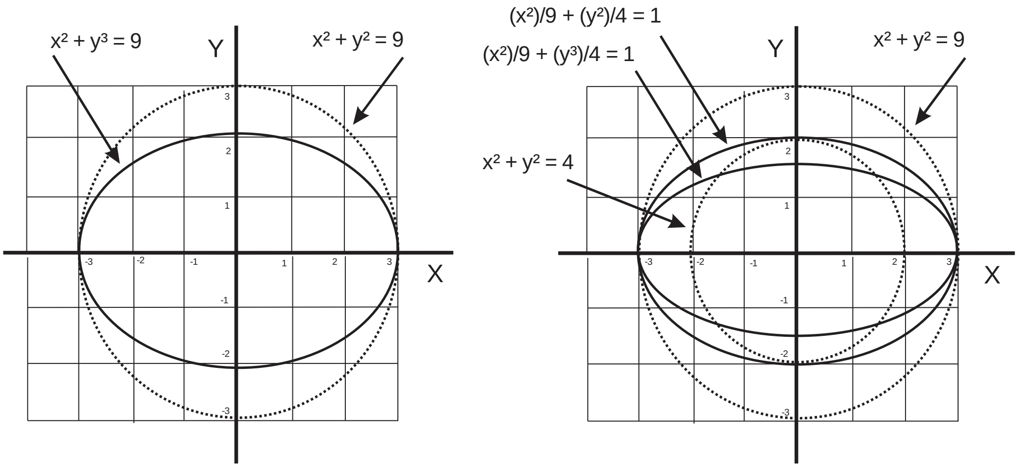
As explained above, in a math expression, the number of variables tells us the “geometric order” of the geometric figure considered, while the exponents of those variables give us a different meaning of the “algebraic degree”. It is also important to mention that the modification of geometric figures we refer in this paper is a kind of modification that affects the geometric properties of the figure, not a single

[2] http://www.globalscientificjournal.com/researchpaper/THE_MATHEMATICS_OF_NUMERICAL_EXPRESSIONS.pdf

displacement in relation to the Cartesian system of axes, nor a different view due to a perspective angle.

The illustration in Figure 05(a) shows a circumference, a typical geometric figure, which follows a geometric law, as well as a math expression ($x^2 + y^2 = r^2$). The same illustration also shows a true oval-shaped form, which follows an altered math expression ($x^2 + y^3 = r^2$), but not a geometric law, it being a modified geometric figure, which results from the alteration of the math expression representing the circumference.

We see an internal oval-shaped form in Figure 05(a) because we increased the exponent of one of the variables and made it greater than "2" (equal to "3"). We would obtain external oval-shaped forms in case we decrease said exponent and make it less than "2". The example shows whole numbers, but the effects would be the same with fractional figures.



(a) (b)

Figure 05: Typical and modified geometric figures

A similar effect would occur if we alter the exponents of the math expression of an ellipse (also a typical geometric figure that follows a geometric law and a math expression), as we see in Figure 05(b). However, the oval-shaped forms generated by altering the math expressions of a circumference or of an ellipse would not be true ellipses, because they do not obey any geometric law, but only follows an altered math expression.

In order to illustrate and emphasize the difference between typical and modified geometric figures, we may state:

> *There are three types of oval-shaped forms: "ellipse" (follows a geometric law and a math expression); "true oval-shaped form" (follows a math expression, only); and "ovoid" (follows neither a geometric law nor a math expression).*

Under the suggested approach, the exponents of the variables in the math expressions of typical geometric figures are equal to "1" and/or "2" (simple application of the Theorem of Pythagoras). The exponents of the variables relating to modified geometric figures may take any value, either integers or fractional figures. As an example, I will show that the so-called “elliptical curve” ($y^2 = x^3 - x + 1$) is a “modified parabola”.

I will not discuss trigonometric functions, cycloids, spirals, catenaries and other flat or three-dimensional geometric figures of few or no interest at all for the purposes of this text.

POLYNOMIALS: FAMILLY OF FLAT AND OPEN CURVES

There is a third type of geometric figures represented by a general polynomial, not necessarily classified as typical or modified geometric figures, which encompasses a family of flat and open geometric figures, as we see in Figure 06. The above-referred elliptical curve, as well as straight lines, parabolas and many others, are components of this family of geometric curves.

