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Improving the Performance of Linear Regression Model: A Residual Analysis approach

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ABSTRACT:

The study considered GDP as dependent variable, agriculture, industry and trade as independent variables respectively. The data was gotten from the central bank of Nigeria from 1983 to 2019. The aim of the study was to apply residual analysis approach to improve the performance of linear regression. The relationship between the dependent variable, GDP, and the independent variables, Agriculture, industry and trade was determined using the ordinary least squares estimation method. The results of the ordinary least squares estimated regression showed that Agriculture, industry and trade contributed significantly to GDP and were able to explain about 89% of the variance in GDP. Furthermore, evidence from Breusch-pagan test confirmed that heteroscedasticity exist in the residuals of the linear regression model while ACF and PACF revealed that the error terms were autocorrelated. The Jarque-Bera normality test revealed that the errors were normally distributed. To account for the autocorrelation in the error terms, we applied two different generalized least squares models, that is regression ARMA model (RAM) and overfitted regression ARMA model (ORAM) with different ARMA components, that is, ARMA (1, 2) and ARMA (1, 3) respectively. Results of our analysis revealed that the estimates of the RAM model were better than those of ORAM. Also based on minimum information selection criteria (AIC, BIC, LOGLIK) RAM was selected as the suitable model. The autocorrelation in the error terms was found to be completely modelled by ARMA (1, 2) process. An ARMA (1,3) model (specification) would be unusually complicated, but in any event the tests support the ARIMA(1,2) specification.

Keywords: GDP, Agriculture, Industry, Trade, ARMA Generalised least squares.

1 Introduction

In developing predictive regression models, a number of concerns need to be addressed. The first is model adequacy, or explanatory power of the independent variable in accounting for the variability of the dependent variable. This is typically measured by the coefficient of determination R^2, which is the percentage of variance in the dependent variable explained by the independent variable(s). A large value of R^2 is always seen as a good suggestion of how well the model fits the data but the unexplained variation which largely make up the residuals and tend to be embedded with useful information is often overlooked [1].Linear regression models are tied to certain assumptions about the distribution of the error terms [2, 3]. The violation of assumptions surrounding linear regression models is reflected in the residuals. The possible model adequacies could be identified through residual analysis.

Several diagnostic methods to check the violation of regression assumption are based on the study of model residuals. [3]. According to [1, 4] model adequacy or diagnostic checking incorporates all relevant information and when calibrated to the data no important significant departures from statistical assumptions made can be found. Actually, model adequacy involves residual analysis and over fitting.

These residuals are obtained by taking the difference between an observed value of a time series and a predicted value from fitting a candidate model to the data. They are useful in checking whether a model has adequately captured the information in the data. Model adequacy is related mainly to the assumption that residuals are independent. Moreover, if the residuals of a given model are correlated, the model must be refined because it does not completely capture the statistical relationship amongst the variables [5]. Furthermore, a model is said to be adequate if the residuals are statistically independent implying that the residual series is uncorrelated. Therefore, in testing for model adequacy, which is mainly to check for independence of the residual series, an autocorrelation function (ACF), Partial autocorrelation function (ACF), Jarque-Bera normality test and Breusch-Pagan heteroscedasticity test are often considered.

Another tool for checking adequacy regression model is over fitting, which has to do with adding another coefficient to a fitted model so as to see if the resulting model is better. If a simple model seems promising, check it out before trying a more complicated model [6].In this study, our aim is to apply the residual analysis approach targeted at improving the performance of linear regression model.

The rest of the paper is organised follows, section two presents the materials and methods adopted in the study, section three takes care of results discussion while conclusion is handled by section four

2 Materials and methods

In this section, we shall discuss the methods and procedures adopted in achieving the aim of the study.

2.1 Method of Ordinary Least Squares Linear Regression

The least squares estimation procedure uses the criterion that the solution must give the smallest possible sum of squared deviations of the observed Y_t from the estimates of their true means provided by the solution. Let $\hat{\beta}_0$and $\hat{\beta}_1$be numerical estimates of the parameters $\beta_o$and β_1 respectively, and

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_t \hat{X}_t. \quad (1)$$

Be the estimated mean of Y_t for each X_t t = 1, …, n.

