Global Scientific JOURNALS

CONTINUOUS FUZZY MAPPINGS IN FUZZY METRIC SPACE

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ABSTRACT

In this paper, we studied the fuzzy metric space as defined by Z.Q.Xia, and F.F.Guo. Defined in different way in the sense of fuzzy scalars instead of fuzzy numbers or real numbers are used to define fuzzy metric. We further define open and closed fuzzy sets in the sense of open sphere and established the basic properties of open and closed fuzzy sets. We also define continuous fuzzy mapping and established the properties of continuous mappings, as per new definition, which is more similar to classic metric space.

KEY WORDS: Fuzzy metric space, open fuzzy sphere, open fuzzy set, closed fuzzy sets, continuous fuzzy mapping.

1. INTRODUCTION

There are so many approaches to define fuzzy metric spaces. The researcher like Kaleva (1980),George (1994),Gregory (2000),etc. They are using real numbers to measure the distance between fuzzy sets. The problem is that they are using different measure in different problems in fuzzy environment. There does not exist a uniform measure that can be used in all kinds of fuzzy environment. In this paper, an attempt has been made to using fuzzy scalars (Fuzzy points defined on real valued space R) to measure the distance between fuzzy points which is consistent with the theory of fuzzy linear spaces in the sense of Xia and Guo (2003). We further define open fuzzy set and closed fuzzy sets, in the sense of open sphere. We also define continuous fuzzy mapping more similar to classic metric space and established the properties of continuous mapping according to this new definition.

2. PRELIMINARIES

FUZZY POINTS 2.1: A fuzzy set in X is called a fuzzy points iff it takes the value '0' for y ∈ X, except one say x ∈ X. If its value at x is λ (0 < λ ≤ 1). We denote this fuzzy point by x_λ, where the point x is called its support.

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$$x_{\lambda}(y)=\begin{cases}\lambda & if \quad x=y \\ 0 & if \quad x\neq y\end{cases}$$

In this paper, fuzzy points are usually denoted by (x,λ), and the set of all fuzzy points defined on X is denoted by $P_F(X)$. In particular, when X = R, fuzzy points are also called fuzzy scalars and the set of all fuzzy scalars is denoted by $S_F(R)$. A fuzzy set A can be regarded as a set of fuzzy points belonging to it, i.e.

$A=\{(x,\lambda):A(x)\geq\lambda\}$ Or a set of fuzzy points on it $A=\{(x,\lambda):A(x)=\lambda\}$

3. FUZZY METRIC SPACES

FUZZY SCALAR 3.1: Suppose (x, λ) and (y, λ) are two fuzzy scalars. The following conditions are satisfied

(a) We say (a, λ) $\succeq$ (b, γ), if $a>b$ or (a, λ) = (b, γ).

(b) (a, λ), is not less than (b, γ) , if a $\geq$ b, denoted by (a, λ) $\succ$ (b, γ), or $(b,\gamma)\prec(a,\lambda)$

(c) (a, λ) is said to be non-negative if $a\geq 0$. The set of all non-negative fuzzy scalars is denoted by $S_F^+(R)$.

The definition (a) and (b) are both partial orders. When R is a subset of $S_F(R)$, (R, $\succ$), and (R, $\succeq$), are the same as (R, $\geq$). Thus both $\succ$ and $\succeq$ can be viewed as some kind of generalization of the ordinary complete order $\geq$. It is obvious that the order defined in (a) is stronger than defined in (b).

FUZZY METRIC SPACE 3.2: Suppose X is a non-empty set and $d_F:P_F(X).P_F(X)\rightarrow S_F^+(R)$, is a mapping. $(P_F(X),d_F)$ is said to be a fuzzy metric space if for any (x,λ),(y, γ),and (z, ρ) $\subset P_F(X)$, d_F satisfy the following three conditions.

(i) d_F ((x,λ),(y, γ)) = 0, iff x = y, and λ = γ = 1

(ii) d_F ((x,λ),(y, γ)) = d_F ((y, γ),(x,λ)) (Symmetric)

(iii) $d_F((x,\lambda),(z,\rho))\prec d_F((x,\lambda)+(y,\gamma))+d_F((y,\gamma)+(z,\rho))$, Triangular inequality

d_F is called a fuzzy metric defined in $P_F(X)$, and $d_F((x,\lambda),(y,\gamma))$ is called fuzzy distance between two fuzzy points.

