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INDEPENDENCE AND CLIQUE POLYNOMIALS OF ZERO-DIVISOR GRAPH OF THE INTEGER MODULO n ($\mathbb{Z}_n$)

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ABSTRACT

The independent set of a graph is the set of vertices which are not adjacent in a graph while the clique of a graph is the set of vertices in the graph which are adjacent in the graph. In this paper we study the independence polynomial and clique polynomial of zero-divisor graphs of the integers modulo n. The independence polynomial of a graph is a polynomial with coefficient as the number of independent sets in the graph. And clique polynomial is a polynomial with coefficients as the number of cliques in the graph. The work considered the zero-divisor graphs of five commutative rings; $\mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15}, \mathbb{Z}_{16}$, where the independence and clique polynomials of each graph are computed.

Keywords : Clique, Clique number, Clique polynomial, Commutative ring, Independence number, Independence polynomial, Independent set, Zero-divisor graph.

INTRODUCTION

Integers modulo n ($\mathbb{Z}_n$, $n \in \mathbb{N}, n \geq 2$) is a ring with the binary operations of addition and multiplication modulo n. $\mathbb{Z}_n$is a commutative ring with unity 1. There are many works on graphs of commutative rings.

Graph denoted as G= (V, E) consists of set of vertices V and edges E; set of vertices in a graph are said to be adjacent if they are joined by an edge while edge is a pair of elements of the vertices. This work considered simple and undirected graphs. Simple graphs are graphs that have no edges joining vertices to themselves. A graph is called undirected if the edges have no orientation. A graph with a certain number of vertices and no single edge is called an empty graph denoted as E_n, an empty graph with zero vertex is a null graph; written as $E_\emptyset$[5].

In 1999 Anderson and Livingston [2] introduced the zero-divisor graph of commutative rings; where they considered the set of vertices as the set of zero-divisors of the ring and in which two vertices x and y are said to be adjacent in the graph iff $xy = 0$. Anderson and Badawi's work in 2008 [3] introduced and investigated the total graph and regular graph of a commutative ring. They considered all the elements of the ring as set of vertices and any two vertices x and y are said to be adjacent in the graph iff $x + y \in Z(R)$, with $Z(R)$ as the set of zero-divisors of the ring R. As for Ahmad Abbasi [7], his work in 2012 was on the T-graph of a commutative ring. Where he considered all the elements of the ring R as vertices, with two vertices $x\ and\ y$ being adjacent iff $x + y \in S(I)$. Using I as a proper ideal of R and$S(I) = \{a \in R/\ ra \in I\ for\ some\ r \in R - I\}$.

In this paper we constructed the zero-divisor graph of integers modulo n and used the concept put forward by Hoede and Li [1] to compute the independence polynomial and clique polynomial of the graphs constructed. There were many researches also on polynomials of graphs of some groups. Najmuddin et al computed the independence polynomials of conjugate graph and non-commuting graph of dihedral groups in 2017 [10], they also computed the independence polynomials of nth- central graphs of dihedral groups in the same year [9]. In 2018 Najmuddin et al worked on the independence and clique polynomials of the conjugacy class graphs of dihedral groups [11].

We presented this paper in four sections; starting with introduction, section 2 gives the preliminaries on the work, section 3 is where the zero-divisor graphs are constructed, the independence and clique polynomials are computed in section 4.

PRELIMINARIES

This section is where we look at some concepts in graph and group theories that aid in computing the independence and clique polynomials of zero-divisor graphs of the ring of integers modulo n ($\mathbb{Z}_n$).

DEFINITION 2.1 INDEPENDENT SET [1]

The independent set of vertices of a graph is the sets of vertices which are not adjacent in the graph.

DEFINITION 2.2 INDEPENDENCE NUMBER [1]

Independence number of a graph is the number of elements in the independent set with the highest cardinality.

