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A study to Investigate the Impact of Price Levels on Product Sales- Latin Square Approach

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Abstract

When two kinds of variations are present in experimental designs that are either, due to the nature or arrangement of experimental units, the Latin Square is an effective design for partitioning those sources of variation from the experimental error. This paper aimed to investigate the effect of three price levels (A, B and C) on sales of an Alcoholic drink using Latin square design by controlling the influence of three types of stores (Small, Medium and Large) and three types of packaging labelled as Packaging I, II and III. The secondary data collected were based on three categories of information. Latin Square Design which involves arranging experimental units into rows and column using Latin letters in such a way that each letter did not repeat itself in either the row or column was adopted. The empirical result shows that the three price levels do not have the same effect on sale of the Alcoholic drink.

Keywords: Latin Square Design, Randomised Complete Block Design (RCBD), ANOVA, Model Specification.

1. Introduction

Latin square design is an extension of randomized complete block design (RCBD) to accommodate two blocking factors. Randomization is applied to assignments of rows, columns and treatments. The Randomized Complete Block Design is commonly used to improve the ability of an experiment to detect real treatment differences by partitioning a known source of variation (blocks) from the experimental error. When this idea is extended to remove two different known sources of variations (i.e. two-way blocking), the resulting design is the Latin square.

Hajihassani (2013) used Latin square design to model the factors affecting banking industry efficiency. The paper aim was to investigate the influence of time, type banks and financial ratios of efficiency accepted in Tehran stock exchange banks. In experimental design, a Latin square is an n × n array filled with n different symbols, each occurring exactly once in each row and exactly once in each column. The paper concluded that time and type bank do not have the same effect on the efficiency of banks but financial ratios have the same effect on the efficiency of banks. The research findings show that the Latin square design is an efficient and effective model for measuring the impact of various factors on the efficiency of banks. Similarly, the Latin square model can be used for industry and other sectors.

Rangriz et al (2012) presented performance Evaluation of Iran cement companies in Iran. The purpose of the study is representing manners which select the problem and solve ranking optimally by multi criteria decision making methods and by high ability. Rapanos, (2009) presented Latin squares and their partial transversals. The paper introduces the theory of Latin squares and their partial transversals.

Martin and Nadarajah (2005) defined a pair of orthogonal Latin squares (each letter combination occurs exactly once) and the extension to sets of mutually orthogonal Latin squares, are useful for constructing experimental designs in several situations in which there are four or more blocking or treatment structures. Euler (2007) presented orthogonal Latin squares as two Latin squares which when superimposed have the properties that the cells contain each of the possible pairs of symbols exactly once. Similar researches on Latin and Graeco Latin squares were investigated by: Klyve and Stemkoshi (2006) and Clausen (2003) who presented Graeco-Latin square as product of juxtaposing or superimposing two Latin squares with treatment denoted by Latin letters and the other with treatment denoted by Greek letters. A Latin square arrangement is an arrangement of s symbols in rows and columns, such that every symbol occurs once in each row and each column. Donovan and Mahmoodian (2000) presented an algorithm for writing any Latin interchange as a sum of intercalates. A Latin interchange is a pair of disjoint partial Latin squares of the same shape and order which are row-wise and column wise mutually balanced.

This study, therefore, employs Latin square Design for more than two sources of variation. Latin square design is preferred over completely randomized design (CRD) and randomized complete block design (RCBD) because CRD is effective for experimental design model with only homogeneous treatments while RCBD is effective when there is only one source of variation present in the design. Latin square designs differs from complete randomized and randomized complete block designs by its capacity to simultaneously handle two known sources of variation among the experimental units. This study is aimed at determining the effect of price level, influence of three types of stores and three types of packaging on sales of product.

2. Materials and Methods

A Latin square arrangement is an arrangement of s symbols in rows and columns, such that every symbol occurs once in each row and each column. The Latin square design consists of two blocking variables refer to as rows and columns and one treatment observation per block Montgomery (1991). The experiment is organized such that each treatment appears exactly once within each row and within each column. Similar to RCBD's with one replication per block-treatment combination, Latin Squares have one replication per row-column treatment combination.

Table 2.1: Layout of the Latin Square design

Random analysis of the price levels to the Latin Square Design where the rows are the store types, columns are the packaging, and Latin letters are price levels (A, B and C).

