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Application of Picture Uncertainty Soft Relations in Multi- attribute Decision Making

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Abstract: In this paper, we first analyse picture Uncertainty soft Collection (PUSC) and discouss some of their relevant operations such as sub-

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set,equal, complement, AND,OR... and so on. We investigate some theorems on picture Uncertainty soft Collection based on union and intersection with counter examples. Also we proved a necessary and sufficent condition for the dual laws of PFSS theory. Finaly, we then introduce an algorithum based on relational picture Uncertainty soft matrix to solve decision making problems.
Key words: Uncertainty collection,soft collection,Uncertainty soft collection,picture Uncertainty soft collection,subset,equal,AND,OR,complement,dual law and decision making.

1 Introduction

A Uncertainty collection was first introduced by Zadeh [23] and then the Uncertainty collection have been used in there consideration of classical mathematics. Yuan.et.al. [22] introduced the concept of Uncertainty subgroup with thresholds. A Uncertainty subgroup with thresholds λ and μ is also called a (λ, μ)-Uncertainty subgroup. A.Solairaju and R.Nagarajan introduced the concept of structures of Q- Uncertainty groups [19]. A.Solairaju and R. Nagarajan studied some structural properties of upper Q-Uncertainty index order, with upper Q- Uncertainty subgroups [20]. Such in accuracies are associated with the membership function that belongs to [0,1]. Through membership function, we obtain information which makes possible for us to reach the conclusion. The Uncertainty collection theory becomes a strong area of making observations in different areas like medical science, social sciences, engineering, management sciences, artificial intelligence, robotics, computer networks, decision making and so on. Due to unassociated sorts of unpredictability occurring in different areas of life like economics, engineering, medical sciences, management sciences, psychology, sociology, decision making and Uncertainty set as noted and often effective mathematical instruments have been offered to make, be moving in and grip those unpredictability. Since the establishment of Uncertainty collection, several extensions have been made such as Atanassov's ([3], [4], [5], [6]) work on bifuzzy Uncertainty collection (IUSC) was quite remarkable as he extended the idea of USs by assigning non-membership degree say "N(x)" along with membership degree say "P(x)" with condition that $0 \leq P(x) + N(x) \geq 1$. Strengthening the idea IFS suggest pythagorean Uncertainty sets which somehow enlarge the space of positive membership and negative membership by introducing some new condition that $0 \leq P^2(x) + N^2(x) \geq 1$. Molodtsov [14] discussed the concept of soft collections that can be seen as a new mathematical theory for dealing with probablity. The soft collection theory has been cited to various different fields with great success. Maji.et.al. ([8],[9][10]) worked on theoretical study of soft collections in detail, and presented an application of soft collection in the decision making problem using the reduction of rough sets. N -picture Uncertainty soft collections studied in [12,13]. Recently, Cuong [7] proposed picture Uncertainty collection (PUC) and investigated the some basic operations and properties of PUS. The picture Uncertainty collection is characterized by three functions expressing the degree of membership, the degree of neutral membership and the degree of non-membership. The only constraint is that the sum of the three degrees must not exceed 1. Basically, PUS based models can be applied to situation requiring human opinions involving more answers of types: yes, abstain, no, refusal, which can't be accurately expressed in the traditional UC and IUS. Until now, some progress has been made in the research of the PUC theory. Singh [17] investigated the correlation coefficients for picture Uncertainty collection and apply the correlation coefficient for clustering analysis with picture Uncertainty information. Son [18] introduce several novel Uncertainty clustering algorithms on the basis of picture Uncertainty sets and applications to time series forecasting and weather forecasting. In this paper,we study picture Uncertainty soft collections (PUSC) and discues some of their relevant operations such as subset,equal, complement, AND,OR... and so on. We investigate some theorems on picture Uncertainty soft collections based on union and intersection with counter examples. Also we proved a necessary and sufficent condition for the dual laws of PUSC theory. Finaly, we then introduce an algorithum based on relational picture Uncertainty soft matrix to solve problems aspected withcertain conditions.

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2 Preliminaries and Basic Concepts

2.1 Definition

Let the universal collection be $R \neq \phi$. Then $A = \{< r, P_{A(r)} > /r \in R\}$ is said to be a Uncertainty collection of R, where $P_A : U \to [0,1]$. is said to be the membership degree of r in R.

2.2 Example

Let $R = \{r_1, r_2, r_3, r_4, r_5\}$ be the reference set of students. Let $\widetilde{A}$ be the Uncertainty collection of " smart students" where 'smart' is Uncertainty term. $\widetilde{A} = \{< r_1, 0.1 >, < r_2, 0.4 >, < r_3, 0.7 >, < r_4, 1 >, < r_5, 0.9 >\}$. Here $\widetilde{A}$ indicates that the smartness of r_1 is 0.1 and so on.

