Recall that (23...n) denotes the cyclic permutation. The following is the main theorem of this paper which gives a one-to-one correspondence between C_n *and* Q_n

Theorem 4.1

Let $S_1 := (2\,3\,...n), S_2 \in Q_n$ *and define* $\varphi(S_2) : Q \to Map(Q\,, Q)$ *by*

$$\left(\varphi\left(S_2\right)\right)_i := \begin{cases} S_1 & i=1 \\ S_2 & i=2 \\ S_2 S_1^{-1} S_2 \;=\; S_3 & \end{cases}$$

Then we have $\varphi\left(S_2\right) \in C_n^{\#}$, *and* hence give a Map $\varphi : Q_n \to C_n^{\#}$.

This induces from Q_n *onto* C_n

Proposition 4.2 recall that $Q_n = \left\{\left(1\,3\right)\right\}$ *and* $Q_5 = \left\{\left(1\,5\,3\,4\right)\right\}$

Proof: The basic strategy is the following. First of all, we list up all elements in $\left(S_n\right)_{n-1}$ satisfying (Q1). These elements are called the members for clarity we then check whether each member satisfies (Q2) or not in these case of n = 3. In the case of n = 3 , the only member is $S_2 = (13)$.

$S_2 S_1^{-1} S_2 = (12) = S_1 S_2^{-1} S_1$. Hence $S_{2,}$ satisfies $Q_{2.}$ This proves the first assertion.

Proposition 4.3 we have $Q_5 = \left\{(1534), (1435)\right\}$

Proof: there are six members as for the set of Q_5;

$S_2 = \left(1345\right), \left(1354\right), \left(1543\right), \left(1435\right), \left(1534\right), \left(1453\right)$ One can observe that $\left(1534\right)$ and $\left(1435\right)$ satisfy Q2. We omit proof for the two cases. Therefore we show that the remaining four members do not satisfy (Q2).

Case (1); $S_2 := \left(1345\right), S_1 = \left(2354\right)$ let m= -1 therefore $\left(1345\right); \left(1253 \neq 1432\right)$ $S_2 S_1^{-m} S_2 \neq S_1 S_2^{-m} S_1$

Case (2); $S_2 = \left(1354\right)$; *let* $m = -1$

$\left(1243\right) \neq \left(1532\right)$; $S_2 S_1^{-m} S_2 \neq S_1 S_2^{-m} S_1$

Case (3); $S_2 = \left(1543\right)$;

$\left(1532 \neq 2345\right)$; $S_2 S_1^{-1} S_2 \neq S_1 S_2^{-1} S_1$

Case (4); $S_2 = \left(1453\right)$

$(1235 \neq 1342)$; $S_2 S_1^{-1} S_2 \neq S_1 S_2^{-1} S_1$

This completes the proof of second assertion. When n = 3 the Latin quandles corresponding to $S_2 := (13)$ is the dihedral quandles with cardinality 3 (prime). When n = 5 , the Latin quandles corresponding to $S_2 = \{(1534),(1435)\}$ is pentahedron quandles. When $n \geq 5$; $n = p$ (prime) .

Lemma the following Lemma is useful to examine whether each cardinality satisfies (Q2) or not

Lemma 4.1 [11], $S_2 \in Q_n$ and m=-1 satisfies $S_1^{-m} = S_2^{-l}$ then we have $S_2 S_1^{-m} S_2 = S_1 S_2^{-l} S_1$

Proof: since $Q_n = \{1, 2, ..., n-2\}$ *and* $S_2 \in Q_n$ satisfies (Q2) there exist $l \in \mathbf{Z}$ such that l= m then $S_2 S_1^{-m} S_2 = S_1 S_2^{-l} S_1$.