Store Size	Packaging		
	I	II	III
Small(1)	A	C	B
Medium (2)	B	A	C
Large (3)	C	B	A

2.1 Specification of Latin Square Model

$$y_{ijk=}\ \mu + \ \alpha_i + \ \beta_j + \ \tau_k + \varepsilon_{ijk} \qquad 2.1$$

Let i, j, k =1, 2, 3,…, n.

Definition of the model in (2.1):

y_{ijk} :Observation on kth treatment ith row and jth block

μ : Overall mean

α_i : Effect of first set of block (ith row).

β_j : Effect of second set of block (jth column).

τ_k : Effect of kth treatment.

ε_{ijk} : Random error

Then y_{ijk} can be expressed as:

$y_{ijk} = \bar{y}_{...} + (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})$ 2.3

= A B_r B_c T R

2.3 Latin Square Partitioning of Total Variability

$$SS_{TOTAL} = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{...})^2 = n\sum_{i=1}^{n}(\bar{y}_{i..} - \bar{y}_{...})^2 + n\sum_{j=1}^{n}(\bar{y}_{.j.} - \bar{y}_{...})^2 + n\sum_{j=1}^{n}(\bar{y}_{..k} - \bar{y}_{...})^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})^2$$

$let\ y_{...} = \sum_i^n\sum_j^n\sum_k^n y_{ijk}$ and $\bar{y}_{...} = \frac{\sum_i^n\sum_j^n\sum_k^n y_{ijk}}{n^2}$ 2.4

$SS_{TOTAL} = \sum_i^n\sum_j^n\sum_k^n({y_{ijk}}^2 - 2y_{ijk}\bar{y}_{...} + {\bar{y}_{...}}^2)$ 2.5

Opening the bracket of 2.5and replacing 2.4 in 2.5 we get:

$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} y_{ijk}^2 - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{y_{ijk}^2}{n^2}$ 2.6

The SS_{TOTAL} is Partition into:

$n\sum_{i=1}^{n}(\bar{y}_{i..} - \bar{y}_{...})^2 + n\sum_{j=1}^{n}(\bar{y}_{.j.} - \bar{y}_{...})^2 + n\sum_{j=1}^{n}(\bar{y}_{..k} - \bar{y}_{...})^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\hat{\epsilon}_{ijk}^2$ 2.7

$SS_{TOTAL} = SS_{Br} + SS_{Bc} + SS_{trt} + SS_{error}$ 2.8

$SS_{Br} = n\sum_{i=1}^{n}(\bar{y}_{i..} - \bar{y}_{...})^2 = n\sum_{i=1}^{n}({\bar{y}_{i..}}^2 - 2\bar{y}_{i..}\bar{y}_{...} + {\bar{y}_{...}}^2)$ 2.9

$let\ \bar{y}_{i..} = \frac{\sum_i^n\sum_j^n\sum_k^n y_{i..}}{n}$ 2.10

Opening the bracket and replacing 2.10 and 2.4 in 2.9 we get:

$\sum_{i=1}^{n}\frac{Br_i^2}{n} - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{y_{ijk}^2}{n^2}$ 2.11

$SS_{Bc} = n\sum_{i=1}^{n}(\bar{y}_{.j.} - \bar{y}_{...})^2 = n\sum_{i=1}^{n}({\bar{y}_{.j.}}^2 - 2\bar{y}_{i..}\bar{y}_{.j.} + {\bar{y}_{...}}^2)$ 2.12

$let\ \bar{y}_{.j.} = \frac{\sum_i^n\sum_j^n\sum_k^n y_{.j..}}{n}$ 2.13

Opening the bracket of 2.12 and replacing 2.13 and 2.4 in 2.12 we get:

$\sum_{i=1}^{n} \frac{Bc_j^2}{n} - \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{y_{ijk}^2}{n^2}$ 2.14

$SS_{trt} = n \sum_{i=1}^{n} (\bar{y}_{..k} - \bar{y}_{...})^2 = n \sum_{i=1}^{n} (\bar{y}_{..k}{}^2 - 2\bar{y}_{i..}\bar{y}_{..k} + \bar{y}_{...}{}^2)$ 2.15