DEFINITION 2.3 INDEPENDENCE POLYNOMIAL [1]

Independence polynomial of a graph G is the polynomial in x whose coefficient on x^k is the number of independent sets of order k in the graph. It is given by;

$$I(G;x) = \sum_{k=0}^{n(G)} C_k x^k$$

Where C_k represent the independent sets of order k in the graph while

$n(G)$ represents the independence number of the graph.

DEFINITION 2.4 INDEPENDENCE POLYNOMIAL OF A NULL GRAPH [1, 8]

The independence polynomial of a null graph is given by;

$$I(E_{\emptyset};x) = 1$$

DEFINITION 2.4 CLIQUE [1]

The clique of a graph is the set of vertices of a graph that are adjacent in the graph.

DEFINITION 2.5 CLIQUE NUMBER [1]

Clique number of a graph is the number of elements in the clique of the graph with highest cardinality.

DEFINITION 2.6 CLIQUE POLYNOMIAL [1]

The clique polynomial of a graph G is the polynomial in x whose coefficient on x^k is the number of cliques of order k in the graph. It is given by;

$$C(G;x) = \sum_{k=0}^{m(G)} P_k x^k$$

Where P_k represents the clique of order k in the graph

$m(G)$ represents the clique number of the graph.

DEFINITION 2.7 CLIQUE POLYNOMIAL OF A NULL GRAPH [1, 8]

The clique polynomial of a null graph is given by;

$$C(E_{\emptyset};x) = 1$$

DEFINITION 2.8 ZERO-DIVISOR OF A RING [6]

A zero-divisor of ring R is a non-zero element $a \in R$ such that there exist a non-zero element $b \in R$ and $ab = 0$.

DEFINITION 2.9 ZERO-DIVISOR GRAPH OF A COMMUTATIVE RING [2]

This is a graph whose set of vertices is the set of zero-divisors of the ring and two vertices $x\ and\ y$ are adjacent iff $xy = 0$.

MAIN RESULT

CONSTRUCTING THE ZERO-DIVISOR GRAPHS

In this section we construct the zero-divisor graphs of five commutative rings; $\mathbb{Z}_{10,}\mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15,}\mathbb{Z}_{16}$.

Let $D(\mathbb{Z}_n)$ denote the set of zero-divisors of the ring $\mathbb{Z}_n$,

G_n^D $denote$ the zero-divisor graph, where n is the order of the ring.

We first get the set of zero-divisors of the rings which will serve as the set of vertices for the graphs then construct the graphs by applying definition 2.8.

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RESULT 3.1 ZERO-DIVISOR GRAPH OF $\mathbb{Z}_{10,}$

Given that $\mathbb{Z}_{10,} = \{0,1,2,3,4,5,6,7,8,9\}$

$D(\mathbb{Z}_{10,}) = \{2,4,5,6,8\}$

The sets of vertices are; $\{2,5\}, \{4,5\}, \{6,5\}, \{8,5\}$

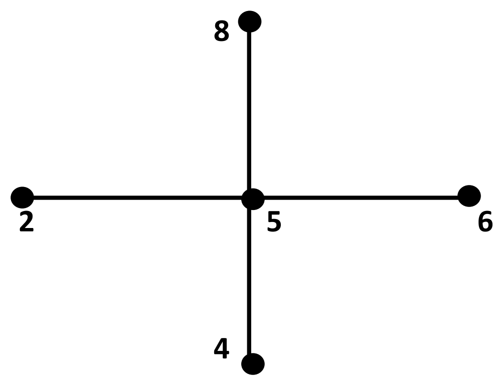

Fig. 1: The zero-divisor graph of $\mathbb{Z}_{10,}$.

RESULT 3.2 ZERO-DIVISOR GRAPH OF$\mathbb{Z}_{12,}$.