Note that $S_2 S_1^{-m} S_2$ is a cyclic permutation of order n- 1, having the imagined fixed point $S_1^{-m}(2)$ similarly, $S_1 S_2^{-l} S_1$ has the imagine fixed point $S_2^{-l}(1)$. Hence combining with the assumption, one has $S_2(1) = S_2^{-m}(2) = S_2^{-l}(1)$

Since $S_2 \in (S_n)_{n-1}$ and it satisfies (Q1), we conclude $l = -1$ this completes the proof.

The above lemma is useful to determine the set Q_n for any n = p (prime). here we apply it to case of $n = 13$.

Proposition 4.4 we have $Q_{13} = \begin{Bmatrix} (1\ 9\ 5\ 7\ 6\ 13\ 3\ 8\ 12\ 10\ 11\ 4),(1\ 8\ 5\ 10\ 6\ 4\ 3\ 9\ 12\ 7\ 11\ 13), \\ (1\ 13\ 11\ 7\ 12\ 9\ 3\ 4\ 6\ 10\ 5\ 8),(1\ 4\ 11\ 10\ 12\ 8\ 3\ 13\ 6\ 7\ 5\ 9) \end{Bmatrix}$

Proof: As for the set Q_{13}, there are four members. $S_{2 =;}$ (1 9 5 7 6 13 3 8 12 10 11 4), (1 8 5 10 6 4 3 9 12 7 11 13), (1 13 11 7 12 9 3 4 6 10 5 8), (1 4 11 10 12 8 3 13 6 7 5 9) One can directly see that all four members satisfy (Q2). That ends the proof.

N	$\#Q$	Q_n
2	0	0
3	1	$\{(13)\}$
5	2	$\{(1534),(1435)\}$
7	2	$\{(167354),(145376)\}$
11	4	$\begin{Bmatrix} (1\ 11\ 9\ 5\ 8\ 3\ 4\ 6\ 10\ 7),(1\ 7\ 10\ 6\ 4\ 3\ 8\ 5\ 9\ 11),(1\ 5\ 4\ 7\ 9\ 3\ 10\ 11\ 8\ 6), \\ (1\ 6\ 8\ 11\ 10\ 3\ 9\ 7\ 4\ 5) \end{Bmatrix}$
13	4	$\begin{Bmatrix} (1\ 9\ 5\ 7\ 6\ 13\ 3\ 8\ 12\ 10\ 11\ 4),(1\ 8\ 5\ 10\ 6\ 4\ 3\ 9\ 12\ 7\ 11\ 13), \\ (1\ 13\ 11\ 7\ 12\ 9\ 3\ 4\ 6\ 10\ 5\ 8),(1\ 4\ 11\ 10\ 12\ 8\ 3\ 13\ 6\ 7\ 5\ 9) \end{Bmatrix}$

Table 1: Latin Quandles of cyclic type with prime cardinality up to 13

The results are summarized in table 1, which gives a classification of Latin quandles of cyclic type with prime cardinality up to 13. Then note that $\#Q_n$ denotes the cardinality of Q_n we note that table 1 agrees with some previous known results [11], [8]and[11]. By looking at the classification we conjecture the following,

Conjecture 4.5: let $n \geq 3$ then there exist a Latin quandles of cyclic type if and only if n is a power of a prime number.

V. PROOF OF THE MAIN THEOREM

In this section we proof theorem 4.1, which gives a bijection from Q_n onto C_n.

Proof :

For this proof, we define auxiliary sets P_n and R_n, then we constructed bijections

$$g_3 : Q_n \to P_n \text{ , } g_2 : P_n, \; g_1 : R_n \to C_n \qquad (5.1)$$

In this subjection, we define a set R_n and constructed bijection from on to R_n onto C_n (5.1) is a bijection from R_n onto C_n. Recall that $Q := \{1,...,n\}$, *and* $(S_n)_{n-1}$ is the subset of S_n consisting of all cyclic permutations of order n-1.

Two subsets $\omega, \omega' \subset S_n$ are said to be conjugate if there exists $g \in S_n$ such that $g^{-1}\omega g = \omega^{-1}$.