2.3 Definition

A pair (δ, A) is said to soft collection over R, where F is a function given by $F : A \to P(R)$.

2.4 Example

Suppose $R = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the the collection of six houses and E={Expensive(e_1), Beautiful(e_2), Wooden(e_3), Cheap(e_4)} then the soft set (δ, E) is $(\delta, E) = \{\{h_2, h_4\}\{h_1, h_3\}, \{h_3, h_4, h_5\}, \{h_1, h_3, h_5\}\}$ where each approximation has two parts

(i) A predicate,P;

(ii) An approximate value collection V.

R	$Expensive(e_1)$	$Beautiful(e_2)$	$Wooden(e_3)$	$Cheap(e_4)$
h_1	0	1	0	1
h_2	1	0	0	0
h_3	0	1	1	1
h_4	1	0	1	0
h_5	0	0	1	1
h_6	0	0	0	0

thus, a sof set $(\delta, E) = \{P_1 = V_1, P_2 = V_2, P_3 = V_3, \ldots P_n = V_n\}$.

2.5 Definition

Let I^R denote the collection of all Uncertainty collections on X and $A \subset E$. A pair (δ, A) is called Uncertainty soft collection over R, where $\delta : A \to I^R$. that is, for each $a \in A$, $\delta_a : A \to \ I^R$ is the Uncertainty collection on R.

2.6 Example

Let $R = \{r_1, r_2, r_3, r_4, r_5\}$ be a universial collection and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters.If $A = \{e_1, e_2, e_4\} \subseteq E$. $P_A(e_1) = \{0.2/r_2, 0.3/r_4, \}$, $P_A(e_2) = R$ $P_A(e_4) = \{0.5/r_1, 0.7/r_3, 0.9/r_5\}$ then the Uncertainty soft collection δ_A is defined as $\delta_A = \{r_1, (0.2/r_2, 0.3/r_4), (r_2, U), (r_4, (0.5/r_1, 0.7/r_3, 0.9/r_5)\}$.

2.7 Definition

Let the universe collection be $R \neq \phi$. Then the collection $A = \{< r,\ P_{A(r)},\ I_{A(r)},\ N_{A(r)} > /r \in R\}$ is said to be a picture Uncertainty collection of R, where $P_A : R \longrightarrow [0, 1]$, $I_A : R \longrightarrow [0, 1]$, $N_A : R \longrightarrow [0, 1]$ are said to be the degree of r in R and the positive membership degree of r in R, and the neutral membership degree of r in R, and the nagative membership degree of r in R respectively. Also $P_{(A)}, I_{(A)}, N_{(A)}$ satisfy the following condition:$(\forall r \in R)$ $(0 \leq P_{A(r)}+I_{A(r)}+N_{A(r)} < 1)$. Then for $r \in R$, $\pi_{A(r)} = 1-\{P_{A(r)}+I_{A(r)}+N_{A(r)}\}$ could be called the degree of refusal membership of r in R. Clearly, if $(r \in R)$, $\pi_{A(r)} \ = 0$, then A will be generated to be a standared bifuzzy Uncertainty collection. If $(\forall r \in R), I_{A(r)} = 0$ and $\Pi_{A(r)} \ = 0$, then A will be generated to be a classical Uncertainty collection. Let $\Pi_{A(r)}$ denote the collection of all Uncertainty collections of R. For the sake of simplicity picture Uncertainty collection is denoted by PUC.

2.8 Example

Basically, the model picture Uncertainty collection may be adequate in situations when we face human opinions involving more answers of type:(i) Yes (ii)abstain (iii) No (iv) Refusal.Voating can be a good example of such a situation as the human voters may be class into four groups of those who: (i)

Vote for (ii) abstain (iii) Vote aganist (iv) Refusal of the voting.

2.9 Definition

For $A, B \in PF(R)$, define

(i)$A \subseteq B \iff P_{A(r)} \leq P_{B(r)},\ I_{A(r)} \leq I_{B(r)} and\ N_{A(r)} \leq N_{B(r)}$.

(ii) $A = B \iff A \subseteq B$ and $B \subseteq A$.

(iii)$A \cup B = \{(r, max\{(P_{A(r)}, P_{B(r)})\},\ min\{(I_{A(r)}, I_{B(r)})\}, min\{(N_{A(r)}, N_{B(r)})\}/r \in R\}$.

(iv)$A \cap B = \{(r, min\{(P_{A(r)}, P_{B(r)})\},\ min\{(I_{A(r)}, I_{B(r)})\}, max\{(N_{A(r)}, N_{B(r)})\}/r \in R\}$.

(v) $A^C = \{(r, N_{A(r)}, I_{A(r)}, P_{A(r)})/r \in R\}$.