Note that fuzzy metric spaces have fuzzy points as their elements, ie they are sets of fuzzy points.

EXAMPLE :1 Suppose (X,d) is an ordinary metric space. The distance of any two fuzzy points $(x,\lambda),(y,\gamma)$ in $P_F(X)$,is defined as

$$d_F((x,\lambda),(y,\gamma) = \{d(x,y), \min(\lambda,\gamma)\}$$

Where $d(x,y)$, is the distance between x and y defined in (x, d). Then $(P_F(X), d_F)$,is a fuzzy metric space.

PROOF : It suffices to prove that d_F satisfied all the condition as defined in 3.2. Suppose that $(x,\lambda),(y,\gamma)$, are two fuzzy points in $P_F(X)$. Since $d(x,y)$ is the distance between x and y also $d(x,y) \geq 0$, if $x \neq y$, it follows from definition (1) that $d_F((x,\lambda),(y,\gamma)) = \{d(x,y), \min(\lambda,\gamma)\}$ is non-negative fuzzy scalars. It is obvious that $d((x,\lambda),(y,\gamma)) = 0$, iff $d(x,y) = 0$ and $\min\{\lambda,\gamma\} = 1$, which is equal to that $x = y$,and $\lambda = \gamma = 1$.

Symmetric: For any $\{(x,\lambda),(y,\gamma)\} \subset P_F(X)$, we have

$$d_F((x,\lambda),(y,\gamma)) = \{d(x,y) \min(\lambda,\gamma)\}$$

$$d_F((x,\lambda),(y,\gamma)) = \{d(y,x), \min(\gamma,\lambda)\}$$

$$d_F((x,\lambda),(y,\gamma)) = d_F((y,\gamma),(x,\lambda))$$

Triangular inequality: For any $\{(x,\lambda),(y,\gamma),(z,\rho)\} \subset P_F(X)$, we have

$$d_F((x,\lambda),(z,\rho)) = \{d(x,z), \min(\lambda,\rho)\}$$

$$d_F((x,\lambda),(z,\rho)) \prec \{d(x,y) + (y,z)), \min(\lambda,\gamma,\rho)\}$$

$$= \{d(x,y), \min(\lambda,\gamma)\} + \{d(y,z)), \min(\gamma,\rho)\}$$

$$= d_F((x,\lambda),(y,\gamma)) + d_F((y,\gamma),(z,\rho))$$

DEFINITION 3.3: Suppose X is a non-empty set and $d_F : P_F(X).P_F(X) \to S_F^+(R)$, is a mapping. Then $(P_F(X), d_F)$,is said to be a strong fuzzy metric space, if it satisfies the first two condition in definition 3.2, and for any $(x,\lambda),(y,\gamma),(z,\rho)$,in $P_F(X)$ one has

(iii)′ $d_F((x,\lambda),(z,\rho)) \preceq d_F((x,\lambda),(y,\gamma)) + d_F((y,\gamma),(z,\rho))$

It is obvious from definition (3.2) and (3.3) that every strong fuzzy metric space is a fuzzy metric space. The following example shows the existence of strong fuzzy metric spaces and the difference between these two kinds of spaces.

EXAMPLE 2: Let L is a fuzzy linear space defined in R^n. The distance between arbitrary two fuzzy points (x,λ) and (y,γ) on L is defined by

$d_{FE}((x,\lambda),(y,\gamma)) = \{d_E(x,y), \min(\lambda,\gamma)\}$ ....(i)

Where d_E is the Euclidean distance. Then (L, d_{FE}) is a strong fuzzy metric space where L denotes the set of fuzzy points on the fuzzy set L.

