

Development of an energy detection algorithm in MATLAB when Noise power is estimated

By

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Abstract

The Development of an energy detection algorithm in MATLAB when Noise power is estimated in a security system to identify a signal. The performance of the energy detector is heavily dependent on the estimate of the noise floor, as an inaccurate estimate will degrade the performance of the detector.

Before the design of the Energy Detector, a spectrum sweep was conducted in Matlab environment to observe the available channels in order to test the accuracy of the design. The noise power is estimated based on the observation of the received signal in the noise. the designed energy detection algorithm in a MATLAB, when Noise power is estimated, was able to filter the noise in the signal by approximation at -40.53dB.

Keyword: Energy Detector, Noise power, Algorithm, Matlab.

1. Introduction

The demand of allocating and using the radio frequency spectra is rapidly growing due to increasing number of wireless applications. Unlicensed user or secondary user may utilise this band when licensed user is absent. To encourage the proficient use of spectrum, the concept of Cognitive Radio (CR) has been proposed by [1]. Energy detection has the advantage of being low in complexity, and efficient for implementation using the fast Fourier transform algorithm. Energy detection algorithms do not require prior information about the signals to be detected, and therefore can always be applied for blind detection. An inaccurate estimate of in-band noise power can reduce performance from the energy detector. For example, if the noise power estimate is too high, the chosen detection threshold will be higher, causing a decreased probability of signal detection. Different studies in [2] quantifies the effect of uncertain noise power estimates on the performance of energy detectors. Noise power estimation can be difficult in practice, especially since (1) the power of noise generated by RF front ends can vary over frequency, (2) the noise floor can also consist of external (background) noise in addition to thermal noise, and (3) thermal noise power varies over temperature.

Cyclo-stationary detection does not require a noise power estimate. Cyclo-stationary detection can also be used for signal classification, as shown from the study [3]. Cyclo-stationary detection is thought of as the more robust technique by many researchers since noise does not exhibit spectral correlation (SCD is zero). However, this only

applies to the “true” SCD, which in theory involves sensing for an infinite amount of time. Practical estimates of the SCD use a finite sensing time causing the SCD estimate for noise to be small but not zero. Increased sensing time increases the performance of cyclo-stationary detection but also increases the performance of energy detection since more Fourier transforms averages produce a flatter noise spectrum. Existing research has shown that cyclo-stationary feature detection requires longer sensing time to achieve the same performance as energy detection [4]. As mentioned in [5 and 3], some cyclostationary detection algorithms require prior knowledge of the signals to be detected, which is not useful for blind detection. Additionally, the entire bandwidth of a signal must be captured for spectral correlation to be seen. Partially captured signals may not exhibit any spectral correlation.

For example, if the receiver’s bandwidth only covers half of the bandwidth of a signal, spectral correlation features may be completely non-existent in the SCD estimate. Computation of the SCD also involves higher computational complexity leading to increased use of hardware/software resources in implementation. Cyclo-stationary detection algorithms include blind detection algorithms and non-blind detection algorithms. In blind detection algorithms, no prior information such as carrier frequency, modulation, and bandwidth are required about the signals to be detected. Non-blind techniques are not useful for the general-purpose spectrum sensing needed by cognitive radios. Existing work in blind detection algorithms including [5], which uses the squared magnitude of the spectral coherence as a detection statistic, then uses a threshold test to determine if a signal is present.

For blind detection, the work reported in [3] suggests using the crest factor of the domain profile as a detection statistic, where the domain profile is defined as the maximum observed magnitude for each. The crest factor (CF) is defined as the ratio of peak amplitude to root-mean-square amplitude:

For non-blind detection, the work in [4] suggests integrating the SCD over a “feature mask” to detect signals, where the feature mask restricts the integration to the range of values for values where peaks/features are expected. This requires the receiver to know before the frequency and theoretical SCD of the input signal, and therefore cannot be used for blind detection.