2.10 Proposition

Let $A, B, C \in PF(R)$.Then

(i) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

(ii)$(A^C)^C = A$

(iii) Operations $\cap$ and $\cup$ are commutatiive, associative and distributive.

(iv) Operations $\cap$ and $\cup$ satisfy demorgons Laws.

3 Propertices of Picture Uncertainty Soft Collections

In this part, we study the idea of PUSC and definition some relvant operation on a PUSC, namely subset, equal, complement, AND, OR and so on. Now we propose the definition of a PUSC and we will see illustrative example it.

3.1 Definition

Let R be an initial universe collection and E a collection of parameters. By a picture Uncertainty soft collection (PUSC) over R we mean a pair (δ, A) where $A \subseteq E$ and δ is a mapping given by $\delta : A \to PF(R)$.

3.2 Example

Consider a PUSC (δ, A) over R, where $R = \{h_1, h_2, h_3, h_4\}$ is the collection of four cases under consideration of a decision making to purchase, and $A = \{e_1, e_2, e_3\}$ is the collection of paramaters where e_1 students for the parameter "Cheap" e_2 students for the parameter "Beautiful" and e_3 students for the parameter "Good location" the PUSC (δ, A) describes the "attractivencss of the house" to this decision maker. Suppose that

R	$Cheap(e_1)$	$Beautiful(e_2)$	$Goodlocation(e_3)$
h_1	$(0.6, 0.3, 0.1)$	$(0.3, 0.5, 0.2)$	$(0.7, 0.5, 0.4)$
h_2	$(0.1, 0.4, 0.6)$	$(0.3, 0.4, 0.5)$	$(0.1, 0.7, 0.9)$
h_3	$(0.3, 0.7, 0.9)$	$(0.4, 0.6, 0.7)$	$(0.2, 0.5, 0.7)$
h_4	$(0.2, 0.5, 0.8)$	$(0.5, 0.7, 0.9)$	$(0.3, 0.5, 0.7)$

For convence of explanation, we can also represent PUSC (δ, A) which is described the above matrix form as follows:

$$(\delta, A)= \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \begin{array}{c} \begin{array}{ccc} \quad e_1 \quad & \quad e_2 \quad & \quad e_3 \quad \end{array} \\ \begin{pmatrix} (0.6, 0.3, 0.1) & (0.3, 0.5, 0.2) & (0.7, 0.5, 0.4) \\ (0.1, 0.4, 0.6) & (0.3, 0.4, 0.5) & (0.1, 0.7, 0.9) \\ (0.3, 0.7, 0.9) & (0.4, 0.6, 0.7) & (0.2, 0.5, 0.7) \\ (0.2, 0.5, 0.8) & (0.5, 0.7, 0.9) & (0.3, 0.5, 0.7) \end{pmatrix} \end{array}$$

3.3 Definition

Let(δ, A) and (Δ, B) be two Picture Uncertainty soft collection's (PUSC's) over U. Then (δ, A) is said to picture Uncertainty soft subset of (Δ, B), denoted by $(\delta, A) \subseteq (\Delta, B)$ if

(i) $A \subseteq B$ and

(ii) $F(e) \subseteq G(e)$ for all $e \in A$.

3.4 Example

Let $U = \{C_1, C_2, C_3, C_4, C_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Suppose (δ, A) and (Δ, B) are two Picture Uncertainty soft set's (PUSC's) over U given by $A = \{e_1, e_2, e_3\}$ and $B = \{e_1, e_2, e_4, e_5\}$.

$$(\delta, A)= \begin{array}{c c} & \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} & \begin{pmatrix} (0.3, 0.2, 0.1) & (0.2, 0.7, 0.3) & (0.1, 0.5, 0.1) \\ (0.2, 0.2, 0.3) & (0.8, 0.6, 0.1) & (0.1, 0.6, 0.7) \\ (0.1, 0.3, 0.6) & (0.7, 0.3, 0.2) & (0.2, 0.3, 0.1) \\ (0.3, 0.1, 0.2) & (0.3, 0.1, 0.5) & (0.3, 0.7, 0.4) \\ (0.2, 0.1, 0.6) & (0.2, 0.3, 0.7) & (0.5, 0.6, 0.2) \end{pmatrix} \end{array}$$

$$(\Delta, B)= \begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & e_4 & e_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} & \begin{pmatrix} (0.6, 0.2, 0.1) & (0.1, 0.2, 0.4) & (0.1, 0.7, 0.6) & (0.7, 0.5, 0.2) \\ (0.5, 0.3, 0.2) & (0.2, 0.4, 0.6) & (0.2, 0.7, 0.5) & (0.1, 0.2, 0.3) \\ (0.1, 0.3, 0.2) & (0.3, 0.0, 0.7) & (0.3, 0.2, 0.1) & (0.3, 0.5, 0.2) \\ (0.1, 0.0, 0.7) & (0.3, 0.7, 0.6) & (0.1, 0.6, 0.2) & (0.7, 0.3, 0.8) \\ (0.0, 0.3, 0.1) & (0.1, 0.2, 0.3) & (0.5, 0.7, 0.2) & (0.1, 0.3, 0.5) \end{pmatrix} \end{array}$$

then δ_A is picture Uncertainty soft subset of Δ_B.