The least squares principle chooses $\hat{\beta}_0$ and $\hat{\beta}_t$ that minimize the sum of squares of residuals (SSE),

$$SSE = \sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2 = \sum_{t=1}^{n} \varepsilon_t^2, \quad (2)$$

where,$\varepsilon_t = (Y_t - \hat{Y}_t)$ is the observe residuals for the ith observation

Also we can express ε_i in terms of Y_t, X_t, $\beta_o$and β_1. Hence, we have

$$\varepsilon_t = Y_t - \beta_0 - \beta_1 X_t \ . \quad (3)$$

Equation (3) becomes

$$SSE = \sum_{t=1}^{n}(Y_t - \beta_0 - \beta_1 X_t)^2. \quad (4)$$

The partial derivative of SSE with respect to the regression constant $\hat{\beta}_0$,th

$$\frac{\delta SSE}{\delta \beta_0} = \frac{\delta}{\delta \beta_0} \left[\sum_{t=1}^{n} (Y_t - \beta_0 - \beta_1 X_t)^2\right]. \quad (5)$$

With some subsequent rearrangement, the estimate of $\hat{\beta}_0$ is obtained as

$$\hat{\beta}_0 = \left[\frac{\sum_{t=1}^{n} Y_t}{n}\right] - \beta_1 \left[\frac{\sum_{t=1}^{n} X_t}{n}\right]. \quad (6)$$

The partial derivative of SSE with respect to the regression coefficient β_1. That is

$$\frac{\delta SSE}{\delta \beta_1} = \frac{\delta}{\delta \beta_1} \left[\sum_{t=1}^{n} (Y_t - \beta_0 - \beta_1 X_t)^2\right]. \quad (7)$$

Rearranging equation (8), we obtained the estimate of β_1.

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{n} Y_t X_t - \frac{\sum_{t=1}^{n} Y_t \sum_{t=1}^{n} X_t}{n}}{\sum_{t=1}^{n} X_t^2 - \frac{(\sum_{t=1}^{n} X_t)^2}{n}}. \quad (8)$$

2.2 Method of Generalized Least Square (GLS)

For a standard linear model

$$y = X\beta + \varepsilon, \quad (9)$$

where, y is the $n \times 1$ response vector; X is $n \times K + 1$ model matrix model matrix, typically with an initial column of 1s for the regression constant; β is a $K + 1 \times 1$ vector of regression coeficients to estimate; and ε is an $n \times 1$ vector of errors. Assuming that $\varepsilon \sim N_n(0, \sigma^2 I_n)$, or at least that the errors are uncorrelated and equally variable, leads to the familiar ordinary-least-squares (OLS) estimator of β,

$$b_{ols} = (X'X)^{-1}X'y. \quad (10)$$

With covariance matrix

$$var(b_{ols}) = \sigma^2(X'X)^{-1}. \tag{11}$$

More generally, we can assume that $\varepsilon \sim N_n(0, \sum)$, where the error covariance matrix $\sum$ is symmetric and positive-definite. Different diagonal entries in $\sum$error variances that are not necessarily all equal, while nonzero off-diagonal entries correspond to correlated errors. Suppose, for the time-being, that $\sum$ is known. Then, the log-likelihood for the model is

$$\log L(\beta) = -\frac{n}{2}log2\pi - \frac{1}{2}\log(det\sum) - \frac{1}{2}(y - X\beta)'\sum^{-1}(y - X\beta), \tag{12}$$

which is maximised by the generalised least square (GLS) estimate of β,

$$b_{GLS} = (X'\sum^{-1}X)^{-1}X'\sum^{-1}y \tag{13}$$

With covariance matrix

$$Var(b_{GLS}) = (X'\sum^{-1}X)^{-1} \tag{14}$$

If we assume that the process generating the regression errors is stationary: That is, all of the errors have the same expectation (already assumed to be 0) and the same variance (σ^2), and the covariance of two errors depends only upon their separation s in time:

$$C(\varepsilon_t, \varepsilon_{t-s}) = C(\varepsilon_t, \varepsilon_{t-s})\sigma^2\rho_s. \tag{15}$$

where ρ_s is the error autocorrelation at lag s.

In this situation, the error covariance matrix has the following structure:

$$\Sigma = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{n-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdots & 1 \end{pmatrix} = \sigma^2\rho \tag{16}$$

Hence, for known values of $\sigma^2$and ρ_s , then GLS estimator of β can be computed in a time series regression. In addition, in the error covariance matrix Σ, the large number (n-1) of different ρ_s makes their estimation impossible without specifying additional structure for the autocorrelated errors [7]. Moreover, this additional could be specified to follow stationary time series models such as Autoregressive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA).