Given that $\mathbb{Z}_{12,} = \{0,1,2,3,4,5,6,7,8,9,10,11\}$

$D(\mathbb{Z}_{12,}) = \{2,3,4,6,8,9,10\}$

The sets of vertices are;

$\{2,6\}, \{6,4\}, \{6,8\}, \{6,10\}, \{3,4\}, \{3,8\}, \{9,4\}, \{9,8\}$

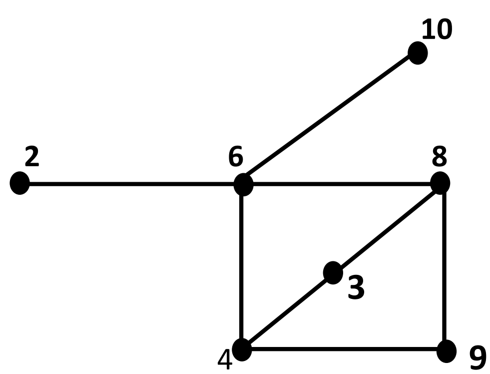

Fig. 2: The zero-divisor graph of $\mathbb{Z}_{12,}$.

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RESULT 3.3 ZERO-DIVISOR GRAPH OF $\mathbb{Z}_{14}$

Given that $\mathbb{Z}_{14} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13\}$

$D(\mathbb{Z}_{14}) = \{2,4,6,7,8,10,12\}$

The sets of vertices are;

$$\{2,7\}, \{4,7\}, \{6,7\}, \{8,7\}, \{10,7\}, \{12,7\}$$

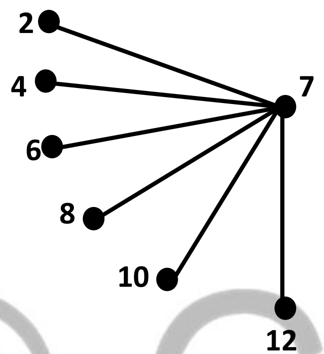

Fig. 3: The zero-divisor graph of $\mathbb{Z}_{14}$

RESULT 3.4 ZERO-DIVISOR GRAPH OF $\mathbb{Z}_{15}$

Given that $\mathbb{Z}_{15} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$

$D(\mathbb{Z}_{15}) = \{3,5,6,9,10,12\}$

The sets of vertices are;

$$\{3,5\}, \{6,5\}, \{9,5\}, \{12,5\}, \{3,10\}, \{6,10\}, \{9,10\}, \{12,10\}$$

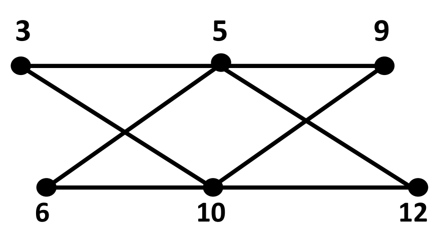

Fig. 4: The zero-divisor graph of $\mathbb{Z}_{15}$.

RESULT 3.5 ZERO-DIVISOR GRAPH OF $\mathbb{Z}_{16}$.

Given that $\mathbb{Z}_{16} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$

$D(\mathbb{Z}_{16}) = \{3,5,6,9,10,12\}$

The sets of vertices are;

$$\{2,8\},\{4,8\},\{6,8\},\{10,8\},\{12,8\},\{14,8\}\{4,12\}$$

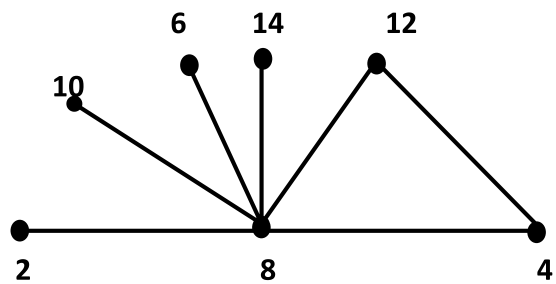

Fig. 5: The zero-divisor graph of $\mathbb{Z}_{16}$.