Proof : The first two condition can be proved just as example.1. Here we only prove the third one. Given arbitrary three fuzzy points on L, $(x,\lambda),(y,\gamma)$ and (z,ρ) .Since (R^n, d_E) is a metric space.

$$d_E(x,z) \le d_E(x,y) + d_E(y,z) \text{ ....(ii)}$$

In the case of that inequality (ii) holds strictly, it is obvious from the definition 3.1(a) that the condition (iii)' is satisfied. In the other case, there must exists some $\lambda \in F$ such that $y = (1-\lambda)x + \lambda z$. Let $\alpha = \min\{\lambda, \rho\}$. We have that $\{x,z\} \subset L_\alpha$. Since L is a fuzzy linear space. L_α is a linear subspace of R^n (see the representation theorem of fuzzy linear space due to Lowen (1980) [13]). It follows that $y \in L_\alpha$ i.e $\gamma = L(y) \ge \alpha = \min\{\lambda, \rho\}$. This implies that $\min\{\lambda,\gamma,\rho\} = \min\{\lambda,\rho\}$ .Thus one has

$$d_{FE}((x,\lambda),(z,\rho)) = \{d_E(x,z), \min\{\lambda,\rho\}\}$$

$$\prec (d_E(x,y) + d_E(y,z), \min\{\lambda,\gamma,\rho\})$$

$$= d_{FE}((x,\lambda),(y,\gamma) + d_{FE}((y,\gamma),(z,\rho))$$

Consequently, condition (iii)' is satisfied.

Note that the strong fuzzy metric space given above is a set of fuzzy points on the same fuzzy linear space. Different from it, the fuzzy metric space in example 2 comprises fuzzy points belonging to a fuzzy linear space. The difference is caused by that '$\prec$'is replaced by the partial order '$\preceq$'which is must stronger than it.

4. OPEN FUZZY SET.

DEFINITION 4.1: The open fuzzy sphere $S_r(x_0,\lambda_0)$ in induced fuzzy metric space $(P_F(x), d_F)$ of metric (X,d) with centre (x_0,λ_0) and radius r is the subset of $P_F(X)$ defined by

$$S_r(x_0,\lambda_0) = \{(x_0,\lambda_0) : (d(x,x_0), \min(\lambda,\lambda_0)) < r\}$$

DEFINITION 4.2: A subset G of induced fuzzy metric space $(P_F(x), d_F)$ of metric (X,d) is called an open set if given any point (x,λ) in G ,there exist a real number r such that $S_r(x,\lambda) \subseteq G$.

THEOREM 4.1: In any induced fuzzy metric space $(P_F(X), d_F)$ of metric (X,d) ,the empty ∅ and the full space $P_F(X)$ are open sets.

Proof: In order to show that ∅ is open, we must show that each point in ∅ is the centre of an open sphere contained in ∅ ,since there is no point in ∅ ,this is obviously satisfied. $P_F\left(X\right)$ is clearly open, since every open sphere centred on each of its points is contained in $P_F\left(X\right)$.

THEOREM 4.2: In any induced fuzzy metric space $(P_F(X), d_F)$ of metric $\left(X, d\right)$,each open sphere is open set.

Proof: Let $S_r\left(x_0, \lambda_0\right)$ be an open sphere in $\left(P_F\left(X\right), d_F\right)$ and let $\left(x, \lambda\right)$ be a point in $S_r\left(x_0, \lambda_0\right)$ .Now we construct an open sphere centred on $\left(x, \lambda\right)$ and contained in $S_r\left(x_0, \lambda_0\right)$, since

$$d_F\left(\left(x_0, \lambda_0\right), \left(x, \lambda\right)\right) = \left\{d\left(x_0, x\right), \min\left(\lambda_0, \lambda\right)\right\} < r$$

Let $r_1 = r - \left\{d\left(x_0, x\right), \min\left(\lambda_0, \lambda\right)\right\} = r - d_F\left(\left(x_0, \lambda_0\right), \left(x, \lambda\right)\right)$, is a positive real number. We show that $S_{r_1}\left(x, \lambda\right) \subseteq S_r\left(x_0, \lambda_0\right)$. If $\left(y, \lambda_1\right)$ is a point in $S_{r_1}\left(x, \lambda\right)$, so that

$$d_F\left(\left(y, \lambda_1\right), \left(x, \lambda\right)\right) = \left\{d\left(y, x\right), \min\left(\lambda_1, \lambda\right)\right\} < r_1,$$