2.3 Autoregressive (AR) Model

Autoregressive models are based on the idea that current values of the series X_t can be explained as a function of past values, $X_{t-1}, X_{t-2}, \dots, X_{t-p}$, where p determines the number of steps into the past needed to forecast the current values. An autoregressive model of order p abbreviated as $AR(p)$ can be written as:

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \cdots + \varphi_p X_{t-p} + e_t, \tag{17}$$

where, X_t is the stationary series, $\varphi_{1,}\varphi_{2,} \dots \varphi_p$ are parameters of AR ($\varphi_p = 0$) unless if otherwise stated, we assumed that e_t is Gaussian white noise with mean zero and variance δ^2. The highest order of p is referred to as the order of the model.

The model in lag operator takes the following form.

$$\left(1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p\right) X_t = e_t, \tag{18}$$

where the lag (backshift) operator is defined as $\beta^p X_t = X_{t-p},\ p = 0, 1, 2.$

More concisely we can express the equation (18) as $\varphi(\beta) X_t = e_t$.

The autoregressive operator $\varphi(B)$ is defined to be $\left(1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p\right)$

The values of φ which make the process stationary are such that the roots of $\varphi(B) = 0$ lie outside the unit circle in the complex plane [8]. If the roots of $\varphi(B)$are larger than one in absolute values, then the process satisfying the autoregressive equation which can be represented as:

$$x_t = \sum_{j=1}^{\infty} \varphi_j X_{t-1} \tag{19}$$

The coefficient of φ_j converges to zero such that

$$\sum_{j=1}^{\infty} |\varphi_j| < \infty$$

If some roots are “exactly” one in modulus, no stationary solution exists. The plot of the ACF of a stationary $AR(p)$ model shows a mixture of damping sine and cosine pattern and exponential decay depending on the nature of its characteristic roots.

Another characteristic feature of $AR(p)$ model is the partial autocorrelation function defined as

$PACF(j) = Corr(x_t, x_{t-j} | x_{t-1}, x_{t-2}, \dots, x_{t-j+1})$, becomes exactly zero for values larger than p[9].

2.4 Moving Average (MA) Models

An alternative to autoregressive representation in which the X_{t-1} on the left-hand side of the equation are assumed to be combined linearly to form the observed data. A series $X_t$is said to follow a moving average process of order q or simply $MA(q)$ process if

$$X_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_p e_{t-q}, \tag{20}$$

where θ_1,θ_2, …,θ_q are the MA parameters to be estimatede_t, e_{t-1}, …, e_{t-q} are error terms. The value of q is called the order of MA model [9]. In order to preserve a unique representation, usually the requirement is imposed that all roots of

$$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q = 0 \tag{21}$$

are greater than one in absolute value. If all the roots of $\theta(B) = 0$ lie outside the unit circle, the MA process has an autoregressive representation of generally infinite order.

2.5 Autoregressive Moving Average (ARMA)

We now proceed with the general development of autoregressive moving average (ARMA) models for stationary time series. In most cases, it is best to develop a mixed autoregressive moving average model when building a stochastic time series. The order of an ARMA model is expressed in terms of both p and q. the model parameters relate to what happen in period t to both the past random errors that occur in the past periods. The ARMA model is defined as follows:

$$X_t = \varphi_1 X_{t-1} + \cdots + \varphi_p X_{t-p} + e_t - \theta_1 e_{t-1} - \cdots - \theta_p e_{t-p} \quad (22)$$

Where the φ's are autoregressive parameters to be estimated and θ's the moving average parameters to be estimated. The e's are series of unknown errors (or residuals) which are assumed to follow the normal probability distribution. [10] used a backshift operator to make the model easier. The backshift operator B has the effect of changing time period t to time period $t-1$.

$BX_t = X_{t-1}, B^2 X_t = X_{t-2}$And so on. Using the backshift notation, the above model may be rewritten as:

$$\left(1 - \varphi_1 B - \cdots - \varphi_p B^p\right) X_t = \left(1 - \theta_1 B - \cdots - \theta_q B^q\right) e_t \quad (23)$$

Equation (23) may be abbreviated further by writing:

$$\varphi_p(B) X_t = \theta_q(B) e_t \quad (24)$$

These formulas show that the operators $\varphi_p(B)$ and $\theta_q(B)$ are polynomials in B of orders p and q respectively. The ARMA model is stable, i.e. it has a stationary solution if all roots of $\varphi(B) = 0$ are larger than one in absolute value.