Definition 5.1 we denoted by $R_n^{\#}$ the set of $\omega \subset (S_n)_{n-1}$ satisfying

$(R1) \forall s \in \omega, S^{-1}\omega s \subset \omega$, *and*

$(R2) \forall x \in Q$; *there exist* $s \in \omega \; s(x) = x$

We also denoted by R_n the set of conjugate classes $[\omega]$ of $\omega \in R_n^{\#}$. Firstly we study $R_n^{\#}$. Note that condition (R1) and (R2) are presented by conjugation. Namely; if $\omega \in R_n^{\#}$ and is conjugate o ω', then one has $\omega' \in R_n^{\#}$. Therefore, the following lemma yields that every $\omega \in R_n^{\#}$ satisfies $\#\omega = n$

Lemma 5.2 Let $\omega \in R_n^{\#}$. for each $x \in Q$, denote by $S_x^w \in \omega$ the unique element with $S_x^w(x) = x.$ Then, the obtained map $S^{\omega}; \; Q \to \omega$ is bijective.

Proof

We show that S^{w} is surjective. Take any $S \in \omega$. Since $s \in (S_n)_{n-1}$, there exist $x \in Q$ such that $S(x) = x$ by definition. Thus x is the unique fixed point of $S_x^w \in (S_n)_{n-1}$. Similarly, y is the unique fixed point of S_y^w. This concludes x = y

Lemma 5.3 the above defined map $S^{\omega} \circ Q \to (S_n)_{n-1}$ satisfies $S^{\omega} \in C_n^{\#}$, that is (Q, S^{ω}) is a Latin quandles of cyclic type.

Proof:

By definition S^{ω} Satisfies (S1). Hence we have only to show (S3). Take any $x, y \in Q$. Condition (R1) yields

$$S_y^w (S_x^{\omega})^{-1} \circ S_y^w \in \omega \tag{5.2}$$

$$S_y^w \circ (S_x^{\omega})^{-1} \circ S_y^{\omega}(S_x^{\omega}(y)) = S_y^{\omega} \circ S_x^{\omega}(y) = S_y^{\omega}(x) \tag{5.3}$$

Therefore from the uniqueness in (R2), we have;

$$S_y^w \circ (S_x^{\omega})^{-1} \circ S_y^{\omega} = S^{\omega}(S_x^{\omega}(y))^{-1} = S^{\omega}_{s_x^{\omega}(y)} \tag{5.4}$$

This proves (S3). This completes the proof.

Lemma 5.4: The following map is surjective

$$\bar{g}_1 : R_n^{\#} \to C_n^{\#} : \omega \to S^{\omega}$$

Proof:

Take any $S \in C_n^{\#}$. let us put

$$\omega := \left\{ s_x \middle| x \in Q \right\} \subset (S_n)_{n-1} \tag{5.5}$$

We prove that $\omega \in R_n^{\#}$ *and* $\bar{g}_1(\omega) = s$

We show that ω satisfies (R1). Take any $s_x, s_y \in \omega$ since s_x^{-1} is an automorphism, one has

$$s_x \circ s_y^{-1} \circ s_x = S_{S_y^{-1}}(x) \in \omega \tag{5.6}$$

This proves $S_y^{-1} \omega s_y \subset \omega$

Next we show that ω satisfies (R2). Take any $x \in Q$. Since s satisfies (S1), $s_x \in \omega$ *satisfies* $s_x(x) = x$. Since $s \in C_n^{\#}$, one has $s_y \in (S_n)_{n-1}$.hence x is the unique fixed point of s_y. Thus (S1) yields that x = y which proves the uniqueness. By definition of $\bar{g}_1$, it is obvious to see that $\bar{g}_1(\omega) = s$ this completes the proof.