Then $d_F\left(\left(y, \lambda_1\right), \left(x_0, \lambda_0\right)\right) = \left\{d\left(y, x_0\right), \min\left(\lambda_1, \lambda_0\right)\right\}$

$$d_F\left(\left(y, \lambda_1\right), \left(x_0, \lambda_0\right)\right) \leq \left\{\left(d\left(y, x\right) + d\left(x, x_0\right)\right), \min\left(\lambda_1, \lambda, \lambda_0\right)\right\}$$

$$d_F\left(\left(y, \lambda_1\right), \left(x_0, \lambda_0\right)\right) \leq \left\{d\left(y, x\right), \min\left(\lambda_1, \lambda\right) + d\left(x, x_0\right), \min\left(\lambda, \lambda_0\right)\right\}$$

$$d_F\left(\left(y, \lambda_1\right), \left(x_0, \lambda_0\right)\right) \leq \left\{r_1 + d\left(x, x_0\right), \min\left(\lambda, \lambda_0\right)\right\}$$

$$d_F\left(\left(y, \lambda_1\right), \left(x_0, \lambda_0\right)\right) \leq r - \left\{d\left(x, x_0\right), \min\left(\lambda, \lambda_0\right)\right\} + \left\{d\left(x, x_0\right), \min\left(\lambda, \lambda_0\right)\right\}$$

$$d_F\left(\left(y, \lambda_1\right), \left(x_0, \lambda_0\right)\right) \leq r$$

So, that $\left(y, \lambda_1\right)$ is in $S_r\left(x_0, \lambda_0\right)$.

THEOREM 4.3: In any induced fuzzy metric space $\left(P_F\left(X\right), d_F\right)$,of metric $\left(X, d\right)$. A subset G of $P_F\left(X\right)$ is open if and only if it is union of open sphere.

Proof: We first suppose that G is an open, and we show that it is union of open sphere. If G is empty, it is the union of the empty class of open sphere. If G is non-empty, then it is open ,only when each of its points is the centre of an open sphere contained in it, and it is the union of all the open sphere contained in it.

We now assume that G is the union of a class S of open spheres. We must show that G is open. If S is empty, then G is also empty, and by previous theorem, G is open. Suppose that S is non-empty. Hence G is also non empty. Let (x,λ) be a point in G . Since G is the union of open spheres in S , (x,λ) belongs to an open sphere $S_r(x_0,\lambda_0)$ in S . Hence by theorem 4.2, (x,λ) is the centre of an open sphere $S_{r_1}(x,\lambda) \subseteq S_r(x_0,\lambda_0)$ .Since $S_r(x_0,\lambda_0) \subseteq G$,implies $S_{r_1}(x,\lambda) \subseteq G$ i.e. an open sphere centred on (x,λ) contained in G .Hence G is open.

5. CLOSED FUZZY SET

DEFINITION 5.1: The closed sphere $S_r(x_0,\lambda_0)$ in induced fuzzy metric space $(P_F(X),d_F)$ of metric (X,d) with centre (x_0,λ_0) and radius r is the subset of $P_F(X)$ defined by

$$S_r(x_0,\lambda_0) = \{(x_0,\lambda_0) : (d(x,x_0), \min(\lambda,\lambda_0)) \le r\}$$

DEFINITION 5.2: A subset A of induced fuzzy metric space $(P_F(X),d_F)$ of metric (X,d) is called closed fuzzy set if any point (x,λ) in $P_F(X)$ called a limit point of A if each open sphere centred on (x,λ) contains at least one point of A different from (x,λ).

THEOREM 5.1: In any induced fuzzy metric space $(P_F(X),d_F)$ of metric (X,d) ,the empty set ∅ and full set $P_F(X)$ are closed set.

Proof: Proof is straight forward.

THEOREM 5.2: In any induced fuzzy metric space $(P_F(X),d_F)$ of metric (X,d) ,A subset F of $P_F(X)$ is closed if and only if its complement F' is open.

Proof: We first suppose that F is closed. We have to show that F' is open. If F' is empty, it is open by theorem 4.1, so we may suppose that F' is non-empty. Let (x,λ) be a point in F' .Since F is closed and (x,λ) is not in F i.e. (x,λ) is not a limit point of F .Then there exist an open sphere $S_r(x,\lambda)$ which is disjoint from F . Hence $S_r(x,\lambda)$ is an open sphere centred on (x,λ) and contained in F' ,since $(x.\lambda)$ be an arbitrary point in F' ,thus F' is open .Therefore F is closed.