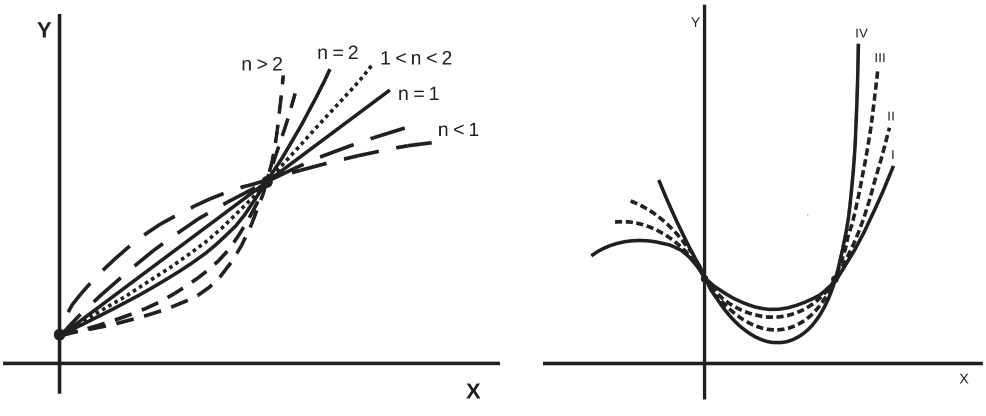
We may express this general polynomial as follows:

$$y^m = \pm a_n x^n \pm a_{n-1} x^{n-1} \pm a_{n-2} x^{n-2} \pm \ldots \pm a_1 x \pm a$$

As examples of families of modified geometric figures, Figure 06(a) shows what may occur when we alter the “x” exponent in the math expression of the straight line ($y = ax^n + b$), and take “$n < 1$” and “$n > 1$”, having the parabola ($n = 2$) and polynomials ($n > 2$) as particular cases.

Figure 06(b) exhibits the well-known math expression of a parabola, “$y = x^2 - x + 1$” (full line, curve I), and when the exponent of “y” increases from “1” to “2”, and the exponent of “x” increases from “2” to “3”, we will reach the famous math expression of the above-referred elliptical curve, “$y^2 = x^3 - x + 1$” (full line, curve IV). Figure 06(b) also shows intermediate curves in between the parabola and the elliptical curve.

The elliptical curve is a modified parabola.



(a) (b)

Figure 06: Family of flat and open curves

The second aspect of the previously referred approach, about variables and their exponents, has a broader effect, which I will discuss below under the proposal of a new “Fundamental Axiom of Algebra”.

FUNDAMENTAL AXIOM OF ALGEBRA

Math communicates by means of math expressions formed with numbers and letters, the latter representing constants and variables. These numbers and letters form the terms of the math expressions under the command of the six arithmetic operations (addition, subtraction, multiplication, division, raising power and root extraction). We complete a math expression by connecting its terms with the help of additions (+), subtractions (-) and equalities (=). Then we have formulas for geometry and sciences, equations and Cartesian representations of geometric figures, different applications of the math language. It is unnecessary to say that constants and variables are also numbers not specified at the beginning.

In any math expression, numbers, constants and variables represent material or immaterial things of nature, as books, monetary values and the like, which are not positive nor negative. Therefore, they do not have signs. It is intuitive that the six arithmetic operations that form the terms of any math expressions deal with numbers and letters taken as absolute values ("modules").

There are two reasons to recognize the incoherence of accepting that numbers and letters in a math expression may be positive or negative figures:

1st) as previously said, they represent things of nature, which are neutral elements;

2nd) a math expression imposes additions (+) and subtractions (-) to its terms, and there is no justification whatsoever to apply a double command of signs to these terms.

Whatever the elected field of application, when using math we end up with algebraic expressions commanded by positive (additions), negative (subtractions) and equality signs. However, in any specific application, as in an equation or a formula, they may refer to things of different meanings or nature, as credits and debts, and in a Cartesian representation they must comply with the rules of an arbitrary representation method, it being the reason why numbers and letters may be used or appear with signs.

This reasoning take us to the most relevant concept that, in my view, I can suggest in this paper: a new "Fundamental Axiom of Algebra", which states that:

> *Numbers, constants and variables which form the terms of math expressions must be understood as modules (the arithmetic operations will be performed with numbers and letters without signs), since they represent things of the real world, which are neither positive nor negative. Consequently, the terms then formed are also modules. The result of any math expression is determined by the additions and subtractions imposed to its terms.*

The meaning, extension and effects of this new Fundamental Axiom of Algebra is much broader than we can initially think. It means that numbers do not have signs, and complex numbers do not exist. It also says that, in determining the terms of any math expression, we will perform arithmetic operations, as multiplication, division, raising power and root extraction, with numbers and letters without signs. Additions and subtractions, as imposed to the terms of a math expression, will be the only operations dealing with signs.

Accordingly, a polynomial, which is an algebraic sum, should read as follows:

$$y = \pm |a_n||x|^{|n|} \pm |a_{n-1}||x|^{|n-1|} \pm \ldots \pm |a_1||x| \pm |a|$$

Once we know the values of the coefficients, the variable “x” (often referred to as the independent variable) and the exponents of that independent variable, as well as the applicable addition and subtraction signs imposed to the polynomial, we may obtain the value of the variable “y” (then referred to as the dependent variable). The value of “y” may be positive, negative or zero, because it is the difference between the sums of two sets of modules, one taken as positive (as credits), the other taken as negative (as debts) as per a previous assumption.