3.5 Definition

Two PUS collections δ_A and Δ_B over U are called to be picture Uncertainty softset equal,if and only if δ_A is a picture Uncertainty soft subset of Δ_B and Δ_B is a picture Uncertainty soft subset of δ_A. That is if $(\delta, A) \subseteq (\Delta, B)$ and $(\delta, B) \subseteq (\Delta, A)$ then $\delta_A = \Delta_B$.

3.6 Definition

Let δ_A be a Picture Uncertainty soft collection PUSC over U. The complement of δ_A, denoted by δ_A^C, where $\delta_A^C : A \to PFS(U)$ is mapping given by $\delta_{(e)}^C = \{\delta_{(e)}\}^C$ for all $e \in A$.

3.7 Example

Consider the PUS collection δ_A then the complement of δ_A is represented as

$$(\delta_A)^C= \begin{array}{c c} & \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} & \begin{pmatrix} (0.1, 0.3, 0.6) & (0.2, 0.5, 0.3) & (0.4, 0.5, 0.7) \\ (0.6, 0.4, 0.9) & (0.5, 0.4, 0.3) & (0.9, 0.7, 0.1) \\ (0.9, 0.7, 0.3) & (0.7, 0.6, 0.4) & (0.7, 0.5, 0.2) \\ (0.8, 0.5, 0.2) & (0.9, 0.7, 0.5) & (0.7, 0.5, 0.3) \end{pmatrix} \end{array}$$

By using the privious idea by Molodtsov, [14] we discuss the AND and OR operation on two PUS collections as proved.

3.8 Definition

Let δ_A and Δ_B are PUS collections over U. Then δ_A AND δ_B, denoted by $\delta_A \cap \delta_B$.

3.9 Example

Let $U = \{C_1, C_2, C_3, C_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$. Take $A = \{e_1, e_2\}$ and $B = \{e_1, e_3, e_5\}$, define

$$\delta_A = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{pmatrix} (0.2, 0.3, 0.4) & (0.1, 0.3, 0.4) \\ (0.1, 0.4, 0.6) & (0.3, 0.2, 0.1) \\ (0.2, 0.0, 0.8) & (0.1, 0.3, 0.6) \\ (0.1, 0.2, 0.4) & (0.2, 0.4, 0.1) \end{pmatrix} \end{array} \text{ and}$$

$$\Delta_B = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_3 & e_5 \end{array} \\ \begin{pmatrix} (0.1, 0.7, 0.6) & (0.2, 0.4, 0.7) & (0.1, 0.3, 0.6) \\ (0.2, 0.4, 0.3) & (0.1, 0.3, 0.8) & (0.2, 0.6, 0.7) \\ (0.1, 0.3, 0.6) & (0.4, 0.6, 0.2) & (0.1, 0.4, 0.6) \\ (0.7, 0.1, 0.8) & (0.7, 0.6, 0.3) & (0.1, 0.0, 0.2) \end{pmatrix} \end{array}$$

Here $\delta_A \cap \Delta_B = (H, C)$, where $C = A \cap B$ and $\forall e \in C$

$$H(e) = \begin{cases} \delta(e), & \text{if } \ \text{x } e \in A - B \\ \Delta(e), & \text{if } \ \text{x } e \in B - A \\ \delta(e) \cap \Delta(e), & \text{if } \ e \in A \cap B \end{cases}$$

in this example

$$\delta_A \cap \Delta_B = (H, C) = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \begin{array}{c} e_1 \\ \begin{pmatrix} (0.2, 0.3, 0.4) \\ (0.1, 0.4, 0.6) \\ (0.2, 0.0, 0.8) \\ (0.1, 0.2, 0.4) \end{pmatrix} \end{array}$$

3.10 Definition

Let δ_A and Δ_B are PUS collections over U. Then δ_A OR Δ_B, denoted by $\delta_A \cup \Delta_B$.