2.6 Detecting Autocorrelation in the Error Terms

Consider the following two variable regression model

$$Y_t = \beta_0 + \beta_1 X_1 + u_{t.} \quad (25)$$

Assume the error term $u_t$follows the p^{th} order autoregressive, AR(p), structure as follows

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \cdots + \rho_p u_{t-p} + \varepsilon_t. \quad (26)$$

Where, ε_t is a white noise term.

The null hypothesis H_0 to be tested is that;

$$H_0 : \rho_1 = \rho_2 = \cdots = \rho_p = 0$$

Thus to obtain the residual $\hat{u}_t$ from equation (26) and regress $\hat{u}_t$ on the original

$Y_t$and$\hat{u}_{t-1}, \hat{u}_{t-2}, \ldots, \hat{u}_{t-p}$ that is

$$\hat{u}_t = \propto_1 + \propto_2 X_t + \hat{\rho}_1 \hat{u}_{t-1} + \hat{\rho}_2 \hat{u}_{t-2} + \cdots + \hat{\rho}_p \hat{u}_{t-p} + \varepsilon_t \quad (27)$$

Then, the R^2 obtained from the above auxiliary regression is used to compute BG test statistic given as

$$(n-p)R^2 \sim X^P{}_p, \tag{28}$$

where, n is the sample size, p is the number of lags of the residual $\hat{u}_t$ included in the auxiliary regression. If $(n-p)R^2$ exceed the critical chi-square value of the chosen level of significance, we reject the null hypothesis, in which case at least one rho is equation (28) is statistically significantly different from zero, hence we conclude that there is autocorrelation in the error term u_t

3 Results and Discussion

In this study, we consider Gross Domestic Product as the dependent variable (denoted by (GDP), Agriculture (denoted by Agric), industry and trade as independent variables. The data were obtained as available from the Central Bank of Nigeria Statistical Bulletin for a period ranging from 1983 to 2019. Each series consists of 37 observations. R package and E-views were the statistical software used for analysis.

Since our aim is to use residual analysis approach to improve the performance of linear regression analysis, we begin by ascertaining the relationship between the dependent and the independent variables via a linear regression. The estimated linear regression model is presented in Table 1

Table 1: Estimates of Linear Regression Model

	Estimates	Std. error	T-value	P-value
Intercept	0.03416	0.03416	3.019	0.004951 **
Agric	0.27966	0.04734	5.907	1.42e-06 ***
Industry	0.27266	0.03757	7.258	3.02e-08 ***
Trade	0.22908	0.05573	4.111	0.000256 ***
R^2	0.8932			
F-statistics	89.22	p.value=1.2e-15		

From Table 1, it is observed that all the independent variables are significant since the p-values corresponding to Agric (1.42e-06), industry (3.02e-08) and trade (0.000256) are less than 5% significance level and were able to explain about 89% ($R^2 = 0.8932$) of the variation in GDP. Also from Table 1, the p-value ($1.2e^{-15}$) corresponding to the F-statistics (89.22) is less than 5% level of significance level which also give credence to the fact that the independent variables jointly influence the variation in the dependent variable. On the other hand, about 10.68% of unexplained variation is embedded in the residuals. This implies that useful information can further be modelled from the residual term, hence, the need for residual analysis.

3.1 Model Diagnosis

Regression model diagnosis is used to evaluate the model assumptions and investigate whether or not there are observations with large, undue influence on the model. Since our interest is geared towards increasing the performance of linear regression model using the residual analysis approach, we shall evaluate the residuals under the three main assumptions (1) No serial correlation (2) homoscedasticity (3) Residuals are normally distributed.

3.1.1 Diagnostic checking for serial correlation.

To diagnose the model for serial correlation we plot the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the residual. If the lags of the ACF and PACF of the residuals of the fitted model are zero, there is no serial correlation. Conversely, if the coefficient of the terms of both ACF and PACF are significant, then there is serial correlation in the residuals of the model. Assessing Figures 1 and 2, we observed that lags 1, lag 2 and lag 3 of the ACF, and lags 1 of the PACF are significant. This indicates that the residuals serially correlated thereby violating the assumption of no serial correlation.

