An infected individual makes contact and is able to transmit the disease to βN others per unit time and the fraction of contacts by an infected with a susceptible is S/N. The number of new infections in unit time per infective then is (βN)(S/N), giving the rate of new infections (or those leaving the susceptible category) as (βN)(S/N)I = βSI [3]. For the second and third equations, consider the population leaving the susceptible class as equal to the number entering the infected class. However, a number equal to the fraction (γ) which represents the mean recovery rate, infective are leaving this class per unit time to enter the removed class. A widely accepted idea is that the rate of contact between two groups in a population is proportional to the size of each of the groups concerned [3,4]. Finally, it is assumed that the rate of infection and recovery is much faster than the time scale of births and deaths and therefore, these factors are ignored in this model.

SIR Model Open Population Model

Open Population Model

The modified model is then extended to open population where birth rate and death rate are introduced with N = S(t) + I(t) + R(t) + b(t) + u(t) where b(t) is the birth rate and u(t) is the death rate.The assumptions are as follows:

1) The population is open (There is immigration and emigration).

2) The only way a person can leave the susceptible group is to become infected. The only way a person can leave the infected group is to recover from the disease. Once a person has recovered, the person receives immunity.

3) Age, sex, social status, and race do not affect the probability of being infected.

4) There is no inherited immunity.

5) The members of the population mix homogeneously (have interactions with one another to the same degree).

6) Birth and Death can occur.

7) Entry into the population is through birth.

8).Death in the population is caused by the disease only.

Here we assume N = S + I + R+ b – u where b and u are the numbers of births and deaths respectively. The equations used in generating the data are;

$$S_{(t+\Delta_t)} = S_{(t)} - \beta\Delta_t S_{(t)} + b\Delta_t N_t - uS_t \qquad \text{.... (iv)}$$

$$\frac{dS}{dt} = -\beta S + bN - uS$$

$$I_{(t+\Delta t)} = I_t + \beta \Delta_t S_t - \gamma \Delta_t I - u \Delta_t I_t \qquad \textbf{.... (v)}$$

$$\frac{dI}{dt} = \beta S - \gamma I - uI$$

$$R_{(t+\Delta t)} = R_{(t)} + \gamma \Delta_t I_t - u \Delta_t R_t \qquad \textbf{.... (vi)}$$

$$\frac{dR}{dt} = I\gamma - uR$$

For Susceptible

$$S_{(t+\Delta_t)} = S_{(t)} - \beta \Delta_t S_{(t)} + b \Delta_t N_t - uS_t$$

$$S_{(t+\Delta_t)} = S_{(t)} - \beta \Delta_t S_{(t)},$$

If we equate equation (i) and (iv)

Then $bN_t = US_t$

U = bNt/S

For Infective

$$I_{(t+\Delta t)} = I_t + \beta \Delta_t S_t - \gamma \Delta_t I - u \Delta_t I_t$$

$$I_{(t+\Delta_t)} = I_t + \beta \Delta_t S_t - \gamma I_t \Delta_t$$

If we equate equation (ii) and (v)

$UI_t = 0$

For Recovery

If we equate equation (iii) and (vi)

$$R_{(t+\Delta t)} = R_{(t)} + \gamma \Delta_t I_t - u \Delta_t R_t$$

$$R_{(t+\Delta t)} = R_{(t)} + \gamma I_t \Delta_t$$

If we equate equation (i) and (iv)

$UR_t = 0$

Since $UI_t = 0$ and $UR_t = 0$

Then $I_t = R_t$

The new model can be written as;

$$\textbf{Since } U = \frac{bN_t}{S_t}, S_t = \frac{bN_t}{U} \quad , \quad b = \frac{US_t}{N_t}$$

$$S_{ttDt} = S_t - \beta S_t - bN_t - US_t$$

$$S_{t+Dt} = S_t - \beta D_t S_t - bD_t N_t - UD_t S_t$$

$$S_{t+Dt} = \frac{bN_t}{U} - \frac{\beta bN_t}{U} - bN_t - \frac{bN_t S_t}{S_t}$$

$$= \frac{bN_t}{U} - \frac{\beta bN_t}{U} - bN_t - bN_t$$

$$= \frac{bN_t}{U} - \frac{\beta bN_t}{U} - 2bN_t$$

$$= bN_t \left[\frac{1}{U} - \frac{\beta}{U} - 2\right]$$

$$= \frac{bN_t}{U}[1 - \beta - 2U]$$

With this model we come up with following assumption;

i. the number recover is equal to the number infected
ii. only birth in the population are susceptible this implies that other peoples in the population are expose to the infection

Where;

S = susceptible

I = Infection

R = Recovery

B = Contact Rate

N = Total Population

b = Birth Rate

u = Death Rate

CONCLUSION

From the result, it shows that only birth in to the population are suscestible which implies that all people in the population were exposed to particular disease Also the equation for the infectives indicate that there is no death and that all people that infected are liable to recover which means that all number that infected were recovered .

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