2. Theoretic Background

After band-pass filtering over a bandwidth (W), the received signal is shown in Eq. (1) [6].

$$r(t) = \begin{cases} v(t) & \mathcal{H}_o \\ s(t) + v(t) & \mathcal{H}_i \end{cases} \quad (1)$$

Given that: $v(t)$ represents the Gaussian noise with one-sided power spectral density, N_o over the considered band and zero elsewhere, $s(t)$ is the bandpass received signal. H_o is the signal not present and H_i is the signal present.

The noise power is given as [6] Eq. (2):

$$\sigma^2 = \mathbb{E}\{v^2(t)\} = N_o W \quad (2)$$

where \mathbb{E} is the epsilon.

The energy detector would measure the received signal energy for a time period T (which for simplicity would start at $t = 0$) and the energy would be compared to a threshold as shown in Eq. (3) [7].

$$\tilde{T} = \int_0^T r(t)^2 dt \geq \xi \tag{3}$$

After down conversion and sampling at time t_i , the discrete-time equivalent low-pass form of Eq. (1) is denoted as shown in Eq. (4) [6].

$$y_i = \begin{cases} n_i & \mathcal{H}_o \\ x_i + n_i & \mathcal{H}_i \end{cases} \tag{4}$$

where $y_i = y(t_i)$, $x_i = x(t_i)$ and $n_i = n(t_i)$ and $y(t)$, $x(t)$ and $n(t)$ are the equivalent low-pass representations of $r(t)$, $s(t)$ and $v(t)$ respectively.

Equation (3) can be expressed as a normalized dimensionless metric by dividing by $N_o/2$ (average spectral energy of noise) so that the discrete form becomes as shown in Eq. (5) [7].

$$\frac{1}{\sigma^2} \sum_{i=1}^N |y_i|^2 \tag{5}$$

Where accuracy improves, with increasing $N \approx 2TW$ assuming $TW \gg 1$ so that the difference between the time discrete and time-continuous versions of the Energy Detection (ED) in Eq. (5) is negligible when compared to Eq. (3). The metric in Eq. (5) is called the test statistics. Hence, in the time-discrete implementation, the (ED) becomes Eq. (6) (Mariani, 2010).

$$V \triangleq \frac{1}{\sigma^2} \sum_{i=1}^N |y_i|^2 \begin{matrix} \mathcal{H}_i \\ \geq \xi \\ \mathcal{H}_o \end{matrix} \tag{6}$$

It should be noted that for a given T , W and a required P_{FA} , the threshold, ξ is set based on the noise power, σ^2 . In a practical situation, σ^2 is estimated as $\hat{\sigma}^2$ [8,9 and 10] such that ED statistics becomes Eq. (7) [6].

$$\hat{V} = \frac{1}{\hat{\sigma}^2} \sum_{i=1}^N |y_i|^2 \tag{7}$$

2.1 Energy Detected with Estimated Noise Power

The receiver can evaluate the maximum likelihood (ML) noise power estimate is shown in Eq. (11) [6].

$$\hat{\sigma}^2 = \frac{1}{2M} \sum_{i=1}^M |n_{-i}|^2 \tag{11}$$

Hence the Energy detector metric or statistics is shown in Eq. (12) [6].

$$\hat{V} = \frac{1}{\hat{\sigma}^2} \sum_{i=1}^N |y_i|^2 = 2N \frac{\frac{1}{2N} \sum_{i=1}^N |y_i|^2}{\frac{1}{2M} \sum_{i=1}^M |n_{-i}|^2} \tag{12}$$

Firstly, it should be noted that the numerator and denominator are different time windows. Also, since y_i and n_{-i} are Gaussian distributed, the metric \hat{V} is the ratio of two chi-squared distribution and can be written with the proper scaling in terms of Fisher distribution (F-distribution). Considering unknown deterministic signal $s(t)$ whose y_i samples is Gaussian distributed, applying proper scaling makes the

hypotheses \mathcal{H}_0 and \mathcal{H}_1 to change slightly such that by specifying with Fisher distribution $\mathcal{F}_{n,m}(x)$ with degrees N, M and non-centrality parameter λ gives Eq. (13) [6].