In the math expression “$y = + 2x^3 – 3x^2 – 1$”, if we make “$x = 0$”, “$x = 2$” and, roughly, “$x = 1.67$” (all “x” values taken as modules), we would find “$y = – 1$”, “$y = + 3$” and “$y = 0$”, respectively. As we see, “y”, as a balance between two different sets of modules (one taken as positive “+”, other taken as negative “– “), may be positive, negative or null, but in no case “x” could be less than zero (a negative value). Considering that the coefficients “2” and “3”, the number “1”, the exponents “2” and “3”, and the independent variable “x” are all modules, the terms “$2x^3$” and “$3x^2$” are modules, too. A negative value for “x” simply does not make sense.

As a side remark, in the given math expression “$y = + 2x^3 – 3x^2 – 1$, whenever numbers with infinite decimals are involved, either as “x” or as “y”, we only find approximate results.

Under the current math structure, the approach to accept negative values for “x” is further complicated because we treat positive and negative numbers in different ways, and due to the improper use of the so-called “rule of signs” discussed below:

(i) We treat even roots differently from odd roots; and

(ii) roots of negative numbers may imply complex numbers.

Under the structure now suggested, these two difficulties disappear.

As we see in Figure 07, under the understanding of math expressions of geometric figures in the Cartesian system here discussed, we only need the first Cartesian quadrant to graphically and mathematically represent symmetrical curves, as in Figure 07(a), and the first and the fourth quadrants to represent a polynomial curve (an algebraic sum), as in Figure 07(b). If wanted, it is possible to represent the entire curve with the use of the four quadrants, as in the case of the lemniscate, as images.

The use of positive and negative coordinates in connection with the Cartesian representation of geometric figures is nothing else but a convention, a requirement of the Cartesian method, and depends on the position we place the geometric figure on the graph.

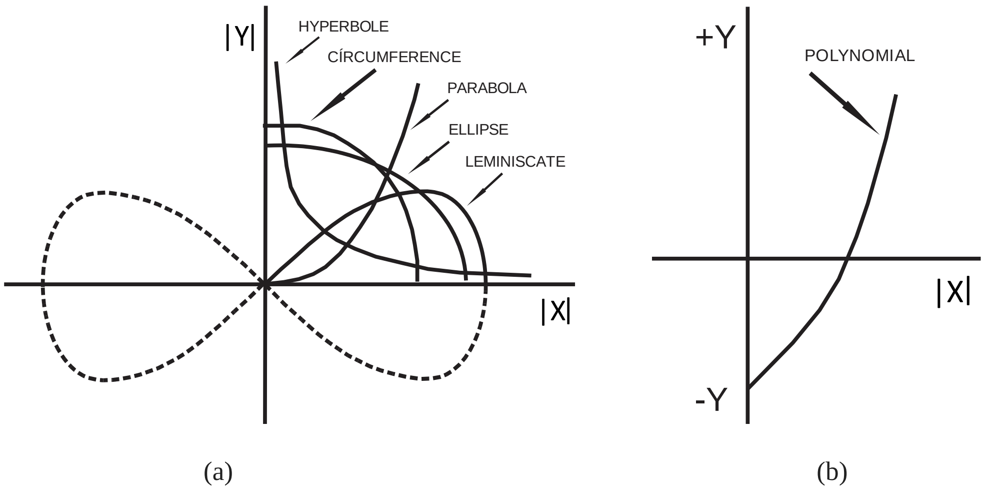


Figure 07: Symmetrical and polynomial curves

Consider the Cartesian representations of a circumference when placed in two different positions, as in Figure 08.

When seen in Figure 08(a), centralized in relation to the Cartesian system, the math expression is:

$$x^2 + y^2 = r^2$$

In this position, to represent the circumference in the graph, we need coordinates with signs (as point “P”).

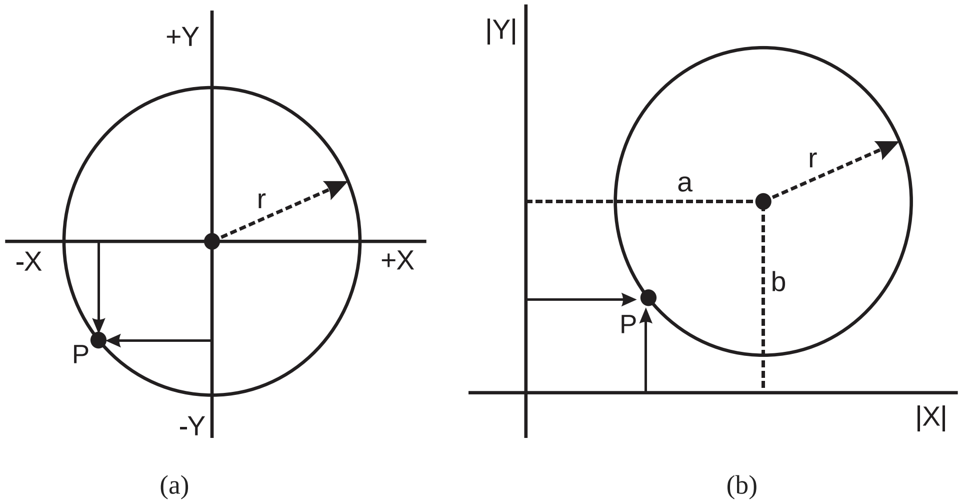


Figure 08: Cartesian coordinates

When seen in Figure 08(b), placed in a specific position of the first quadrant, the math expression is:

$$(x - a)^2 + (y - b)^2 = r^2$$

We see that, in this new position, we do not need coordinates with signs to represent the circumference (as point “P”).