THEOREM 5.3: Ina any induced fuzzy metric space $(P_F(X),d_F)$ of metric (X,d) ,each closed sphere is closed set.

Proof: Let $S_r(x_0,\lambda_0)$ be a closed sphere in $P_F(X)$ by theorem 5.2, it is sufficient to show that its complement $S_r(x_0,\lambda_0)'$ is open. $S_r(x_0,\lambda_0)'$ is open if it is empty, so we suppose that $S_r(x_0,\lambda_0)'$ is non-empty. Let (x,λ) be a point in $S_r(x_0,\lambda_0)'$

Since, $d_F((x_0,\lambda_0),(x,\lambda)) = \{d(x_0,x), \min(\lambda_0,\lambda)\} > r$

Let $r_1 = \{d(x_0, x), \min(\lambda_0, \lambda)\} - r = d_F((x_0, \lambda_0), (x, \lambda)) - r$, is a positive real number. We take r_1 as radius of open sphere $S_{r_1}(x, \lambda)$ centred on (x, λ), to show that $S_r(x_0, \lambda_0)'$ is open we have to show that $S_{r_1}(x, \lambda) \subseteq S_r(x_0, \lambda_0)'$. Let (y, λ_1) be a point in $S_{r_1}(x, \lambda)$, so that

$d_F((y, \lambda_1), (x, \lambda)) = \{d(y, x), \min(\lambda_1, \lambda)\} < r_1$, on the basis of this we have.

$$d_F((x_0, \lambda_0), (x, \lambda)) \le d_F((x_0, \lambda_0), (y, \lambda_1)) + d_F((y, \lambda_1), (x, \lambda))$$

i.e. $d_F((y, \lambda_1), (x_0, \lambda_0)) \ge d_F((x_0, \lambda_0), (x, \lambda)) - d_F((y, \lambda_1), (x, \lambda))$

$$d_F((y, \lambda_1), (x_0, \lambda_0)) \ge d_F((x_0, \lambda_0), (x, \lambda)) - r_1$$

$$d_F((y, \lambda_1), (x_0, \lambda_0)) \ge d_F((x_0, \lambda_0), (x, \lambda)) - \{d_F((x_0, \lambda_0), (x, \lambda)) - r\}$$

$d_F((y, \lambda_1), (x_0, \lambda_0)) \ge r$, so that (y, λ_1) is in $S_r(x_0, \lambda_0)'$

Hence the theorem.

6. CONTINUOUS FUZZY MAPPINGS.

DIFINITION 6.1: Let $P_{F_1}(X)$ and $P_{F_2}(Y)$ be any fuzzy metric spaces induced by metrics d_1 and d_2, and let f be a mapping of $P_{F_1}(X)$ into $P_{F_2}(Y)$. The mapping f is said to be continuous at a point (x_0, λ_0) in $P_{F_1}(X)$ if either of the following equivalent conditions is satisfied.

(i) for each $\varepsilon > 0$, there exists $\delta > 0$ such that
$d_{F_1}((x, \lambda), (x_0, \lambda_0)) < \delta \Rightarrow d_{F_2}((f(x, \lambda), f(x_0, \lambda_0))) < \varepsilon$

(ii) for each open sphere $S_\varepsilon(f(x_0, \lambda_0))$ centred on $f(x_0, \lambda_0)$ there exists an open sphere $S_\delta(x_0, \lambda_0)$ centred on (x_0, λ_0) such that $f(S_\delta(x_0, \lambda_0)) \subseteq S_\varepsilon(f(x_0, \lambda_0))$.

THEOREM 6.1: Let $P_{F_1}(X)$ and $P_{F_2}(Y)$ be any two fuzzy metric spaces and f is a mapping of $P_{F_1}(X)$ into $P_{F_2}(Y)$. Then f is continuous if and only if for any open set G in $P_{F_2}(Y)$, there exists an open set $f^{-1}(G)$ in $P_{F_1}(X)$.