Lemma 5.5 the following map is well-defined;

$g_1 : R_n \to C_n : [\omega] \to [s^{\omega}]$

Proof:

Let $\omega, \omega^{'} \in R_n^{\#}$, and assume that $[\omega] = [\omega^{'}]$. Hence there exists $g \in S_n$ such that ω=$g^{-1}\omega^{-1}g$. In order to show$[S^{\omega}] = [S^{\omega^{'}}]$, it is enough to prove that the following map is a quandles Isomorphism;

$g : (Q, S^{\omega}) \to (Q, S^{\omega^{'}})$ This is obvious bijective. We show that g is a quandles homomorphism. Take any $x \in Q$. By definition, one has

$$S^{\omega^{'}}_{g(x)}(g(x)) = g(x) \qquad (5.8)$$

This means that

$$S^{\omega^{'}}_{y(x)} \circ g^{-1} \circ S^{\omega}_{y(x)} = x \qquad (5.9)$$

on the other hand one has

$$S^{\omega^{'}}_{y(x)} \circ g^{-1} \circ g \in \omega g^{-1} g = \omega \qquad (5.10)$$

Hence , from the uniqueness in (R2). We have

$$S^{\omega^{'}}_{g(x)} \circ g^{-1} g = S^{\omega}_{x} \qquad (5.11)$$

This proves that g is a quandles homomorphism. We now show that the above defined map $g_1$is bijective. The following is the main result of this subsection.

Proposition 5.6 The map $g1 : R_n \to C_n$ is bijective

Proof:

Recall that g1 is surjective, since, so is $\bar{g}1$from lemma (5.4). it remains to show that g1 is injective. Let $[\omega],[\omega^{'}]\in R_n$, and assume that $([\omega])=g1([\omega])$. By definition, one has $[S^{\omega}]=[S^{\omega^{'}}]$, that is there exist a quandles isomorphism.

$$g:(Q\,,S^{\omega^{'}})\rightarrow(Q\,,S^{\omega^{'}}) \tag{5.12}$$

Since g is bijective, we have $g\in S_n$. Since g is a homomorphism, we have for any $x\in Q$ that

$$S_x^{\omega}=S_{y(x)}^{\omega}\circ g^{-1}\circ S_{y(x)}^{\omega}(x)\in\omega^{'}g^{-1}\omega \tag{5.13}$$

this proves $\omega\subset\omega^{'}g^{-1}g$. Recall that $\#\omega=n=\#\omega^{'}$holds from lemma 5.2. therefore, we have $\omega=\omega g^{-1}g$, and thus $[\omega]=[\omega^{'}]$. this concludes that g1 is injective. (5.2) is a bijection from P_n onto R_n. We denoted by

$$S_n,(1,2):=\left\{u\in S_n\,\middle|\,u(1)=1,u(2)=2\right\} \tag{5.14}$$

two elements $(u_1,u_2),(v_1,v_2)\in(S_n)_{n-1}\times(S_n)_{n-1}$are said to be $S_n,(1,2)-$ conjugate if $(u_1,u_2)=(\lambda^{-1}v_1\lambda,\lambda^{-1}v_2\lambda)$ for some $\lambda\in S_n(1,2)$.

Definition 5.7 we denote by $P_n^{\#}$ the set of $(u_1,u_2)\in(S_n)_{n-1}\times(S_n)_{n-1}$satisfying.

(P1) $u_1(1)=1,u_2(2)=2$ and
(P2) $\left\{u_2u_1^{-1}u_2\right\}=\left\{u_1u_2^{-1}u_1\right\}$

We also denote by p_n the set of $S_n(1,2)-$conjugacy classes $[(u_1,u_2)]$ of $(u_1,u_2)\in P_n^{\#}$ firstly of all, we construct a map from $P_n^{\#}$ to $R_n^{\#}$.