Considering that we have a same circumference, it becomes clear that in both expressions, the number "2", the variables "x" and "y", and the constants "a", “b” and "r" are modules, they do not have signs. The use of the Cartesian system when we place the circumference in the position of Figure 08(a) requires coordinates with signs. A mere convention.

Traditional math refers to a “rule of signs”, and states that different signs (“+” and “-”) yield a negative sign (“-”), while equal signs (“+” and “+” or “-” and “-”) yield a positive sign (“+”), often justified by using the distributive property of multiplication. In fact, there is no rule of signs. It is not possible to multiply a credit by another credit or raise a debt to any power. It will be an operation with no meaning.

Under these situations, a positive element works as an absolute value, a multiplier or a divisor. That positive element informs the need to preserve the sign attributed to the other element (as a credit or a debt), whatever that sign (“+” or “-“). In case we have two negative signs (“-” and “-”), it means we are dealing with images, and will have to change both signs, what becomes (“+” and “+”).

Then, if for any reason, we want to perform an algebraic operation “$y = (x – a)(x - a)$”, the result will be “$y = + x^2 – 2ax +a^2$”, but “x” and “a” are neutral elements, without signs. However, given values to “x” and “a”, the result for “y” may be positive, negative or zero, depending upon the signs (±) imposed to the terms of the math expression.

As a lateral subject relating to the concepts here discussed, I will make some comments on Fermat’s Conjecture.

FERMAT’S CONJECTURE

Almost 400 years ago, when reading the book “Diophantus Arithmetic”, Pierre de Fermat proposed a conjecture, which states:

> *With whole numbers, it is impossible to express a third power as the sum of two third powers or a fourth power as the sum of two fourth powers or, in general, any number to a power greater than the second power as the sum of two powers with the same exponent.*

Fermat also added:

> *I found a truly marvelous demonstration for that statement, but the margin of this book is too narrow to write it.*

The conjecture initially named as Fermat’s Conjecture became Fermat’s Last Theorem after a complex and long demonstration made by Andrew Wiles. If Fermat in fact had a proof for his Conjecture, nobody knows, since he did not disclose it.

I spent some time on this Conjecture, mathematically expressed as “$x^n + y^n = z^n$”, and offered my own demonstrations to it with the sole help of information available at Fermat’s time. I proposed a first demonstration in a book previously published[3], and

[3] AMUI, Sandoval, A Circunferência, Pitágoras e Fermat, Ed. Catalivros, Rio de Janeiro – RJ, 2019.

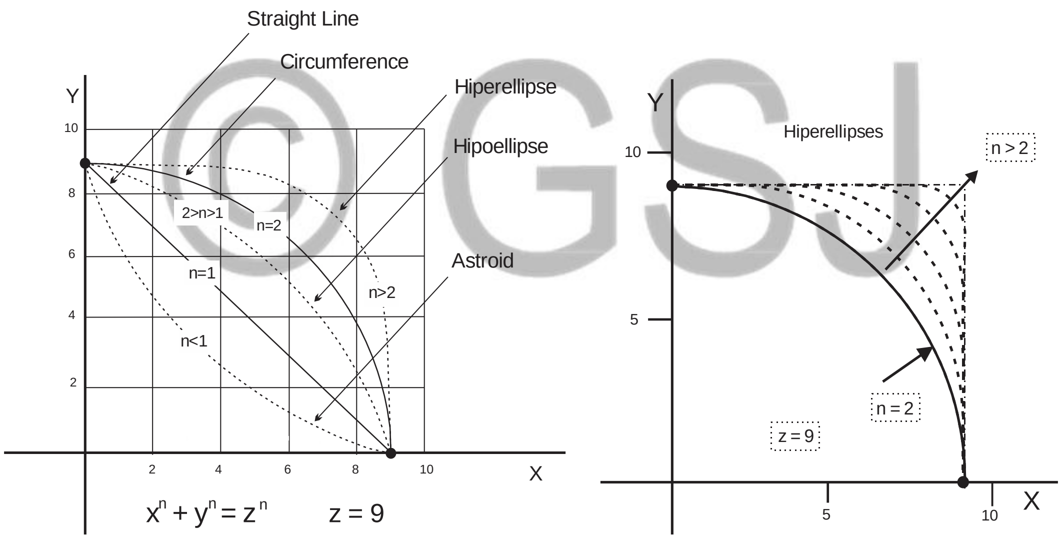
presently I offer a different approach, also based on the combination of algebra with geometry. It seems that, otherwise, it is not possible to prove Fermat's Conjecture.