Proof : We first suppose that f is continuous. If G is an open set $P_{F_2}(Y)$. Then we have to show that $f^{-1}(G)$ is open in $P_{F_1}(X)$. If $f^{-1}(G) = \phi$, then $f^{-1}(G)$ is obviously open. Let us suppose that $f^{-1}(G) \ne \phi$, and (x, λ) be a point in $f^{-1}(G)$, then $f(x, \lambda)$ is in G, since G is open, there exists an open sphere $S_\varepsilon(f(x, \lambda))$ centred on $f(x, \lambda)$, contained in G. Since f is continuous, there exists an open sphere $S_{\delta}(x,\lambda)$ such that $f(S_{\delta}(x,\lambda)) \subseteq S_{\varepsilon}(f(x,\lambda))$,since $S_{\varepsilon}(f(x,\lambda)) \subseteq G$,we also have $f(S_{\delta}(x,\lambda)) \subseteq G$,hence $(S_{\delta}(x,\lambda)) \subseteq f^{-1}(G)$.Therefore $S_{\delta}(x,\lambda)$ is an open sphere centred on (x,λ) contained in $f^{-1}(G)$, so $f^{-1}(G)$ is open.

Conversely, suppose that $f^{-1}(G)$ is open in $P_{F_1}(X)$ whenever G is open in $P_{F_2}(Y)$.We have to show that f is continuous. Now to show that f is continuous at an arbitrary point (x,λ) in $P_{F_1}(X)$. Let $S_{\varepsilon}(f(x,\lambda))$ be an open sphere centred on $f(x,\lambda)$. This open sphere is open set, so its inverse image is an open set which contain (x,λ) there exists an open sphere $S_{\delta}(x,\lambda)$, which contained in this inverse image. It is clear that $f(S_{\delta}(x,\lambda))$ is contained in $S_{\varepsilon}(f(x,\lambda))$,so f is continuous at (x,λ),since (x,λ) be arbitrary, hence f is continuous mapping.

THEOREM 6.2: Let $P_{F_1}(X)$ and $P_{F_2}(Y)$ be metric spaces and f be a mapping of $P_{F_1}(X)$ into $P_{F_2}(Y)$. Then f is continuous if and only if $f^{-1}(F)$ is closed in $P_{F_1}(X)$ whenever F is closed in $P_{F_2}(Y)$.

Proof: Let the mapping $f: P_{F_1}(X) \to P_{F_2}(Y)$ is continuous. Let F be a closed set in $P_{F_2}(Y)$.Then $F^{'}$ is open set in $P_{F_2}(Y)$, now we have to show that $f^{-1}(F^{'})$ is open in $P_{F_1}(X)$. If $f^{-1}(F^{'}) = \phi$,then $f^{-1}(F^{'})$ is obviously open. Let $f^{-1}(F^{'}) \neq \phi$ and (x,λ) be any arbitrary point in $f^{-1}(F^{'})$.Since $f^{-1}(F)$ is closed and (x,λ) is not in $f^{-1}(F)$, therefore (x,λ) is not a limit point of $f^{-1}(F)$. Then there exists an open sphere $S_r(x,\lambda)$ which is disjoint from $f^{-1}(F)$. Hence $S_r(x,\lambda)$ is an open sphere centred on (x,λ) and contained in $f^{-1}(F^{'})$,since (x,λ) be arbitrary point of $f^{-1}(F^{'})$,hence $f^{-1}(F^{'})$ is open, therefore $f^{-1}(F)$ is closed.

Conversely, suppose that $f^{-1}(F^{'})$ is open in $P_{F_1}(X)$ whenever $F^{'}$ is open in $P_{F_2}(Y)$,for any closed set F in $P_{F_2}(Y)$. We have to show that f is continuous. Now to show that f is continuous at arbitrary point (x,λ) in $f^{-1}(F^{'})$.Since $f^{-1}(F^{'})$ is open set in $P_{F_1}(X)$. Let $S_{\varepsilon}(f(x,\lambda))$ be an open sphere centred on $f(x,\lambda)$.This open sphere is open set, so its inverse image is an open sphere $S_{\delta}(x,\lambda)$ which contained in this inverse $f^{-1}(F^{'})$. It is clear that $f(S_{\delta}(x,\lambda))$ is contained in $S_{\varepsilon}(f(x,\lambda))$.So f is continuous at (x,λ),since (x,λ) is arbitrary, therefor f is continuous mapping.

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