Lemma 5.8: let$(u_1,u_2)\in P_n^{\#}$. Then one has

$$\omega_{(u_1,u_2)}:=\left\{u_1,u_2\right\}\cup\left\{u_2u_1^{-1}u_2\right\}\in R_n \tag{5.15}$$

Proof :

we need to show that $\omega_{(u_1,u_2)}$satisfies (R1) and (R2). In order to show (R1), it is enough to prove

$$u_2\omega_{(u_1,u_2)}u_2\subset\omega_{(u_1,u_2)},u^{-1}\omega_{(u_1,u_2)}u_1\subset\omega_{(u_1,u_2)} \tag{5.16}$$

Note that u_1 has order n-1. Then one has

$$u_1u_1^{-1}u_1 = u_1 \in \omega_{(u_1,u_2)}$$

$$u_2u_1^{-1}u_2 = u_1u_2^{-1}u_1 \in \omega_{(u_1,u_2)} \tag{5.17}$$

This proves (5.16) and (P2) yield that

$$\omega_{(u_1,u_2)} = \{u_1, u_2\} \cup \{u_2u_1^{-1}u_2\} \tag{5.18}$$

Next (R2). Take any $x \in Q$. *if* $x = 1, 2$, then it is fixed by $u_1, u_2 \in \omega_{(u_1,u_2)}$, respectively. Assume $x \neq 1, 2$, by (P1) and $u_1 \in (S_n)_{n-1}$, there exists -1 such that $x = u_1^{-1}(2)$. then one has

$$u_2u_1^{-1}u_2(x) = u_2u_1^{-1}u_2(u_2(1)) = u_2u_1^{-1}(1) = u_1^{-1}(2) = x \tag{5.19}$$

This completes the proof of the existence. On the other hand, by definition, one has $\#\omega_{(u_1,u_2)} \leq n$.

This shows the uniqueness.

Lemma 5.9: the following map is defined

$$g2 : p_n \to R_n : [(u_1, u_2)] \to [\omega_{(u_1,u_2)}]$$

Proof: let $\left[\left(u_1, u_2\right)\right], \left[\left(u_1^{'}, u_2^{'}\right)\right] \in p_n$ and assume that $\left[\left(u_1, u_2\right)\right] = \left[\left(u_1^{'}, u_2^{'}\right)\right]$, then there exist $\lambda \in S_{n,(1,2)}$ such that

$$u_1 = u_1^{'}\lambda^{-1}\lambda \ , \ u_2 = u_2^{'}\lambda^{-1}\lambda \tag{5.20}$$

$$w_{(u_1,u_2)}\lambda^{-1}\lambda = w_{(u_1,u_2)} \tag{5.21}$$

Since $w_{\left(u_1^{'},u_2^{'}\right)}, w_{(u_1,u_2)} \in R_n^{\#}$, hence $\#w_{\left(u_1^{'},u_2^{'}\right)} = n = w_{(u_1,u_2)}$ by lemma (5.2). this complete the proof of $\left[w_{(u_1,u_2)}\right] = \left[w_{\left(u_1^{'},u_2^{'}\right)}\right]$

Next we need to prove that g2 is bijective by constructing inverse map from $R_n^{\#}$ *to* $P_n^{\#}$. Recall that

$$g1 : R_n^{\#} \to C_n^{\#} \, ; w \to S^w \tag{5.22}$$

Lemma 5.10: Let $w \in R_n^{\#}$ then one has $\left(s_1^w, s_2^w\right) \in P_n$

Proof: we put $s_x := s_x^w$ for which $x \in Q$ by definition, (s_1, s_2) obviously satisfies $(P1)$ we need to show $(P1)$

Firstly we claim that

$$\{s_2 s_1^{-1} s_2\} = \{s_x | x = 3, 4, \dots n\} \quad (5.23)$$

Since w satisfies $(R1)$ we have

$$s_2 s_1^{-1} s_2 = s_2 w s_2^{-1} \subset w \quad (5.24)$$

Thus, it follows from $s_2 s_1^{-1} s_2 (s_2(1)) = s_2(1)$ and the uniqueness in $(R2)$ *that*

$$s_2 s_1^{-1} s_2 = S_{s_2}(1) \quad (5.25)$$

Since $s_2(1) = 1$ *and* $s_2 \in (s_n)_{n-1}$ *then*

$$\{s_2(2)\} = \{3, 4, \dots, n\} \quad (5.26)$$

This completes the proof of the claim. The above lemma gives a map from $R_n^{\#}$ *to* $P_n^{\#}$. Next is to show that the map induces a map from R_n *to* P_n