Having in mind that we are not able to express any figure with an infinite number of decimals, mathematically speaking only the Pythagorean right triangles are true right triangles; all others are approximations.

By rewriting the above-referred math expression "$x^n + y^n = z^n$", as "$(x^{n/2})^2 + (y^{n/2})^2 = (z^{n/2})^2$", we may say that, in order to be a true equality (exact figures), that math expression must represent a circumference. It also implies that "$x^{n/2}$", "$y^{n/2}$" and "$z^{n/2}$" should be whole numbers and "$n = 2$", in a manner that "$x^{n/2}$" and "$y^{n/2}$" would be the legs, and "$z^{n/2}$" the hypotenuse of a Pythagorean right triangle.

Figure 09(a) illustrates the superellipse, a geometric figure also called Lamé's curve or rounded square. In the form "$x^n + y^n = z^n$", we see that it has as particular cases the straight line ($n = 1$), the circumference ($n = 2$), the hipoellipse ($1 < n < 2$), and the hiperellipse ($n > 2$). When "$n = 2/3$" the curve is named astroid.

Figure 09(b) indicates that when "n" increases, keeping "z" constant, the curve tends to a square with rounded corners, it being the reason why the superellipse is also known as "rounded square".



(a) (b)

Figure 09: Superellipse

The math expression "$x^n + y^n = z^n$", which represents Fermat's Conjecture, is also a particular case of a superellipse when "n" is greater than "2". As in Figure 10, said math expression corresponds to a circumference when "$n = 2$" and "z" is the radius, "$r = 9$", of a circumference, as well as to a hiperellipse when "$n = 3$" and "$z = 9$".

> *We see from Figure 10 that in order to contradict Fermat's Conjecture we would need a Pythagorean right triangle with two different hypotenuses or a same circumference with two different radius represented by whole numbers. Geometrical impossibilities.*

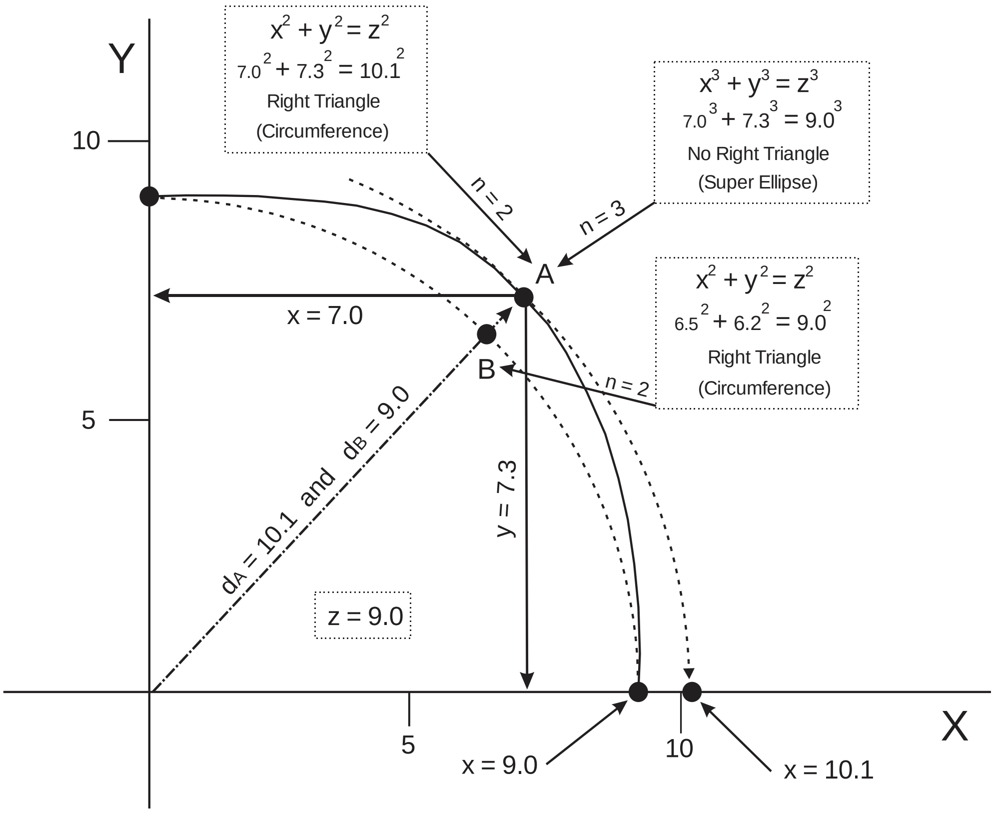


Figure 10: Circumference and hiperellipse

My analysis about Fermat's Conjecture in connection with the superellipse and the relationship between the circumference and the right triangle allows me to state that:

The right triangle that could contradict Fermat's Conjecture does not exist.