COMPUTING THE INDEPENDENCE POLYNOMIALS

This section is where we compute the independence polynomial and clique polynomial by generating the independent sets and cliques of the graphs then apply Hoede and Li's idea to compute the two polynomials.

Theorem 4.1

Given that $\mathbb{Z}_{10} = \{0,1,2,3,4,5,6,7,8,9\}$.

Then the independence polynomial of the zero-divisor graph of $\mathbb{Z}_{10}$, G_{10}^{D} is;

$I(G_{10}^{D};x) = 1 + 5x + 6x^2 + 4x^3 + x^4.$

Proof

By applying definitions 2.1 and 2.2 the graph G_{10}^{D} has 5 independent sets of order 1, 6 of order 2, 4 of order 3 and 1 of order 4 as shown below.

Order 1;

$$\{2\},\{4\},\{5\},\{6\},\{8\}$$

Order 2;

$$\{2,4\},\{2,6\},\{2,8\},\{4,6\},\{4,8\},\{6,8\}$$

Order 3;

$$\{2,4,6\},\{2,4,8\},\{4,6,8\},\{2,6,8\}$$

Order 4;

$$\{2,4,6,8\}$$

Therefore,

$$n(G_{10}^{D}) = 4.$$

By definition 2.3;

$$I(G_{10}^{D}\,;x) = \sum_{k=0}^{4} C_k x^k$$

$$= C_0x^0 + C_1x^1 + C_2x^2 + C_3x^3 + C_4x^4$$

$$= 1 + 5x + 6x^2 + 4x^3 + x^4.$$

Theorem 4.2

Given that $\mathbb{Z}_{12} = \{0,1,2,3,4,5,6,7,8,9,10,11\}$

Then the independence polynomial of the zero-divisor graph of $\mathbb{Z}_{12}$, G_{12}^{D} is;

$I(G_{12}^{D};x) = 1 + 7x + 13x^2 + 9x^3 + 2x^4.$

Proof

By applying definitions 2.1 and 2.2 the graph G_{12}^{D} has 7 independent sets of order 1, 13 of order 2, 9 of order 3 and 2 of order 4 as shown below.

Order 1;

$$\{2\},\{3\},\{4\},\{6\},\{8\},\{9\},\{10\}$$

Order 2;

$$\{2,3\},\{2,4\},\{2,8\},\{2,9\},\{2,10\},\{3,6\},\{3,9\},\{3,10\},\{4,8\},\{4,10\},\{6,9\},\{8,10\},\{9,10\}$$

Order 3;

$$\{2,3,9\},\{2,3,10\},\{2,4,8\},\{2,4,10\},\{3,6,9\},\{3,9,10\},\{4,8,10\},\{2,10,9\},\{2,10,8\}$$

Order 4;

$$\{2,3,9,10\},\{2,4,8,10\}$$

Thus,

$$n(G_{12}^{D}) = 4.$$

By definition 2.3;

$$I(G_{12}^{D};x) = \sum_{k=0}^{4} C_k x^k$$

$$= C_0x^0 + C_1x^1 + C_2x^2 + C_3x^3 + C_4x^4$$

$$= 1 + 7x + 13x^2 + 9x^3 + 2x^4.$$

Theorem 4.3

Given that $\mathbb{Z}_{14} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13\}$

Then the independence polynomial of the zero-divisor graph of $\mathbb{Z}_{14}$, G_{14}^{D} is;

$I(G_{14}^{D};x) = 1 + 7x + 15x^2 + 20x^3 + 13x^4 + 5x^5 + x^6.$

Proof

By applying definitions 2.1 and 2.2 the graph G_{14}^{D} has 7 independent sets of order 1, 15 of order 2, 20 of order 3, 13 of order 4, 5 of order 5 and 1 of order 6 as shown below.