$\delta_A \cup \Delta_B =$

$$\begin{array}{c c} & \begin{array}{cccccc} (e_1,e_1) & (e_1,e_3) & (e_1,e_5) & (e_2,e_1) & (e_2,e_3) & (e_2,e_5) \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \begin{pmatrix} (0.2,0.3,0.4) & (0.2,0.3,0.7) & (0.1,0.3,0.6) & (0.1,0.3,0.6) & (0.1,0.3,0.7) & (0.1,0.3,0.6) \\ (0.1,0.4,0.6) & (0.1,0.3,0.8) & (0.1,0.4,0.7) & (0.2,0.2,0.3) & (0.1,0.2,0.8) & (0.2,0.2,0.7) \\ (0.1,0.0,0.8) & (0.2,0.0,0.8) & (0.1,0.0,0.8) & (0.1,0.3,0.6) & (0.1,0.3,0.6) & (0.1,0.3,0.6) \\ (0.1,0.1,0.8) & (0.1,0.2,0.4) & (0.1,0.0,0.4) & (0.2,0.1,0.8) & (0.2,0.4,0.3) & (0.1,0.0,0.2) \end{pmatrix} \end{array}$$

Here $\delta_A \cup \Delta_B = (H, C)$, where C=$(A \cup B)$ and for all $e \in C$.

$$K(e) = \begin{cases} \delta(e), & \text{if} \quad \text{x } e \in A - B \\ \Delta(e), & \text{if} \quad \text{x } e \in B - A \\ \delta(e) \cup \Delta(e), & \text{if} \quad e \in A \cap B \end{cases}$$

$(\delta_A \cup \Delta_B) = (H, C)$=

$$\begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & e_3 & e_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \begin{pmatrix} (0.2,0.3,0.4) & (0.1,0.3,0.4) & (0.2,0.4,0.7) & (0.1,0.3,0.6) \\ (0.1,0.4,0.6) & (0.4,0.2,0.1) & (0.1,0.3,0.8) & (0.2,0.6,0.7) \\ (0.0,0.0,0.8) & (0.1,0.3,0.6) & (0.4,0.3,0.2) & (0.1,0.4,0.6) \\ (0.1,0.2,0.4) & (0.2,0.4,0.1) & (0.7,0.6,0.3) & (0.1,0.0,0.2) \end{pmatrix} \end{array}$$

3.11 Theorem[Demorgon's Law]

Let δ_A and Δ_B be two PUS collections over U. Then (i) $(\delta_A \cap \Delta_B)^C = {\delta_A}^C \cup {\Delta_B}^C$ (ii) $(\delta_A \cup \Delta_B)^C = {\delta_A}^C \cap {\Delta_B}^C$.

Proof

$$\begin{aligned}
(i)\text{Suppose that } (\delta, A) \cap (\Delta, B) &= (K, A \times B) \\
(\delta, A) \cap (\Delta, B)^C &= (K, A \times B)^C \\
&= (K^C, A \times B) \\
\text{Now}(\delta, A)^C \cup (\Delta, B)^C &= (\delta^C, A) \cup (\Delta^C, B) \\
&= (H, A \times B).
\end{aligned}$$

Take $(\alpha, \beta) \in A \times B$, Therefore

$$\begin{aligned}
K^C(\alpha, \beta) &= \{K(\alpha, \beta)\}^C \\
&= \{\delta(\alpha) \cap \Delta(\beta)\}^C \\
&= \delta^C(\alpha) \cup \Delta^C(\beta) \\
\text{agian } H(\alpha, \beta) &= \delta^C(\alpha) \cup \Delta^C(\beta) \\
H^C(\alpha, \beta) &= H(\alpha, \beta)
\end{aligned}$$

The theorem is proved

(ii) The result can be proved in a similar way.

3.12 Theorem

Union of two PUS collections δ_A and Δ_B is a PUS collection.
Proof
We know that, Let (δ_A) and (Δ_B) are PUS collections over U. Then δ_A OR Δ_B, denoted by $\delta_A \cup \Delta_B$. and $\forall e \in C, e \in A \to B, or\ e \in B \to A$,then $K(e) = \delta(e)$ or K(e)=$\Delta(e)$. So, in either case,we have K(e) is a picture Uncertainty soft collection. If $e \in A \cap B$, for a fixed $x \in U$ without loss of generality, suppose $\lambda_{\delta(e)}(x) \leq \lambda_{\Delta(e)}(x)$,
we have $\lambda_{K(e)}(x) + \mu_{K(e)}(x) + \gamma_{K(e)}(x)$=
$\max\{\lambda_{\delta(e)}(x), \lambda_{\Delta(e)}(x)\}$+$\min\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\}$ + $\min\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$
=$\lambda_{\Delta(e)}(x)$ + $\min\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\}$ + $\min\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$
$\leq \lambda_{\Delta(e)}(x) + \mu_{\Delta(e)}(x) + \gamma_{\Delta(e)}(x) \leq 1$
Therefore (K, C) is a picture Uncertainty soft collection (PUSC).
Hence the proof.