Lemma 5.11: The following map is well-defined;

$$f_2 : R_n \to P_n : [w] \to [(s_1^w, s_2^w)] \quad (5.27)$$

Proof: let $[w], [w^{'}] \in D_n$, and assume that $[w] = [w^{'}]$ by definition, there exist $g \in s_n$ such that $w = g^{-1} w^{'} g$. It then follows from lemma 5.5 that

$$g : (Q, s^w) \to (Q, s^{w^{'}}) \quad (5.28)$$

Is a quandles isomorphism. Note that $(Q, s^{w^{'}})$ is of cyclic type, and hence two points homogeneous. Therefore, since $g(1) \neq g(2)$, there exist $h \in \mathrm{I}nn(Q, s^{w^{'}})$ such that

$$h \circ g(1), h \circ g(2) = (1, 2). \quad (5.29)$$

These yields $h \circ g \in s_{n,(1,2)}$. note that $h \circ g$ is a Latin quandles isomorphism from (Q, s^w) *onto* $(Q, s^{w^{'}})$. Thus we have

$$\left(h\circ g\right)\circ s_1^{w}\circ\left(h\circ g\right)^{-1}=s_{h\circ g(1)}^{w'}=s_1^{w'} \tag{5.30}$$

This completes the proof of $\left[\left(s_1^{w},s_2^{w}\right)\right]=\left[\left(s_1^{w'},s_2^{w'}\right)\right]$. By showing that f_2 is the inverse of g_2. We have the following main result of this subsection.

Proposition 5.12: The map $g_2 : P_n \to R_n$ is bijective

Proof: we show that f_2 is the inverse map of g_2. It is clear that the composition $f_2\circ g_2$ is identity mapping. Consider $g_2\circ f_2 : R_n \to R_n$, and take any $[w]\in R_n$ then one has $f_2([w])=\left[\left(s_1^{w},s_2^{w}\right)\right]$. One also has $g_2\circ f_2([w])=\left[w'\right]$ where

$$w:=\left\{s_1^{w},s_2^{w}\right\}\cup\left\{s_2^{w}\left(s_1^{w}\right)^{-1}s_2^{w}\right\} \tag{5.31}$$

Since s^{w} is a quandles structure, one can see $w'\subset w$ thus we have $w'=w$ for cardinality reason. This shows that $g_2\circ f_2$ is the identity mapping. Recall (5.3) is a bijection from Q_n *onto* R_n. We lastly construct a bijection from Q_n *onto* P_n, let $s_1:=(2,3,\ldots n)$ and recall that Q_n Is the set of $s_2\in\left(s_n\right)_{n-1}$ satisfying $Q(1)$ *and* $(Q2)$

Proposition 5.13: the following map is bijective

$$g_3 : Q_n \to P_n : S_2 \to \left[\left(s_1,s_2\right)\right]$$

Proof: we show that g_3 is surjective. Take any $\left[\left(u_1,u_2\right)\right]\in P_n$ since $u_1\in\left(s_n\right)_{n-1}$ *and* $u_1(1)=1$ we can write $u_1=\left(2a_3a_4\ldots a_n\right)$. Let us define $g\in s_n,(1,2)$ by

$$g:\begin{pmatrix}1 & 2 & 3\ldots & n\\ 1 & 2 & a_3\ldots & a_n\end{pmatrix} \tag{5.32}$$

An easy computation show $g\circ g^{-1}u_1=s_1$ let $s_2:=g\circ g^{-1}u_2$. Then s_2 obviously satisfies $(Q1)$. Furthermore, since $\left(u_1,u_2\right)$ satisfies (Pn). One can see that s_2 satisfies $(Q2)$. We thus have $s_2\in Q_n$. This concludes that g_3 is surjective since