Let $\bar{y}_{..k} = \frac{\sum_i^n \sum_j^n \sum_k^n y_{.j..}}{n}$ 2.16

Opening the bracket of 2.15 and replacing 2.16 and 2.4 in 2.15 we get:

$\sum_{i=1}^{n} \frac{T_k^2}{n} - \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{y_{ijk}^2}{n^2}$ 2.17

$SS_{error} = \quad SS_{TOTAL} - SS_{Br} - \; SS_{Bc} - \; SS_{trt}$ 2.18

A = $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} y_{ijk}^2 / n^2 = \bar{y}_{...}$ = grand mean 2.19

Using equation 2.6, 2.11, 2.14, 2.17 and 2.18 we set up the Latin square ANOVA table in table 2.2

Table 2.2 ANOVA of Latin Square

Source of variation	**Degree of freedom**	**Sum of squares (SS)**	**Mean square (MS)**	F_{ratio}
Row(block)	(n-1)	SS_{Br}	$MS_{Br} = \frac{SS_{Br}}{n-1}$	$F_{ratio} = \frac{MS_{Br}}{MS_{error}}$
Column(block)	(n-1)	SS_{Bc}	$MS_{Bc} = \frac{SS_{Bc}}{n-1}$	$F_{ratio} = \frac{MS_{Bc}}{MS_{error}}$
Treatment	(n-1)	SS_{trt}	$MS_{trt} = \frac{SS_{trt}}{n-1}$	$F_{ratio} = \frac{MS_{trt}}{MS_{error}}$
Error(Residual)	(n-1) (n-2)	SS_{error}	$MS_{error} = \frac{SS_{error}}{(n-1)(n-2)}$	
Total	$n^2 - 1$	SS_{TOTAL}		

Test of hypothesis is

$H_o : \alpha_i = 0$ (rows not significantly different) vs $H_1 : \alpha_i \neq 0$ for at least one i (significantly different)

$H_o : b_j = 0$ (columns not significantly different) vs $H_1 : b_j \neq 0$ for at least one j (significantly different)

$H_o : \tau_k = 0$ (treatments not significantly different) vs $H_1 : \tau_k \neq 0$ for at least one k (significantly different)

Decision Rule

In all the tests reject the null hypothesis if F_{cal} is greater than tabulated F at the appropriate degree of freedom, that is, if $F_{Ratio} > F_{Critical}$ value ($F\alpha$, v_1 v_2), reject the Null hypothesis (H_o) and accept the alternative hypothesis (H_1).

Model Specification

The research work is to determine the effect of three price levels of Alcoholic drink represented by(A, B and C) on the sales of its product in a Latin square design by controlling the influence of three types of stores and three types of packaging labelled as Packaging I, II, III . The research assumes that the data are random samples collected from a normal population; which randomization of treatment to the observations in the experiment ensures that the assumptions are approximately met.

The linear analysis model can be stated as:

$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \qquad 2.20$$

i= 1,2,...,n
j= 1,2,...,n
k= 1,2,...,n

Equation 2.11 is the same with equation 2.1

Where:

y_{ijk} : The observation in the *i*th row and *k*th column for the *j*th treatment.

μ : The overall mean,

α_i : The *i*th row effect (store types)

τ_j : The *j*th Latin-treatment effect (Price levels)

β_k : The *k*th column effect (Packaging types)

ε_{ijk} : The random error.

The model above shows that the effects are additive, this means that each observed value of i, k for each experiment unit is a linear combination of the mean, the contribution of the i^{th} level of A j^{th} level of factor B, k^{th} level of the block factor and the i^{th} level of the treatment.

The error is independent and normally distributed with mean μ and variance σ^2 respectively. The samples are independent random sample from defined population constant variance of $\sum y_{ijk}$ for all factors and treatment.

3. Data Presentation and Analysis

In this study, the data was collected from Alcoholic drink dealer in Sapele, Delta State. The data are based on three categories of information to evaluate the effect of three price levels (A, B and C) on sales of an Alcoholic drink using Latin square design by controlling the influence of three types of stores (Small, Medium and Large) and three types of packaging labeled as Packaging I, II and III. The three stores were randomly selected and the price levels were varied. The data consists of two extraneous variables, the store size and the type of packaging which could also influence sales. The two extraneous variables equal the number of categories of treatment (price levels). The total number of experimental units for this design is 3 x 3. The treatments were assigned to the 3x3 units through randomization procedure in such a way that each treatment occur once in each row (store) and each column (packaging). The significance test was evaluated at 5%.