3.13 Theorem

Intersection of two PUS collectionss δ_A and Δ_B is a PFS set.

Proof

we know that, Let δ_A and Δ_B are PUS collections over U. Thenδ_A OR Δ_B, denoted by $\delta_A \cap \Delta_B$. and $\forall e \in U$, without loss of generality, suppose $\lambda_{\delta(e)}(x) \leq \lambda_{\Delta(e)}(x)$,

we have $\lambda_{K(e)}(x) + \mu_{K(e)}(x) + \gamma_{K(e)}(x)$=

Min$\{\lambda_{\delta(e)}(x); \lambda_{\Delta(e)}(x)\}$+Max$\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\}$ + Max$\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$

=$\lambda_{\Delta(e)}(x)$ + Min$\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\}$ + Max$\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$

$\leq \lambda_{\Delta(e)}(x) + \mu_{\Delta(e)}(x) + \gamma_{\Delta(e)}(x) \leq 1$

There fore (K, C) is a PUS collection. Hence the proof.

3.14 Theorem

Let δ_A, Δ_B and K_C be PUS collections over U.Then (i) $\delta_A \cap \delta_B = \delta_A$ (ii) $\delta_A \cup \delta_A = \delta_A$.

(iii) $(\delta_A \cap \Delta_B) = (\Delta_B \cup \delta_A)$ (iv) $(\delta_A \cup \Delta_B) = (\Delta_B \cap \delta_A)$.

(v) $(\delta_A \cup \Delta_B) \cup K_C = \delta \cup (\Delta_B \cup K_C)$ (vi) $(\delta_A \cap \Delta_B) \cap K_C = \delta \cap (\Delta_B \cap K_C)$.

Proof

The proofs are strightforward by using the definitions((3.5),(3.8),(3.10)) and Theorem(3.11)

(i)Let δ_A and Δ_B are PUS collections over U. Then δ_A AND δ_B, denoted by $\delta_A \cap \delta_B$.

(ii)Let δ_A and Δ_B are PUS collections over U. Then δ_A OR δ_B, denoted by $\delta_A \cup \delta_B$

(iii) Let $A, B, C \in PF(R)$,then (i) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

(ii)$(A^C)^C = A$

(iii) Operations $\cap$ and $\cup$ are commutative, associative and distributive.

3.15 Theorem[Distributive Law]

Let δ_A, Δ_B and K_C be PUC collections over U.Then (i) $\delta_A \cap (\Delta_B \cup K_C) = (\delta_A \cap \Delta_B) \cup (\delta_A \cap K_C)$ (ii)$\delta_A \cup (\Delta_B \cap K_C) = (\delta_A \cup \Delta_B) \cap (\delta_A \cup K_C)$

Proof

The proofs are strightforward by using the definition((3.5),(3.8),(3.10)) and Theorem(3.14).

(i)Let δ_A and Δ_B are PFS sets over U. Then δ_A AND δ_B, denoted by "$\delta_A \cap \delta_B$".

(ii)Let δ_A and Δ_B are PUC collections over U. Thenδ_A OR δ_B, denoted by "$\delta_A \cup \delta_B$"

(iii) Let $A, B, C \in PF(R)$,then (iv) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

(v)$(A^C)^C = A$

(iii) Operations $\cap$ and $\cup$ are commutatiive, associative and distributive.

3.16 Theorem[Dual Law]

Let δ_A, Δ_B and K_C be PUS collections over U.Then (i) $(\delta_A \cap \Delta_B)^C = {\delta_A}^C \cup {\Delta_B}^C$, if and only if A=B

(ii) $(\delta_A \cup \Delta_B)^C = {\delta_A}^C \cap {\Delta_B}^C$. iff A=B

Proof

(i) If A=B, Then we have $\delta_A \cup \Delta_B = \delta_A \cup \Delta_A = (K, A)$. Now for all $e \in A$, $K(e) = \delta(e) \cup \Delta(e)$ Hence $(\delta_A \cap \Delta_B)^C = (\delta_A \cap \Delta_A)^C = (K, A)^C = (K^C, A)$ and $K^C(e) = (\delta(e) \cup \Delta(e))^C = \delta^C(e) \cap \Delta^C(e)$Ȧgain suppose that $(\delta_A \cap \Delta_B)^C = (\delta_A \cap \Delta_A)^C = (I, A)^C = (I^C, A) = (I, A)$ for all $e \in A$. $I(e) = \delta^C(e) \cup \Delta^C(e)$ we see that for all $e \in A.I(e) = K^C(e)$. Therefore this result is true.