$$\frac{\hat{V}}{2N} \sim \begin{cases} \mathcal{F}_{2N,2M}(0), \mathcal{H}_0 \\ \mathcal{F}_{2N,2M}(\lambda), \mathcal{H}_1 \end{cases} \quad (13)$$

Therefore, \hat{V} has probability density function (pdf) given as Eq. (14) and Eq. (15)

$$f_{\hat{V}}(x) = \frac{(2M)^M x^{N-1}}{B(N,M)(x+2M)^{N+M}} \quad (14)$$

and

$$f_{\hat{V}}(x) = f_{\hat{V}}(x) e^{-\lambda/2} {}_1F_1(N+M; N; \frac{\lambda x}{2x+2M}) \quad (15)$$

Where, ${}_1F_1(.,.;.)$ is the confluent hypergeometric function, $u(.)$ is the unit step function and $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ is the beta-function.

2.2 Distribution of Test Statistics

Under \mathcal{H}_0 and \mathcal{H}_1 , the probability density function has been given hence the probability of false alarm is shown in Eq. (16) [6].

$$P_{FA} = \frac{B(M,N,2M/2M+\xi)}{B(M,N)} = \tilde{B}(M,N,2M/2M+\xi) \quad (16)$$

Where $B(a, b, z) = \int_0^z x^{a-1}(1-x)^{b-1} dx$ with $(\Re\{a\} > 0, \Re\{b\} > 0$ and $|z| < 1)$ is the beta regularized function.

Also, \mathcal{H}_1 considering the non-central Fisher distribution, the probability of detection is given as Eq. (17).

$$P_D = e^{-\lambda/2} \sum_{k=0}^{\infty} \frac{(N+M)_k}{(M)_k} \left(\frac{\lambda}{2}\right)^k \frac{1}{k!} \tilde{B}(N+k, M, 2M/2M+\xi) \quad (17)$$

Given that $(a)_k$ is the Pochhammer symbol given as $(a)_k = a(a+1) \dots (a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)}$ and Eq. (17) was obtained by the series version of the confluent hypergeometric function.

3. Energy Detection Algorithm in MATLAB

The noise power is implemented with MATLAB using RTL-SDR in the FM spectrum as in real-time systems, where noise power is estimated based on the observation of the received signal in the noise. It was assumed that the receiver has available a number of noises only samples M . For simplicity of notation, assume that the samples before the detection window t_1, \dots, t_N do not contain the signal so that n_{-1}, \dots, n_{-M} , can be denoted as the noise only samples used for noise power estimation making reference to Eq. (10) to Eq. (16) to get the probability of false alarm.

For the case of estimated noise power (σ^2) then the calculated threshold is show in Eq. (18)

$$\xi^* = \{[\tilde{B}^{-1}(P_{FA}; M, N)]^{-1} 2M\} - 2M \quad (18)$$

Given that $P_{FA} = f(M, N, \xi^*)$ where $f(\dots) = \tilde{B}\left(M, N, \frac{2M}{2M + \xi}\right)$

While for minimum SNR the estimated noise power is giving as Eq. (19):

$$SNR_{\min} = \left(\frac{D - F}{F(1 - D)} \right) \tag{19}$$

Where $F = \text{Inv}\tilde{B}(M, N, P_{FA}^{DES})$, $D = \text{Inv}\tilde{B}(M, N, P_D^{DES})$ and $\text{Inv}\tilde{B}(\dots)$ is an inverse beta regularized function (if $w = \tilde{B}(a, b, z)$, then $\text{Inv}\tilde{B}(a, b, w) = z$).