Test of Hypothesis for the analysis

1. H_o : There is no significant difference in the price level on the sales of Alcoholic drink

   H_1 : There is significant difference in the price level on the sales of Alcoholic drink

2. H_o : There is no significant difference in the store size on the sales of Alcoholic drink

   H_1 : There is significant difference in the store size on the sales of Alcoholic drink

3. H_o : There is no significant difference in the packaging type on the sales of Alcoholic drink

   H_1 : There is significant difference in the packaging type on the sales of Alcoholic drink

In this study the data collected are:

The 3 x3 design:

A165	C151	B159
B155	A168	C145
C152	B158	A173

Random analysis yield of the Latin Design where the rows are store sizes, columns are the packaging, Latin letter are price levels (A, B and C) are shown below:

Table 3.1 using the statistical model stated at equation 2.1 to compute

Store Size	Packaging			
	I	II	III	Y . . .
Small(1)	165 A	151C	159B	475
Medium (2)	155B	168A	145C	468
Large (3)	152C	158B	173A	483
Y . . .	472	477	477	1426=Y . . .

Hence using equation 2.4 to 2.19 the analysis of variance was obtain for the data as summarized in the ANOVA table 3.2

SPSS OUTPUT

Univariate Analysis of Variance

Between-Subjects Factors

		N
Price	A	3
	B	3
	C	3
Store	1	3
	2	3
	3	3
Packaging	1	3
	2	3
	3	3

Tests of Between-Subjects Effects

Dependent Variable: Sales

Source		Type III Sum of Squares	Df	Mean Square	F	Sig.
Price	Hypothesis	566.222	2	283.111	21.058	.045
	Error	26.889	2	13.444[a]		
Store	Hypothesis	37.556	2	18.778	1.397	.417
	Error	26.889	2	13.444[a]		
Packaging	Hypothesis	5.556	2	2.778	.207	.829
	Error	26.889	2	13.444[a]		

a. MS(Error)

Table 3.2: Summary of the Latin square ANOVA

Source of variation(SV)	Sum of square(SS)	Degrees of freedom	Mean of square (MS)	Computed F_{ratio}
Treatment (Price levels)	566.222	2	283.111	21.058
Row(Store types)	37.556	2	18.778	1.397
Column(Packaging)	5.556	2	2.778	0.207
Error	26.889	2	13.444	
Total		8		

The table value of F(0.05) with 2 degrees of freedom in both numerator and denominator at level of significant $\alpha = 0.05$ is given by 19.00 for all the factors.

4. Discussion of Results

For Latin letter-Treatment (Price levels): Since F_{ratio}= 21.058 is $>$ F_{tab}=19 reject the null hypothesis (H_o) which implies that the three price levels do not have the same effect on sale of Alcoholic drink.

For Store size (row): Since F_{ratio} = 1.397 is < F_{tab}= 19 we accept the null hypothesis (H_o) which implies that there is no significant difference among the effect of the three types of stores (small, medium and large) on sale of Alcoholic drink.

For Packaging (Column): Since F_{ratio}= 0.207 is < F_{tab}=19 accept the null hypothesis (H_o) which implies that there is no significant difference among the various packaging types on the sale of Alcoholic drink.

5. Conclusion

The principle aim of this research work is to determine if any significance difference exist in the effect of three price levels (A, B and C) on sales of an Alcoholic drink using a Latin square design by controlling the influence of three types of stores (Small, Medium and Large) and three types of packaging labelled as Packaging I, II and III. Latin Square Design which involves arranging experiments units in rows and column using Latin letters in such a way that each letter did not repeat itself in either the row or column.

Based on the analysis carried out on the data collected, we can infer that the packaging and store types do not impact on the sales of the product from the empirical results. However, the three price levels do not have the same effect on sale of the Alcoholic drink. Therefore, the three price levels influences the sales of the Alcoholic drink. Pricing is very important in the sales of any product, this paper recommend that manufacturers and producers of any product should ensure they carry out research to determine proper pricing of their products to enhance sales and profitability.

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