Order 1;

$$\{2\},\{4\},\{6\},\{8\},\{10\},\{12\},\{14\}$$

Order 2;

$$\{2,4,\},\{2,6\},\{2,8\},\{2,10\},\{2,12\},\{4,6\},\{4,8\},\{4,10\},\{4,12\},\{6,8\},\{6,10\},\{6,12\},\{8,10\}\{8,12\}\{10,12\}$$

Order 3;

$\{2,4,6\},\{2,4,8\},\{2,4,10\},\{2,4,12\},\{2,6,8\},\{2,6,10\},\{2,6,12\},\{2,8,10\},\{2,8,12\},\{2,10,12\},\{4,6,8\},$

$$\{4,6,10\},\{4,6,12\},\{4,8,10\}\{4,8,12\}\{4,10,12\}\{6,8,10\}\{6,8,12\}\{6,10,12\}\{8,10,12\}$$

Order 4;

$$\{2,4,6,8\},\{2,4,6,10\}\{2,4,6,12\}\{2,4,8,10\}\{2,4,8,12\}\{2,6,8,10\}\{2,,6,8,12\}$$

$$\{2,6,10,12\}\{2,8,10,12\}\{4,6,8,10\}\{4,6,8,12\}\{4,8,10,12\}\{6,8,10,12\}$$

Order 5;

$$\{2,4,6,8,10\}\{2,4,6,8,12\}\{2,6,8,10,12\}\{2,4,8,10,12\}\{4,6,8,10,12\}$$

0rder 6

$$\{2,4,6,8,10,12\}$$

So that

$$n(G_{14}^{D}) = 6.$$

By definition 2.3;

$$I(G_{14}^{D};x) = \sum_{k=0}^{6} C_k x^k$$

$$= C_0x^0 + C_1x^1 + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 + C_6x^6$$

$$= 1 + 7x + 15x^2 + 20x^3 + 13x^4 + 5x^5 + x^6.$$

Theorem 4.4

Given that $\mathbb{Z}_{15} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$.

Then the independence polynomial of the zero-divisor graph of $\mathbb{Z}_{15}$, G_{15}^{D} is;

$$I(G_{15}^{D};x) = 1 + 6x + 7x^2 + 4x^3 + x^4.$$

Proof

By applying definitions 2.1 and 2.2 the graph G_{15}^{D} has 6 independent sets of order 1, 7 of order 2, 4 of order 3 and 1 of order 4 as shown below.

Order 1;

$$\{3\},\{5\},\{6\},\{9\},\{10\},\{12\}$$

Order 2;

$$\{3,6\},\{3,9\},\{3,12\},\{5,10\},\{6,9\},\{6,12\},\{9,12\}$$

Order 3;

$$\{3,6,9\},\{3,6,12\},\{3,9,12\},\{6,12,9\}$$

Order 4;

$$\{3,6,9,12\}$$

And

$$n\left(G_{15}^{D}\right)=4.$$

By definition 2.3

$$I\left(G_{15}^{D};x\right)=\sum_{k=0}^{4}C_k x^k$$

$$=C_0x^0+C_1x^1+C_2x^2+C_3x^3+C_4x^4$$

$$=\ 1+6x+7x^2+4x^3+x^4.$$

Theorem 4.5

Given that $\mathbb{Z}_{16}=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$.

Then the independence polynomial of the zero-divisor graph of $\mathbb{Z}_{16}$, G_{16}^{D} is;

$I(G_{16}^{D};x)=\ 1+7x+12x^2+14x^3+7x^4+2x^5.$

Proof

By applying definitions 2.1 and 2.2 the graph G_{16}^{D} has 7 independent sets of order 1, 12 of order 2, 14 of order 3, 7 of order 4 and 2 of order 5 as shown below.