4. Implementation in MATLAB

Before the design of the ED was implemented, a spectrum sweep was done in the primary spectrum in latitude 6.3999 longitude 5.6138 location from **87.5MHz to 108MHz**. This spectrum consists of 100 channels as a channel should spans 200kHz. This spectrum sweep was done in MATLAB to observe the available channels in latitude 6.3999 longitude 5.6138 location in order to test the accuracy of the design.

The spectrum sweep was performed with a single RTL-SDR (those based on R820T and Realtek RTL2832U DVB-T coded orthogonal FDM or COFDM) which receives from 25 MHz to 1.75GHz portion of the RF spectrum. Here, “sweep” as a repetitive process of timing and retuning the RTL-SDR to different frequencies in order to obtain spectral information is defined. Each centre frequency f_c , selected during the retuning process would be 3.2MHz higher than the previous centre frequency $f_{c(i+1)} = f_{c(i)} + 3.2\text{MHz}$ and then fifteen retunes would result in a 48MHz wide band of information with none of the data captured frequency bands overlapping.

Now moving on to the ED designed using the RTL-SDR in the primary spectrum, the energy detection algorithm was designed for 100 channels of individual centre frequencies in a linear switching manner as such **CH₁, CH₂ ... CH₁₀₀**. This was done in order to show the results in the frequency domain (frequency band spectrum sensing) in which the results were compared to that of the spectrum sweep information of the sweeping FFT MATLAB code (both codes for FFT spectrum sweep and that of energy detection are written in the same MATLAB program).

Having implemented an energy detector with a deterministic signal of noise power estimated, it is now time to test it on RF signal of unknown noise power. In addition to the energy detection and FFT sweeping design, some active channels in latitude 6.3999 longitude 5.6138 were tested with the energy detector algorithm for 38400 samples from the RTL-SDR (resulting in a time period of 38400/sample rate which matched approximately with 2 seconds) to determine the primary station’s time activity. The information of time domain sensing is needed to optimise the channel selection algorithm. The ED for the RF signal received by RTL-SDR is simply an extension of the ED algorithm previously discussed and implemented in MATLAB for a deterministic unknown signal. Assuming an AWGN channel while satisfying IEEE 802.22 WRAN condition, $P_{FA}^{DES}, P_{MD}^{DES} \leq 0.1$ or 10%. Hence, the following parameters were computed as before while considering the case of estimated noise power: Number of samples, N; Threshold, ξ ; and Number of noise-only samples M for SNR wall of -10dB.

5. Result

Spectrum sweep and energy detection estimated noise using RTL-SDR

Figure 1 shows the spectrum sweep across the band of FM broadcast in latitude 6.3999 longitude 5.6138. The blue plot is the frequency plot whose amplitude is measured in dB relative to 50 Ohms load while the plot in orange is that of linear scale amplitude. With plot in place a proper zoom an average will do the trick of an estimated noise level at 25dB gain. The approximation is given as -40.53dB as shown in appendix A11. The sweep was done to compare the efficiency of the energy detector; the plots are Figures 2 to 3 for different sample rates.

The results showed in Figures 1 and 2 compares the energy detector considering 400 kHz bandwidth of received FM samples and 150 kHz bandwidth of samples. This is done to show the compromise done in measurement and how correlated the result remain. Considering 200 kHz of channel bandwidth, the minimum sampling rate is 400 kHz or 400 kS/s however for the RTL-SDR used the allowable sample rates are split in two ranges 225000 S/s to 300000 S/s (only 300000 S/s inclusive) and 900000 S/s to 3.2 MS/s (only 3.2 MS/s inclusive). With these range of allowed sample rate, the best choice of sampling rate is a little more than 900 kHz and this means two channels will be considered which was notice in Figures 1 and 2.