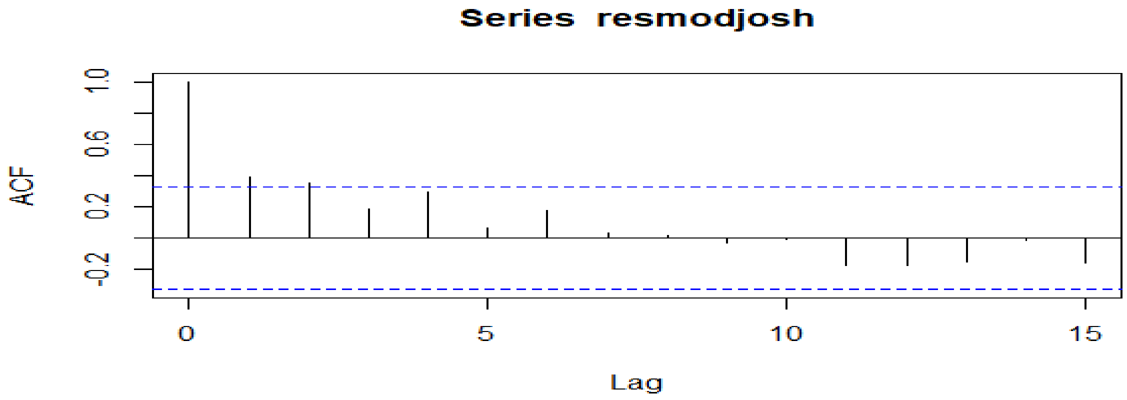

Figure 1: ACF of Linear Regression Model

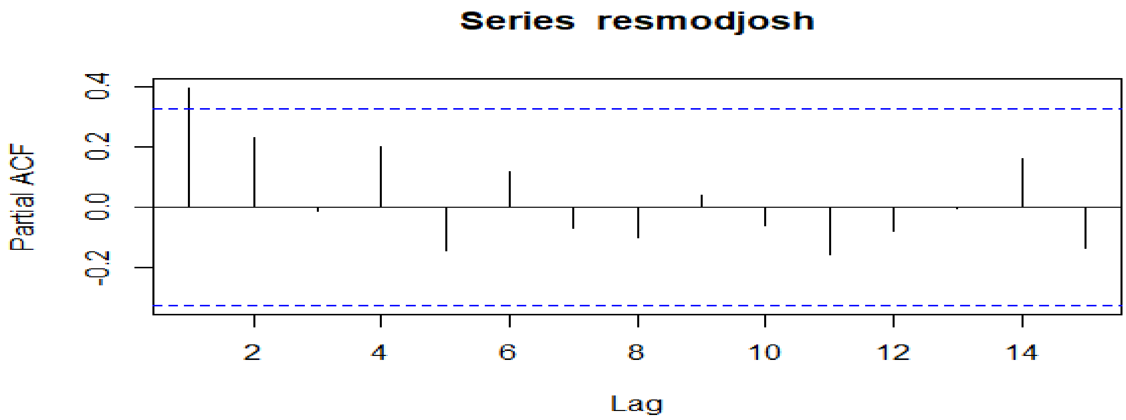

Figure 2: PACF of Linear regression model

3.1.2 Diagnostic Checking for Heteroscedasticity.

In testing for heteroscadaticity in the residuals of the model, we applied Breusch pagan test. Observing Table 2, we notice that the p- value (0.0001) corresponding to the test statisctic (18.09862) is less than 5% level of significance which confirms the presence of heteroscadaticity in the model. Therefore, the assumption of homoscedasticity is violated

Table 2: Breusch-pagan Heteroscedasticity Test

F-statistic	14.84171	Prob. F(2,31)	0.0000
Obs*R-squared	18.09862	Prob. Chi-Square(2)	0.0001