Order 1;

$$\{2\},\{4\},\{6\},\{8\},\{10\},\{12\},\{14\}$$

Order 2;

$$\{2,4,\},\{2,6\},\{2,10\},\{2,12\},\{2,14\},\{4,6\},\{4,10\},\{4,14\},\{6,10\},\{6,12\},\{6,14\},\{12,14\}$$

Order 3;

$$\{2,4,6\},\{2,4,10\},\{2,4,14\},\{2,6,12\},\{2,6,14\},\{2,6,10\},\{2,10,14\},\{2,10,12\},$$

$$\{2,14,12\},\{10,6,14\},\{10,6,12\}\{10,6,4\},\{6,14,12\}\{6,14,4\}$$

Order 4;

$$\{2,4,6,10\},\{2,4,6,12\}\{2,4,6,14\}\{2,6,10,12\}\{2,6,10,14\}\{4,6,10,14\}\{6,10,12,14\}$$

Order 5;

$$\{2,4,6,10,14\},\{2,6,10,12,14\}$$

So

$$n(G_{16}^D) = 5.$$

By definition 2.3;

$$I(G_{16}^D;x) = \sum_{k=0}^{5} C_k x^k$$

$$= C_0x^0 + C_1x^1 + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5$$

$$= 1 + 7x + 12x^2 + 14x^3 + 7x^4 + 2x^5.$$

On the clique polynomial we have

Theorem 4.6

Given that $\mathbb{Z}_{10} = \{0,1,2,3,4,5,6,7,8,9\}$

Then the clique polynomial of the zero-divisor graph of $\mathbb{Z}_{10}$, G_{10}^D is;

$C(G_{10}^D;x) = 1 + 5x + 4x^2.$

Proof

By applying definitions 2.4 and 2.5 the graph G_{10}^{D} has 5 cliques of order 1 and 4 of order 2 as shown below.

Order 1;

$$\{2\},\{4\},\{5\},\{6\},\{8\}$$

Order 2;

$$\{2,5\},\{5,6\},\{4,5\},\{5,8\}$$

And

$$m(G_{10}^{D}) = 2.$$

By definition 2.6,

$$C(G_{10}^{D};x) = \sum_{k=0}^{2} P_k x^k$$

$$= P_0x^0 + P_1x^1 + P_2x^2$$

$$= 1 + 5x + 4x^2.$$

Theorem 4.7

Given that $\mathbb{Z}_{12} = \{0,1,2,3,4,5,6,7,8,9,10,11\}$

Then the clique polynomial of the zero-divisor graph of $\mathbb{Z}_{12}$, G_{12}^{D} is;

$C(G_{12}^{D};x) = 1 + 7x + 8x^2.$

Proof

By applying definitions 2.4 and 2.5 the graph G_{12}^{D} has 7 cliques of order 1 and 8 of order 2 as shown below.

Order 1;

$$\{2\},\{3\},\{4\},\{6\},\{8\},\{9\},\{10\}$$

Order 2;

$$\{2,6\},\{6,10\},\{6,4\},\{6,8\},\{3,4\}\{3,8\}\{4,9\}\{8,9\}$$

And

$$m(G_{12}^{D}) = 2.$$

By definition 2.6

$$C(G_{12}^{D}; x) = \sum_{k=0}^{2} P_k x^k$$

$$= P_0 x^0 + P_1 x^1 + P_2 x^2$$

$$= 1 + 7x + 8x^2.$$

Theorem 4.8

Given that $\mathbb{Z}_{14} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13\}$

Then the clique polynomial of the zero-divisor graph of $\mathbb{Z}_{14}$, G_{14}^{D} is;

$C(G_{14}^{D}; x) = 1 + 7x + 6x^2.$

Proof

By applying definitions 2.4 and 2.5 the graph G_{14}^{D} has 7 cliques of order 1 and 6 of order 2 as shown below.