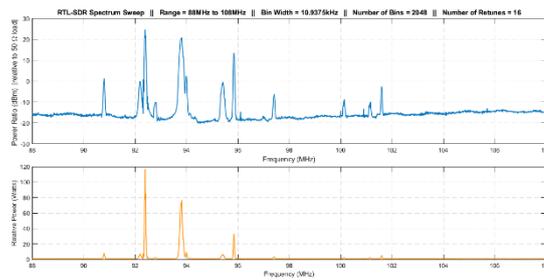


Figure 1: Spectrum sensing in the FM spectrum relative to 50 Ohms load

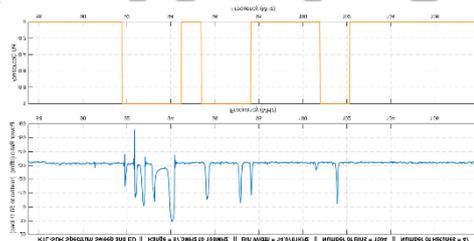


Figure 2: Spectrum sweep and energy detector with estimated noise power considering approximately 400 kHz bandwidth

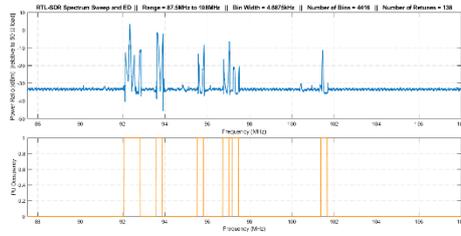


Figure 3: Spectrum sweep and energy detector with estimated noise power considering 150 kHz bandwidth

Apart from testing the energy detector in the frequency domain, test was also done in time domain as well to measure the noise at different channel. The following are some results for some FM stations in latitude 6.3999 longitude 5.6138 location and their respective duty cycles measured for 2 seconds are shown in Figures 4 to 12.