NEW MATH FOUNDATION

It seems we need a different mathematics, better aligned with the real world and tailored to suit the day-to-day needs of any person. A new approach that yields the same useful results, without the inconsistencies, strange responses and poorly explained questions found in connection with the traditional math. A new math foundation that favors the enjoyment of math and the improvement of students' performance.

I dare to state that it is possible to achieve this target by simply assigning different interpretations to some of the existing math fundamentals as I discussed in this paper and will summarize below:

(a) Math is a language at the service of other areas of human knowledge; contrarily to geometry, physics, astronomy and other sciences, math does not have a pre-existing and proper object of studies, other than its own premises, rules and procedures;

(b) In accordance with the new Fundamental Axiom of Algebra introduced in this paper, in any math expression numbers and letters representing constants and variables are neither positive nor negative values, they are modules and do not have signs;

(c) Inevitably, the terms formed by these numbers, constants and variables in a math expression are also modules;

(d) The so-called “rule of signs”, which states that operations with different signs yield a negative sign (“-“), and operations with equal signs yield a positive sign (“+”) is simply a practical procedure to indicate that, in said operation, one of the elements is an operator (module) and work as a multiplier or a divisor, in a manner to preserve the sign of the other element, in order to conform with a previous assumption;

(e) An equation is formed as an equality imposed to two algebraic sums of a same variable, which becomes the sole unknown value to be determined;

(f) Math representations of geometric figures placed in the Cartesian system of axes, including polynomials, are not equations and do not form systems of equations;

(g) The number of variables in any math expression informs if the represented geometric figure is a flat or a spatial figure (the geometric order), while the exponents of the variables indicate if we are dealing with a typical or modified geometric figure (the algebraic degree);

(h) The use of the number zero as a common figure is awaiting for clarifications and improvements;

(i) Math needs a better way to represent the continuity that exists in nature, other than the concept of real numbers.

EFFECTS OF THE NEW MATH FOUNDATION

As exemplified below, the new interpretations of math premises and concepts discussed in this paper may have significant impact and effects on the applications and results obtaining in connection with math expressions. Even if they are accepted and implemented, math will still need further revisions and improvements in addition to the ones here proposed.

1) As a language at the service of other areas of the human knowledge, and without violating its principles, math must comply with the laws ruling the object of the served area. Geometry is not a branch of math, but an independent area of studies. When at the service of geometry, math must comply with the laws of geometry;

2) Numbers and letters in any math expression are neutral elements (modules), but they may be used or appear as positive or negative figures, depending upon the specific application of the math expression, as well as the addition and subtraction commands imposed to the terms of said math expression. Then:

   (i) in an equation, we impose an equality between two algebraic sums of a same variable (a module), which becomes the equation unknown; the module must assume the single real value, either positive, negative or zero, which makes the equality a true statement;

(ii) by assigning values to the constants and to the independent variable (modules) forming the terms of a polynomial, these terms will also be modules, but the polynomial, as an algebraic sum that represents the balance between two sets of modules of different nature, may be positive, negative or null;

(iii) When mathematically representing geometric figures placed in the Cartesian system, we may use positive and negative coordinates in function of the position the geometric figure may occupy in relation to the Cartesian axes. The Cartesian system is an arbitrary method to represent geometric figures and the numbers behind positive or negative coordinates remain as modules;

(3) There is no need for a theory of complex numbers, and together with all the associated developments, it may be discarded;

(4) Math expressions of Cartesian representations of geometric figures are not equations and have neither real nor imaginary roots;

(5) Except additions and subtractions, the arithmetic operations with numbers, constants and variables forming the terms of math expressions will involve no signs, because these elements are modules; there will be no need to multiply a positive constant by a negative variable, nor obtain the square root of a negative number;

(6) However, it is possible, under a previous assumption, to multiply a credit “C” by a number “m” (and obtain a positive term “mC”) or divide a debit “D” by another number “n” (and obtain a negative term “D/n”), in a manner that the balance “B” will be “B = + mC – D/n”. Note that “C” and “D” are modules of different nature (credit and debt by assumption), while “m” and “n” are modules used as operators (multiplier and divisor); the signs (+) and (–) are imposed by the math expression;

(7) We may fully represent symmetrical geometric figures centralized in the Cartesian graph with the sole use of the first quadrant. Polynomials, as a resulting module balance, will require the first and the fourth quadrants of the Cartesian system; the abscissa variable “x” will always be a module;

(8) The math representation of nature continuity is perhaps the most important problem that challenges mathematicians, since the real numbers only yield approximate values. True values only occur when the input data and the output responses are whole numbers;

(9) Indeterminations involving the number zero is a common difficulty faced by math students, particularly when dealing with limits. However, limit indeterminations are a peculiarity of math expressions of Cartesian representations of geometric figures. It would be convenient to review the properties and uses of the number zero, a matter that deserves further thoughts;

(10) Some proposed theorems and theories, as well as unproven conjectures and unsolved problems may simply be improper statements resulting from imperfect math premises and assumptions. Building cracks resulting from structural defects.