Conversely, hypotheses $A \neq B$. Suppose that $\delta_A \cup \delta_B =$(K,C) where $C = A \cup B$ and for all $e \in C$.

$$K(e) = \begin{cases} \delta(e), & \text{if} \quad \text{x } e \in A - B \\ \Delta(e), & \text{if} \quad \text{x } e \in B - A \\ \delta(e) \cup \Delta(e), & \text{if} \quad e \in A \cap B \end{cases}$$

Thus $(\delta_A \cup \Delta_B)^C = K_C^C$ and

$$K^C(e) = \begin{cases} \delta^C(e), & \text{if} \quad \text{x } e \in A - B \\ \Delta^C(e), & \text{if} \quad \text{x } e \in B - A \\ \delta^C(e) \cap \Delta^C(e), & \text{if} \quad e \in A \cap B \end{cases}$$

Again suppose that $\delta_A^C \cap \Delta_B^C = (I, J)$.Where $J = A \cap B$ and $\forall e \in J$. $I(e) = \delta^C(e) \cap \Delta^C(e)$. obviously, where $A \neq B$, we have $C = A \cup B \neq A \cap B = J$, so $K_C^C \neq I_J$. This is controdiction of over condition. $(\delta_A \cap \Delta_B)^C = {\delta_A}^C \cup {\Delta_B}^C$.

Hence A=B.
(ii) This result can be proved in a similar way.

3.17 Remark

From the above theorem for dual theorem,we know that Demorgon's laws are invalied for PUS collections with the different parameters collections, but they are true for PUS collections with the identical parameter set.

4 Picture Uncertainty Soft Relations and its Decision Making

In this part,we construct picture Uncertainty soft operator and a decision making method on relations.
Now we construct a decision making method on picture Uncertainty soft relation by the following algorithum:
step-1 Input the picture Uncertainty soft collections A and B
step-2 Obtain the picture Uncertainty soft matrix R corresponding to cartesion product of A and B respectively.
step-3 Compute the comparision table using the following formula $P_A(r) + I_A(r) - N_A(r)$.
step-4 Select the hight numerical grades from comparison table for each row.
step-5 Find the solve table which having the following form

R	(x_1, y_1)	...	...	(x_n, y_n)
$(Objects)$	h_1			
$(Highestgrade)$				

where X_n denotes the parameter of A and y_n denotes the parameter of B.
step-6 Compute the solve of each objects by taking the sum of these numerical grades.
step-7 Find m, for which $S_m = maxS_j$. then S_m is the hight score,if m has most then one values,you can choose any one value S_j.
Now we use this algorithm to find the best choice in decision making system.

4.1 Example

Let $U = \{u_1, u_2, u_3, u_4\}$ be the set of four cars. Suppose that two friends wont to buy a car for a muutal friend among these four cars according to their choice parameters $E_1 = \{x_1, x_2, x_3\}$={Expensive, moderate, inexpensive} and $E_2 = \{y_1, y_2, y_3\}$={Green,Black,Red} respectively, then we select a car on the basis of the collections of friends parameters by using the picture Uncertainty soft collection of relation decision making method.

step-1 We input the picture Uncertainty soft collection A and B as

$$A= \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} \quad u_1 \quad & \quad u_2 \quad & \quad u_3 \quad & \quad u_4 \quad \end{array} \\ \begin{pmatrix} (0.3, 0.6, 0.4) & (0.2, 0.4, 0.5) & (0.3, 0.6, 0.7) & (0.3, 0.5, 0.7) \\ (0.1, 0.3, 0.4) & (0.1, 0.3, 0.3) & (0.2, 0.4, 0.5) & (0.1, 0.3, 0.9) \\ (0.3, 0.3, 0.4) & (0.4, 0.6, 0.7) & (0.3, 0.4, 0.5) & (0.3, 0.7, 0.9) \end{pmatrix} \end{array}$$

$$\text{and } B= \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} \quad u_1 \quad & \quad u_2 \quad & \quad u_3 \quad & \quad u_4 \quad \end{array} \\ \begin{pmatrix} (0.4, 0.6, 0.7) & (0.2, 0.4, 0.5) & (0.4, 0.4, 0.6) & (0.3, 0.5, 0.9) \\ (0.2, 0.3, 0.5) & (0.3, 0.5, 0.6) & (0.2, 0.4, 0.5) & (0.1, 0.5, 0.9) \\ (0.2, 0.3, 0.4) & (0.2, 0.8, 0.8) & (0.3, 0.6, 0.6) & (0.1, 0.4, 0.7) \end{pmatrix} \end{array}$$

step-2 we obtain the picture Uncertainty soft matrix R corresponding to cartesion product of A and B respectively