3.1.3 Diagnostic checking for normality of the residual

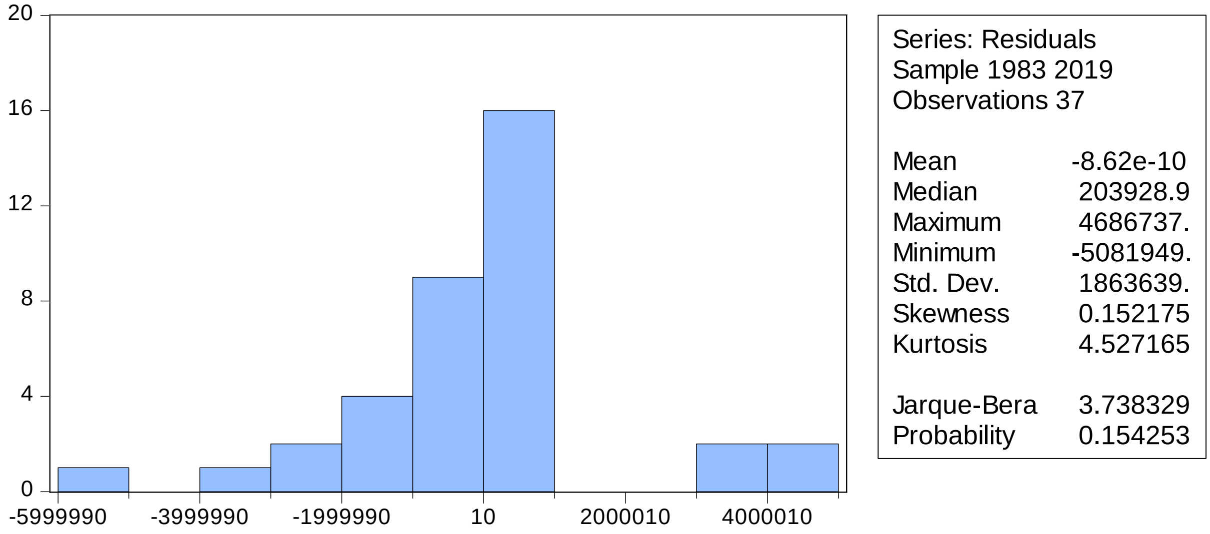

Figure 3: Jarque – Bera Histogram Normality Test

Considering Jarque – Bera normality test in Figure 3, the p- value (0.1543) corresponding to Jarque-Bera test (3.7383) is more than 5% level of significance which is desirable because the errors are normally distributed and the assumption of normality of the error is not violated.

Having identified and established the presence of autocorrelation and heteroscadaticity in the residual series, we moved to ascertain the order of Autoregressive Moving Average (ARMA) model that could capture the information in the autocorrelated errors. Observing the ACF and PACF in Figures 1 and 2, respectively, there is a cut off at lag 2 in ACF while there is a cut-off at lag 1 in PACF. This implies that a mixed model can entertained. Hence, ARMA (1,2) model could be identified. Also, overfitting approach is employed by adding one parameter to the fitted model to see if the model is adequate.

To entertain an improved model that could account for both explained and unexplained variations in the dependent variable, the regression and ARMA models are combined to achieve a single model that best describes the variation in the dependent variable and is presented in Table 3.

Table 3: Regression-ARMA Model versus Overfitted Regression-ARMA Model

Regression Model Components	**Regression-ARMA Model (RAM)**				**Overfitted Regression-ARMA Model (ORAM)**			
	β_{0RAM}	β_{1RAM}	β_{2RAM}	β_{3RAM}	β_{0ORAM}	β_{1ORAM}	β_{1ORAM}	β_{3ORAM}
Parameter	0.0362	0.2693	0.2550	0.0300	0.0367	0.2738	0.2251	0.1924
Std error	0.0185	0.0368	0.0283	0.0441	0.0212	0.0660	0.0285	0.0431
t-value	1.9557	7.3280	9.0037	4.6044	1.7272	7.4813	8.8221	4.4615
p-value	0.0593	0.0000	0.0000	0.0001	0.0938	0.0000	0.0000	0.0001

ARMA Model Components	ARMA (1,2)			ARMA(1,3)			
	φ_{1RAM}	φ_{2RAM}	θ_{1RAM}	φ_{1ORAM}	φ_{2ORAM}	φ_{3ORAM}	θ_{1ORAM}
Parameter	0.8487	-0.8549	0.1451	0.8573	0.1219	-0.9359	0.0024

AIC	-134.3614	-132.2299
BIC	-121.6932	-117.9782
LOGLIK	75.18067	75.1149

From the regression model component, it is observed that all the independent variables for both models are significant with their corresponding p- values less than 5% significance level. Since the overfitting is not directly affecting the regression component of the joint model, there are significant changes observed in the parameters. For the ARMA component, the ARMA(1,2) model appeared to be adequate given that the corresponding information criteria are smaller than those of ARMA(1,3) model. Thus, by overfitting approach, it is indicative that ARMA (1,2) model is enough to improve the regression model fitted earlier on.