FINAL COMMENTS

Under the new interpretation of some currently accepted premises, concepts and theories, algebra may lose some of its present charm, but it will be for the benefit of the whole math. We will be free from the inconvenience of dealing with negative and complex numbers, strange roots of equations and the like. In addition to that, it is reasonable to believe that, under an approach more in line with the real world, the students will have a better understanding of math uses and limitations and will improve their academic performance when attending math classes. This new scenario may favor the appreciation and avoid unpleasant feelings towards math.

Some math specialties, as probability and combinatorial analysis, will not require modifications due to the new math foundation I proposed in this paper. Others, as calculus, will need some adjustments. However, there subjects, as equations and Cartesian representations of geometric figures, which will require an approach completely different.

Besides, it is mandatory to emphasize that geometry is not a branch of math, but an independent area of studies. As a language at the service of other fields of human work, when at the service of geometry, math must comply with the laws of geometry.

Further, as mentioned in this text, math needs to review the strange situations relating to the use of zero as a common number, and overcome another difficult problem: the poor representation of nature continuity with the real numbers. Otherwise, math will stay as the science of indeterminations and approximate results.

EXAMPLES

Just to illustrate the application and effects of the new math foundation here suggested, I will present three simple examples:

1st) Analyze and comment the math expression "$x + \sqrt{(6 - x)} = 0$".

In accordance with the prevailing concepts "$x + \sqrt{(6 - x)} = 0$" is an irrational equation. If developed under the standard procedure, we get the new expression "$x^2 + x - 6 = 0$" and, once solved, we find two roots: one, that satisfy the given math expression ($x_1 = - 3$) and other, which does not satisfy said math expression ($x_2 = + 2$). Then, "$x_2 = + 2$" is taken as a "strange root".

Under the approach suggested in this paper, this exercise indicates we are dealing with two different geometric figures, represented by different math expressions, as illustrated in Figure 11: "$y = x^2 + x – 6$" and "$y = x + \sqrt{(6 - x)}$".

The first math expression represents a parabola and we see that, when made equal to zero ($x^2 + x – 6 = 0$), it intercepts the abscissas axis when "$x = + 2$" and "$x = - 3$". The second math expression represents a modified geometric figure, which made equal to zero (x + √(6 - x) = 0), only intercepts the abscissas axis when “x = - 3”. The two curves have a second common point around “x = 2.8”.

Under the premises of this paper, this modified geometric figure only exists in the interval “0 ≤ x ≤ + 6”, as per the continuous line in Figure 11. The pair of “x” and “y” values, “x = + 2” and “y = 0”, does not correspond to a point in the given geometric figure, as it actually does in the parabola. According to the prevailing concepts, we would enter the realm of complex numbers for “x > 6”.

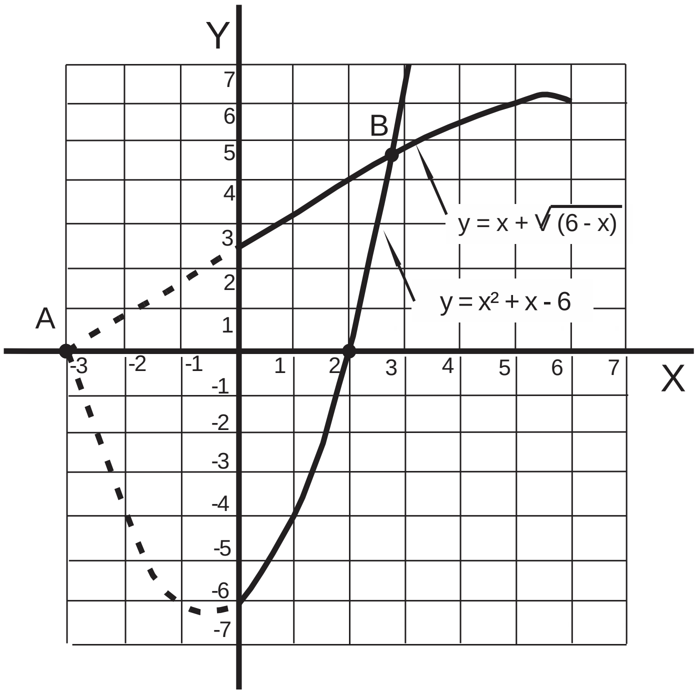


Figure 11: Parabola and a modified geometric figure

Math does not fully accepts the applied operation of going from the given math expression “x + √(6 - x) = 0” to the other, “x^2 + x – 6 = 0”, it being the reason for the occurrence of a strange root, which in fact does not exist. That is an approach to solve the so-called irrational equation, but the equality between the two math expressions only occurs in respect of the two common points “A” and “B” in the geometric figures. The interpretation that “x = + 2” is a strange root of the math expression “x + √(6 - x) = 0” is a mistake caused by the acceptance of an invalid assumption that we are dealing with two equal math expressions, “x + √(6 - x) = 0” and “x^2 + x – 6 = 0”, representing a same geometric figure.