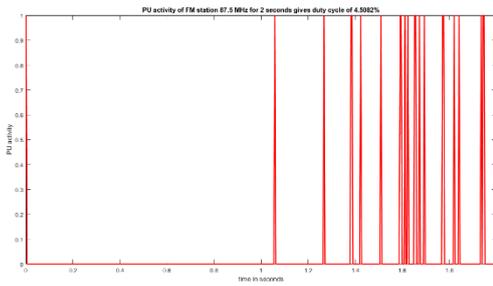


Figure 4: Primary User activity in 87.5MHz FM station showing 4.5082% activity

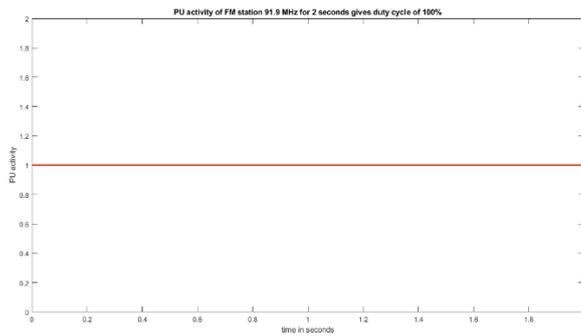


Figure 5: Primary User activity in 91.9 MHz FM station showing 100% activity

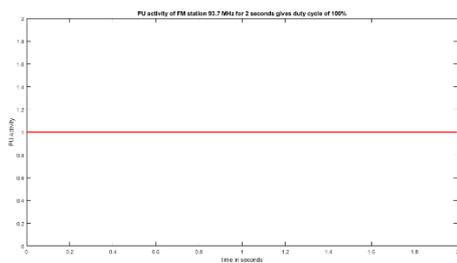


Figure 6: Primary User activity in 93.7 MHz FM station showing 100% activity

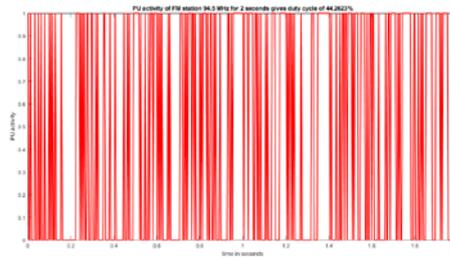


Figure 7: Primary User activity in 94.5 MHz FM station showing 44.2623% activity

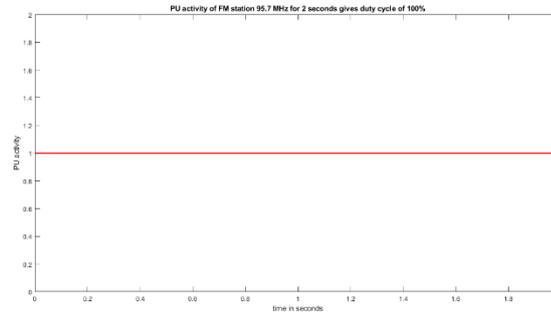


Figure 8: Primary User activity in 95.7 MHz FM station showing 100% activity

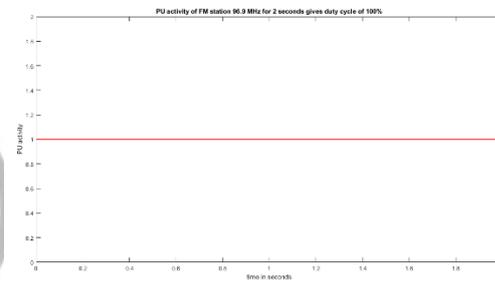


Figure 9: Primary User activity in 96.9 MHz FM station showing 100% activity

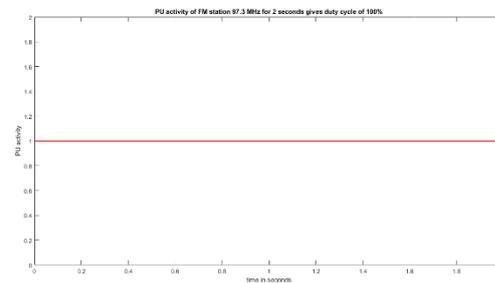


Figure 10: Primary User activity in 97.3MHz FM station showing 100% activity

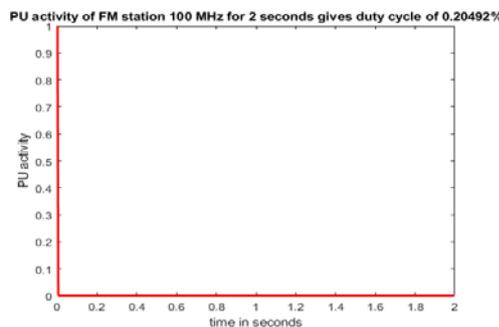


Figure 11: Primary User activity in 100MHz FM station showing 0.20492% activity

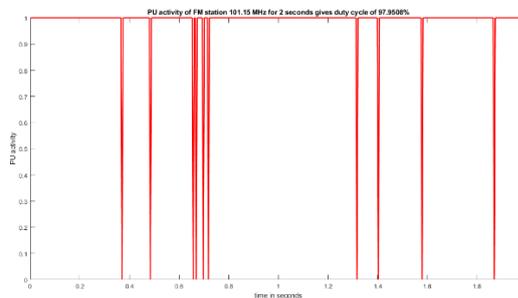


Figure 12: Primary User activity in 101.15MHz FM station showing 97.9508% activity

Alternatively, 150kHz was also consider of channel bandwidth and take the risk of aliasing some high frequency signals and this can be seen in appendix A15, the FFT of a channel is split. A15 MATLAB window showing error message when 400 kHz sampling rate was used with the RTL-SDR. Based on the results, a few other observations were made, the performance of the energy detector majorly depended on the accuracy of the estimated noise floor which changes with time and also with the gain of the RTL-SDR. This leads to an interesting question. How did we estimate the noise level or noise power? The Figures 13 to 17 contain the answer but not without some explanation. There was a separate MATLAB file for visualization of an FM channel at 97.3 MHz and noise estimation. The visualization file help to visualize the capture samples using the spectrum analyzer in frequency domain and in both frequency and time domain with a colour plot (called spectrogram). This plot is shown in Figure 13. It was notice that there are two channels within the plot due to sample rate of 901 kHz, but the noise level exists between these channels. So, firstly, the sampled unwanted channel (not 97.3 MHz) was filtered out and the result can be seen for a single snapshot of received samples from the SDR in Figure 14.