Order 1;

$$\{2\}, \{4\}, \{6\}, \{7\}, \{18\}, \{10\}, \{12\}$$

Order 2;

$$\{2,7\}, \{4,7\}, \{6,7\}, \{8,7\}, \{10,7\}\{12,7\}$$

And

$$m(G_{14}^{D}) = 2.$$

By definition 2.6

$$C(G_{14}^{D}; x) = \sum_{k=0}^{2} P_k x^k$$

$$= P_0 x^0 + P_1 x^1 + P_2 x^2$$

$$= 1 + 7x + 6x^2.$$

Theorem 4.9

Given that $\mathbb{Z}_{15} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$

Then the clique polynomial of the zero-divisor graph of $\mathbb{Z}_{15}$, G_{15}^{D} is;

$C\left(G_{15}^{D}; x\right) = 1 + 6x + 8x^2.$

Proof

By applying definitions 2.4 and 2.5 the graph G_{15}^{D} has 6 cliques of order 1 and 8 of order 2 as shown below.

Order 1;

$$\{3\}, \{5\}, \{6\}, \{9\}, \{10\}, \{12\}$$

Order 2;

$$\{3,5\}, \{3,10\}, \{5,6\}, \{5,9\}, \{5,12\}\{9,10\}, \{6,10\}, \{10,12\}$$

And

$$m\left(G_{15}^{D}\right) = 2.$$

By definition 2.6

$$C\left(G_{15}^{D}; x\right) = \sum_{k=0}^{2} P_k x^k$$

$$= P_0 x^0 + P_1 x^1 + P_2 x^2$$

$$= 1 + 6x + 8x^2.$$

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Theorem 4.10

Given that $\mathbb{Z}_{16} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$

Then the clique polynomial of the zero-divisor graph of $\mathbb{Z}_{16}$, G_{16}^{D} is;

$$C(G_{16}^{D};x) = 1 + 7x + 7x^2 + x^3.$$

Proof

By applying definitions 2.4 and 2.5 the graph G_{16}^{D} has 7 cliques of order 1, 7 of order 2 and 1 of order 3 as shown below.

Order 1;

$$\{2\},\{4\},\{6\},\{8\},\{10\},\{12\},\{14\}$$

Order 2;

$$\{2,8\},\{4,8\},\{6,8\},\{10,8\},\{12,8\}\{14,8\},\{4,12\}$$

Order 3;

$$\{4,8,12\}$$

And

$$m(G_{16}^{D}) = 3.$$

By definition 2.6

$$C(G_{16}^{D};x) = \sum_{k=0}^{3} P_k x^k$$

$$= P_0x^0 + P_1x^1 + P_2x^2$$

$$= 1 + 7x + 7x^2 + x^3..$$

CONCLUSION

The zero-divisor graphs for the rings of integers modulo n were constructed and the independence and clique polynomials for the graph of each ring are computed. The table below shows the computed polynomials.

S/N	Ring	Independence Polynomial
1	$\mathbb{Z}_{10}$	$1 + 5x + 6x^2 + 4x^3 + x^4$
2	$\mathbb{Z}_{12}$	$1 + 7x + 13x^2 + 9x^3 + 2x^4$
3	$\mathbb{Z}_{14}$	$1 + 7x + 15x^2 + 20x^3 + 13x^4 + 5x^5 + x^6$
4	$\mathbb{Z}_{15}$	$1 + 6x + 7x^2 + 4x^3 + x^4$
5	$\mathbb{Z}_{16}$	$1 + 7x + 12x^2 + 14x^3 + 7x^4 + 2x^5$

Table 1. Table showing Independence Polynomials

S/N	Ring	clique Polynomial
1	$\mathbb{Z}_{10}$	$1 + 5x + 4x^2$
2	$\mathbb{Z}_{12}$	$1 + 7x + 8x^2$
3	$\mathbb{Z}_{14}$	$1 + 7x + 6x^2$
4	$\mathbb{Z}_{15}$	$1 + 6x + 8x^2$
5	$\mathbb{Z}_{16}$	$1 + 7x + 7x^2 + x^3$

Table 2. Table showing the Clique Polynomials.

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