4 Conclusion

This study applied residual analysis approach to improve the performance of linear regression. The relationship between the dependent variable, GDP, and the independent variables, Agriculture, industry and trade was determined using the ordinary least squares estimation method.

The results of the ordinary least squares estimated regression showed that Agriculture, industry and trade contributed significantly to GDP and were able to explain about 89% of the variance in GDP. Furthermore, evidence from Breusch-pagan test revealed that heteroscedasticity exist in the residuals of the linear regression model while ACF and PACF revealed that the error terms were autocorrelated. The JarqueBera normality test revealed that the errors were normally distributed. To account for the autocorrelation in the error terms, we applied two different generalized least squares model, that is RAM and ORAM with different ARMA components, that is, ARMA (1, 2) and ARMA (1, 3) respectively. Results of our analysis revealed that the estimates of the RAM model were better. Also based on minimum information selection criteria (AIC, BIC, LOGLIK) RAM was selected as the suitable model. The autocorrelation in the error terms was found to be completely modelled by ARMA (1, 2) process. An ARMA (1,3) model (specification) would be unusually complicated, but in any event the tests support the ARIMA(1,2) specification.

Therefore, our study showed that where the error terms of ordinary least squares estimated regression model are correlated, the model parameters become ineffective, the standard errors biased; and the t-statistics and the p-values no more valid. On the other hand, this study evidently proved that residual analysis can be applied to improve the performance of a regression model and that generalized least squares is a solution for the violation of assumption serial correlation of the linear regression model.

Moreover, the findings of this study are in agreement with the study of [13] that agriculture a has a significant impact on GDP but differs in terms of approach adopted in this study. Furthermore, it is suggested that this study be extended to cover the possible violation of assumption of the homoscedasticity.

References

[1] Moffat, I. U. and Akpan, E. A. (2019). White Noise Analysis: A Measure of Time Series Model Adequacy. *Applied Mathematics, 10, 989-1003*.

[2] Wei, W.W.S.[ed](2006).**Time series analysis univariate and multivariate methods**. AdisonWestley

[3] Akpan, E. A, &Moffat I. U. (2018).ModelingAutocorrelated Errors in Time Series Regression: A Generalised Least Squares Approach. *Journal of advances in mathematics and computer science,* 26(4):1-15

[4] Green, E. H. (2002). Econometric Analysis. 5th ed., New York: Prentice Hall. P.269.

[5] Jack E.M & Brian R. H. (2007) Evaluating Aptness of a Regression Model, *Journal of Statistics Education, 15:2, , DOI: 10.1080/10691898.2007.11889469*

[6] McLead, A.I. (1993) Parsimony, Model Adequacy and Periodic Correlation in Forecasting Time Series. *International Statistical Review, 61, 387-393.* *https://doi.org/10.2307/1403750*

[7] Pankratz A. (1983). Forecasting with univariate Box-Jenkins model: concept and cases. *John Wiley and sons, New York.doi.org/10.1002/9780470316506*

[8] Cryer, J.D. and Chan, K. (2008) Time Series Analysis with Application in R. 2ndEdition, Springer, New York, 249-260. https://doi.org/10.1007/978-0-387- 75959-3

[9] Fox J, Weisberg S. Time-series regression and generalized least squares in R: An appendix to an R companion to applied regression. 2nd ed; 2010. Available:Citeseerx.ist.psu.edu›Viewdoc›download

[10] Box, G.E.P. and Pierce, D. (1970) Distribution of Residual Autocorrelations in Autoregressive Integrated Moving Average Time Series Models. Journal of the *American Statistical Association, 65, 1509-1526.* *https://doi.org/10.1080/01621459.1970.10481180*

[11] Chatfield, C [ed]. (2000). **Time Series Forecasting.** New York: Chapman and Hall CRC

[12] Tsay, R.S. (2010) Analysis of Financial Time Series. 3rd Edition, John Wiley & Sons, New York. https://doi.org/10.1002/9780470644560

[13] Muhammad, M. A Saba, F &Ghulam Y. K (2015). Agriculture sector performance: An analysis through the role of agriculture Sector share in GDP *The International Journal of Business & Management.***3,** 270-275