$g_3\left(s_2\right)=\left[\left(s_1,s_2\right)\right]=\left[\left(g\circ g^{-1}\circ u_1,g\circ g^{-1}\circ u_2\right)\right]=\left[\left(u_1,u_2\right)\right]$ we show that g_3 is injective. Let $s_2,s_2'\in Q_n$ and suppose that $g_3\left(s_2\right)=g_3\left(s_2'\right)$. Hence there exist $h\in s_n,(1,2)$ such that

$$\left(s_1,s_2'\right)=\left(h^{-1}\circ s_1\circ h,h^{-1}\circ s_2\circ h\right) \tag{5.33}$$

By definition one has $h(1)=1$ and $h(2)=2$ then it follows from $h(2)=2$ that

$$3 = s_1(2) = h \circ s_1^{-1} \circ h(2) = h^2 \circ s_1^{-1}(2) = h(3) \quad (5.34)$$

Similarly, this yields that

$$4 = s_1(3) = h \circ s_1^{-1} \circ h(3) = h^2 \circ s_1^{-1}(3) = h(4) \quad (5.35)$$

One can show inductively that $x = h(x)$ for $x = any\ x \in Q$ this means that $h = id$, *and thus* $s_2^{'} = s_2$ this shows that g_3 is injective.

a. CONSTRUCTING QUANDLES OF CYCLIC TYPE FROM Qn

In the previous subsections. We have constructed the following bijections

$$g_3 : Q_n \to P_n,\ \ g_2 : P_n \to R_n, g_1 : R_n \to C_n \quad (5.36)$$

In this subsection, we describe $g_1 \circ g_2 \circ g_3(s_2)$ for each $s_2 \in Q_n$. Take any $s_2 \in Q_n$. Recall that $s_1 := (2, 3...n)$ and

$$w_{(s_1,s_2)} := \{s_1, s_2\} \cup \{s_2 s_1^{-1} s_2\} \quad (5.37)$$

Then one has $g_2 \circ g_3(s) = \left[w_{(s_1,s_2)} \right]$. We put

$$\psi(s_2) := s^w(s_1, s_2) \in C_n^{\#} \quad (5.38)$$

This means $g_1 \circ g_2 \circ g_3(s_2) = [\psi(s_2)]$. Note that $(\psi(s_2))_i \in w_{(s_1,s_2)}$ is defined as the unique element fixing $i \in Q$. This immediately yields

$$(\psi(s_2))_1 = s_1, (\varphi(s_2))_2 = s_2 \quad (5.39)$$

Let $i \in \{3, ...n\}$. Then one has $i = s_1^{-1}(2)$ and hence

$$s_2 s_1^{-1} s_2 = s_1 s_2^{-1} s_1 = s_3 \quad (5.40)$$

This concludes that

$$(\psi(s_2))_i = s_2 s_1^{-1} s_2 \quad (5.41)$$

This concludes the proof of theorem 4.1.

VI. CONCLUSION

Theorem 4.1 presents abstract construction of Latin quandles of prime order which gave rise to Latin quandles of cyclic type. These constructions help to established the concept of isomorphism in definition 2.3 between any given Latin quandles of the same order . The Latin quandle order structure is also presented on table 1 using cyclic permutation of order n-1 as shown in Q_n which is the inner automorphism. Note that $s_x\left(x \in Q\right)$ is an automorphism of (Q,s) the subgroup of Aut (Q,s) generated by $\left\{s_x \middle| x \in Q\right\}$ is called the inner automorphism group of (Q,s) and denoted by Inn(Q,s).

The results are summarized in table 1, which gives a classification of Latin quandles of cyclic type with prime cardinality up to 13. All Latin quandles presented were thoroughly verified using Maple software.

However, the classification of Latin quandle of cyclic type of order p > 13 up to isomorphism is still very open for future research especially for its fruitful application on cryptography.

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