If analyzed according to the new concepts, and keeping in mind that: (i) there are not negative numbers; and (ii) the independent variable “x” is an absolute value (a positive value), we easily see there is no “x” value that satisfy the expression “x + √(6 - x)= 0”.

The math expression “x + √(6 - x) = 0” is not, as accepted by the current concepts, an equation and does not have any strange root. At the most, it accepts the pair of “x” and “y” values, “x = - 3” and “y = 0”, as a point that belongs to the geometric figure represented in the Cartesian graph. Besides that, what could possibly be the real use of that math expression?

2nd) Develop and analyze the math expression “y = (x + 1)(x – 2)(x – 1)”.

Without considering the relevance or application that a math expression of said type may have, we see we face a product of three factors, which depend upon a same independent variable "x" (actually, the math expressions of three straight lines).

According to the prevailing concepts, the result of the product is "$y = x^3 – 2x^2 – x + 2$", understood as a 3rd-degree polynomial illustrated in Figure 12. This polynomial would have the roots "x = - 1", "x = + 2" and "x = + 1", the values of the independent variable "x" which makes the polynomial equal to zero, a simple consequence of the three factors, "(x + 1)", "(x – 2)" and "(x – 1)".

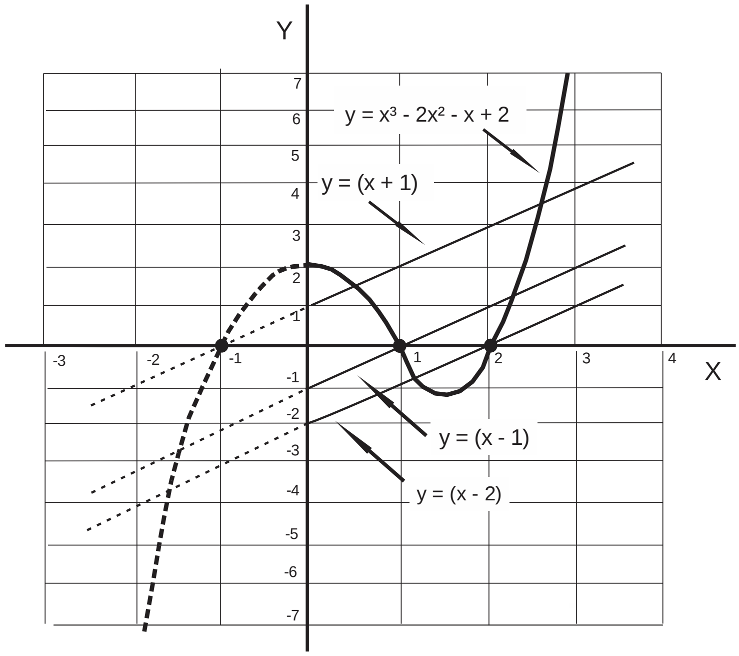


Figure 12: Polynomial and roots

Under the approach adopted in this paper, when made equal to zero (y = 0), the polynomial is not an equation and has no roots. The independent variable "x" is an absolute value that represents things of the world and cannot be negative. The dependent variable "y", as an algebraic result, may assume any value, positive, negative or zero, but it only exists for "$x \geq 0$" (the continuous line in Figure 12).

3rd) Analyze the math expression "$y = x^{-2}$" and find its derivative.

Apparently, we are dealing with negative numbers, since the math expression exhibits a negative exponent. Under the prevailing concepts and rules the derivative of said expression is:

$$y' = -2x^{-3} \text{ or } y' = - (2/x^3).$$

However, if we apply the same derivative rules to the math expression "$y = 1/x^2$", we will get the same result:

$$y' = [(0)(x^2) - (1)(2x)]/x^4 = - (2/x^3).$$

As we see in Figure 13, "$y' = -(2/x^3)$" is not the derivative of "$y = x^{-2}$", but it is the derivative of the existing expression "$y = 1/x^2$". In the given math expression "$y = x^{-2}$", the number "2" is not a negative number. The derivative is negative because we have a decreasing function, but "2", "3" and "x" are absolute values.

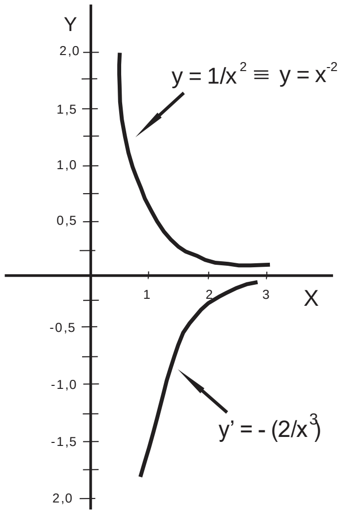


Figure 13: Derivative

In fact, the math expression "$y = x^{-2}$" has no derivative because it does not exist, except as a convention, a different notation for a true math expression "$y = 1/x^2$". There is no negative number involved in the given math expression.