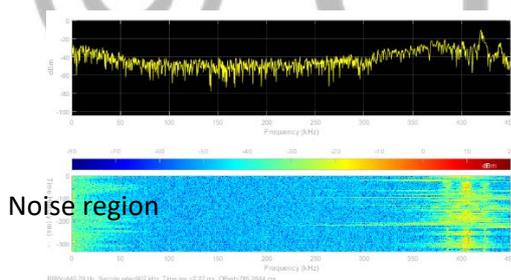


Figure 13: Visualization of FM channel at 97.3 MHz to estimate noise floor

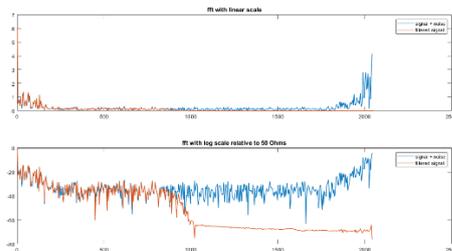


Figure 14: FFT of a single frame 4096 sample for analysis of noise floor showing linear and log scale plots

With plot in place a proper zoom an average will do the trick of an estimated noise level at 25dB gain. The approximation is given as -40.53dB as shown in A11 by the data point on the noise level line in Figure 15.

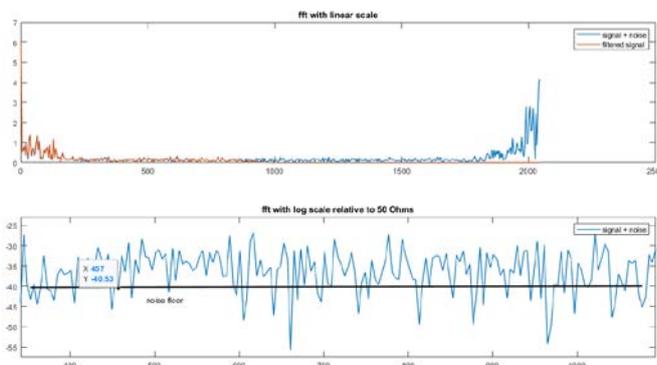


Figure 15: FFT of single frame 4096 samples showing noise floor estimate and value in dB

Figures 16 and 17 show the probability distribution plot of the 901 kHz sampled signal shown by the red plot (the spikier plot), followed by the filtered FM channel signal at 97.3 MHz and lastly the estimated -40.53dB noise signal (the orange plot) all plotted side by side. There is a low SNR between the 97.3 MHz signal \mathcal{H}_1 and the estimated noise \mathcal{H}_0 and our energy detector senses it.

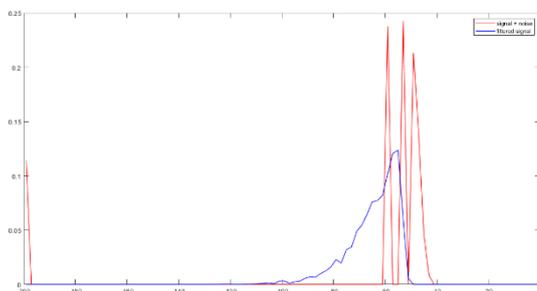


Figure 16: Plot of probability distribution of filtered and unfiltered received signal

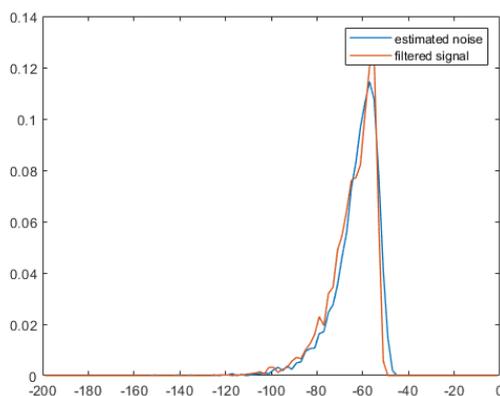


Figure 17: Plot of probability of filtered received signal versus estimated noise at -40.53dB