R	u_1	u_2	u_3	u_4
(x_1, y_1)	$(0.3, 0.6, 0.7)$	$(0.2, 0.4, 0.5)$	$(0.3, 0.5, 0.7)$	$(0.3, 0.5, 0.7)$
(x_1, y_2)	$(0.1, 0.45, 0.4)$	$(0.2, 0.45, 0.6)$	$(0.2, 0.5, 0.7)$	$(0.1, 0.45, 0.9)$
(x_1, y_3)	$(0.2, 0.45, 0.7)$	$(0.2, 0.6, 0.8)$	$(0.3, 0.6, 0.7)$	$(0.1, 0.45, 0.7)$
(x_2, y_1)	$(0.1, 0.45, 0.7)$	$(0.1, 0.4, 0.6)$	$(0.2, 0.4, 0.6)$	$(0.1, 0.4, 0.9)$
(x_2, y_2)	$(0.1, 0.3, 0.5)$	$(0.1, 0.4, 0.6)$	$(0.2, 0.4, 0.5)$	$(0.1, 0.4, 0.9)$
(x_2, y_3)	$(0.1, 0.3, 0.4)$	$(0.1, 0.055, 0.8)$	$(0.2, 0.5, 0.6)$	$(0.1, 0.35, 0.9)$
(x_3, y_1)	$(0.3, 0.4, 0.7)$	$(0.2, 0.5, 0.7)$	$(0.3, 0.4, 0.6)$	$(0.3, 0.06, 0.9)$
(x_3, y_2)	$(0.2, 0.3, 0.5)$	$(0.3, 0.055, 0.7)$	$(0.2, 0.4, 0.5)$	$(0.3, 0.6, 0.7)$
(x_3, y_3)	$(0.2, 0.3, 0.4)$	$(0.2, 0.7, 0.8)$	$(0.3, 0.5, 0.6)$	$(0.3, 0.6, 0.9)$

Table -2 picture Uncertainty soft relational matrix R.

Step-3 By using table-1, we compute the comparison table as

R	u_1	u_2	u_3	u_4
(x_1, y_1)	0.2	0.1	0.1	0.1
(x_1, y_2)	-0.15	0.05	0	-0.35
(x_1, y_3)	-0.05	0.00	0.2	-0.15
(x_2, y_1)	-0.15	-0.1	0.00	-0.4
(x_2, y_2)	-0.1	-0.1	0.1	-0.4
(x_2, y_3)	0.00	-0.645	0.1	-0.45
(x_3, y_1)	0.05	0.00	0.1	-0.54
(x_3, y_2)	0.00	-0.345	0.1	0.2
(x_3, y_3)	0.1	0.1	0.2	0.2

Table -2 comparsion table (P+I-N).

step-4 we select the highest numerical grade from step-3 for each row

R	u_1	u_2	u_3	u_4
(x_1, y_1)	0.2	0.1	0.1	0.1
(x_1, y_2)	-0.15	0.05	0	-0.35
(x_1, y_3)	-0.05	0.00	0.2	-0.15
(x_2, y_1)	-0.15	-0.1	0.00	-0.4
(x_2, y_2)	-0.1	-0.1	0.1	-0.4
(x_2, y_3)	0.00	-0.645	0.1	-0.45
(x_3, y_1)	0.05	0.00	0.1	-0.54
(x_3, y_2)	0.00	-0.345	0.1	0.2
(x_3, y_3)	0.1	0.1	0.2	-0.745

.

Table-3 Highest value of each row

step-5 we find the score table which have the following form

R	u_1	u_2	u_3	u_4
(x_1, y_1)	0.2	$-$	$-$	$-$
(x_1, y_2)	$-$	0.05	$-$	$-$
(x_1, y_3)	$-$	$-$	0.2	$-$
(x_2, y_1)	$-$	$-$	0	$-$
(x_2, y_2)	$-$	$-$	0.1	$-$
(x_2, y_3)	$-$	$-$	0.1	$-$
(x_3, y_1)	$-$	$-$	0.1	$-$
(x_3, y_2)	$-$	$-$	$-$	0.2
(x_3, y_3)	$-$	$-$	0.2	$-$

.

Table -4 is score table.

step-6 we compute the score of each objects by taking the form of numerical grades as;
$u_1 = 0.2$,
$u_2 = 0.05$,
$u_3 = 0.2 + 0 + 0.1 + 0.1 + 0.2 = 0.7$,
$u_4 = 0.2$
step-7 The maximum value of the score value is $S_j = 0.7$, so the two friends will select the car with the highest score, hence they will choose car u_3. with parameter either expensive car with red or expensive car with red.

5 Conclusion

We have defined a picture Uncertainty soft collection with some special operations and proved various result based on the picture Uncertainty soft collection. Finaily we study the decision making approach for solving picture Uncertainty soft matrix under relational concepts. One can obtian the similar result in fermatean Uncertainty soft collection and pythogoner Uncertainty soft collections.

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