A few observations can be made from Figures 4 – 12 in section results. First, observed was that figures 5, 6, 8, 9 have 100% duty cycle that means at the time of the analysis signal was present, it was observed that Figures 4, 7, 11 and 12 at PU activity 1 (H_1) the

activity did not get to 100% of signal present, that means the duty cycle correlated to the SNR of the received signal, for instance the SNR of FM station 101.15MHz is relatively low as shown in Figure 18, the higher the SNR the higher the duty cycle for the estimated noise level.

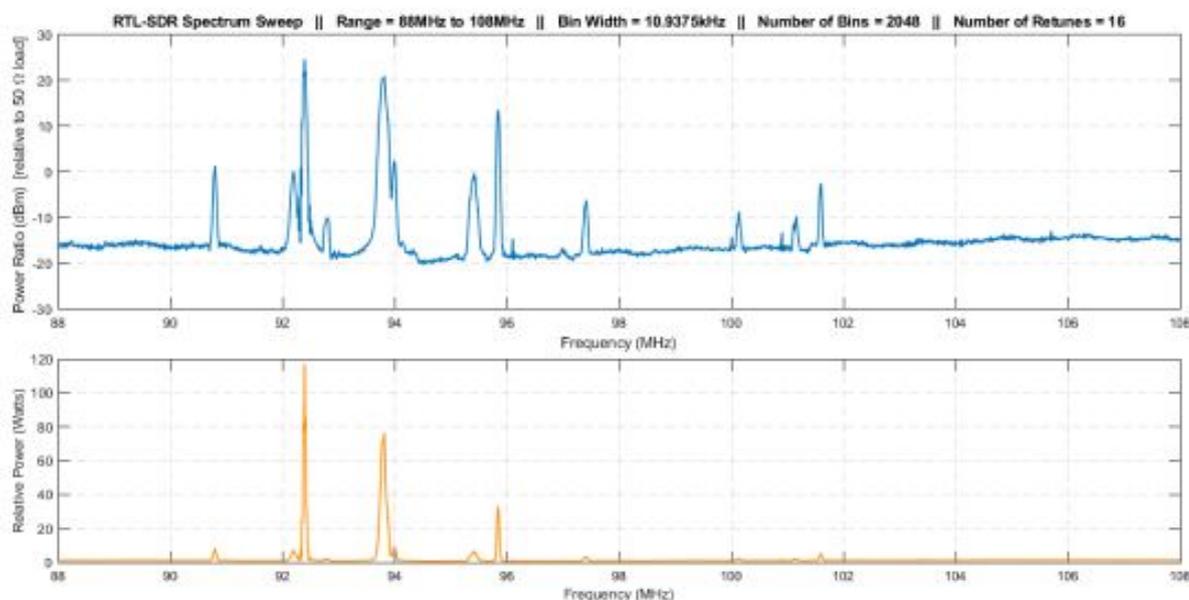


Figure 18: Spectrum sweep showing the FM station 101.15MHz

Conclusion

Based on the results, a few other observations were made, the performance of the energy detector majorly depended on the accuracy of the estimated noise floor which changes with time and also with the gain of the RTL-SDR. There was a separate MATLAB file for visualization of an FM channel at 97.3 MHz and noise estimation. The visualization file helps to visualize the capture samples using the spectrum analyzer in frequency domain and in both frequency and time domain with a colour plot (called spectrogram). This plot shows two channels within the plot due to sample rate, but the noise level exists between these channels. So, firstly, the sampled unwanted channel was filtered out and the result can be seen for a single snapshot of received samples from the SDR.

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