A finite quandle is therefore connected if, for every a, b ∈ A, there exist x1, . . . , xn ∈ A such that
b = x1∗(x2∗(. . . (xn ∗ a))) (compare to unique right division!). Connected quandles are arguably the most important class of quandles, both from the algebraic and topological points of view. Indeed, Latin quandles are connected, and the class of connected quandles is a very natural generalization of left distributive quasigroups: many structural properties of left distributive quasigroups extend to connected quandles.
To illustrate the power of connectedness, let us prove the following implication for quandles that are (both left and right) distributive.

Proposition 3.1 [3] Finite connected distributive quandles are quasigroups.
Proof. Assume the contrary, and let (Q, ∗) be the smallest counter example. Right distributivity says that every right translation Ra is a homomorphism, hence, its image, Ra(Q), forms a subquandle that is also connected and distributive (both properties project to homomorphic images). For every a, b ∈ Q, the subquandles Ra(Q) and Rb(Q) are isomorphic: connectedness of (Q, ∗) provides an automorphism α ∈ LMlt(Q, ∗) such that α (a) = b, and it follows from α (x ∗ a) = α (x) ∗α (a) = α (x) ∗ b that α restricts to an isomorphism between Ra(Q) and Rb (Q). Therefore, by minimality, all subquandles Ra(Q) are proper subquasigroups. Now we prove that Ra(Q) ⊆ Rx∗a(Q) for every x, a ∈ Q. Let y ∗ a ∈ Ra(Q). Since Ra(Q) is a quasigroup, there is z ∗ a ∈ Ra(Q) such that y ∗ a = (z ∗ a)∗(x ∗ a). Hence y ∗ a ∈ Rx ∗a (Q). By induction, Ra (Q) ⊆ Rx_1 ∗a(Q) ⊆ Rx2∗(x1∗a)(Q) ⊆ . . ., and thus, from connectedness, Ra(Q) ⊆ Rb(Q) for every a, b ∈ Q. Hence all subquasigroups Ra(Q) are equal, and since x ∈ Rx(Q) for every x ∈ Q, all of them are equal to Q, a contradiction.

3.2 Loops. A loop is a quasigroup (Q, ·) with a unit element 1, i.e. 1 · a = a · 1 = a for every a ∈ A. Loops will be denoted multiplicatively. To avoid parenthesizing, we
shortcut x · yz = x · (y · z) etc., and we remove parentheses whenever the elements associate, that is we shall write xyz whenever we know that x · y z = x y · z. For all unproved statements, we refer to any introductory book on loops, such as [4, 11].
Let (Q, ·) be a loop. Inner mappings are those elements of the multiplication group Mlt(Q, ·) that fix the unit element. For example, the conjugation mappings Tx(z) = xz/x are inner and, in a way, measure the non-commutativity in the loop. The left inner mappings are defined by Lx,y(z) = (xy)\(x · yz) and measure the non-associativity from the left.
The most common example of loops are groups (i.e. associative loops), and most classes of loops studied in literature are those satisfying a weak version of associativity or commutativity.

We list a few weak associative laws (note that all the conditions hold in groups): a loop is called
• diassociative if all 2-generated sub loops are associative;
• left alternative if x · x y = x_2 y;
• power-associative if all 1-generated subloops are associative;
• Moufang if (xy · x)z = x(y · xz) (the dual law is equivalent in loops);
• left Bol if (x · yx)z = x(y · xz);
• automorphic if all inner mappings are automorphisms.
• left automorphic if all left inner mappings Lx,y are automorphisms.

Moufang's theorem [13] says that in a Moufang loop, every subloop generated by three elements that associate, is associative. In particular, Moufang loops are diassociative, since a(ba) = (ab)a for every a, b, as directly follows from the Moufang law. Bol loops are power-associative.
The nucleus of a loop (Q, ·) is the set of all elements a ∈ Q that associate with all other elements, That is N = {a ∈ Q : a · x y = a x · y, x · a y = x a · y, x · y a = x y · a for all x, y ∈ Q}.

An element of a loop is called nuclear if it belongs to the nucleus. A mapping f : Q → Q is called k-nuclear if $x_k f(x) \in N$ for every $x \in Q$.

4.0 QUASIGROUPS AND LATIN QUANDLES

A Latin square is an n x n array on n symbols having the property that each symbol/ element appears exactly once in each row and column. A set with a binary operation that whose multiplication table is Latin square is called a quasigroup [Definition 2.10].

Smith [16], a quasigroup (Q, .) is a set equipped with binary multiplication . such that in the equation $x\ .y = z,$ (1)

Knowledge of any two of $x,\ y,\ z$ specifies the third uniquely. It follows that for each element $x \in Q$, the right multiplication R(x): Q → Q; $y \to y\,.x$ (2)

And left multiplication L(x): Q → Q; $y \to x\,.x$ (3)

are permutations of Q. Therefore a quasigroup that is self distributive is a Latin quandle [3]. And for Latin quandles, the concept of left (a\b) and the right (b/a) divisibility operations are unique which they inherit from their parent structure (quasigroup). Therefore, Latin quandles are special quandles. Axiomatically we will define Latin quandles below;

4.1 Definition: [3, 14] An algebraic structure (Q, ∗) is called a Latin quandle if it obeys the following laws simultaneously:

i. $x*x = x$ (idempotent law)
ii. $a*x = b$ (left divisibility law)
iii. $y*a = b$ (right divisibility law)
iv. $a*(x*y)=(a*x)*(a*y)$ (left distributive law)
v. $(x*y)*a = (x*a)*(y*a)$ (right distributive law)
vi. $(x*y)*(u*v) = (x*u)*(y*v)$ (mediality property)
vii. $x*(x*y) = y$ left involutory or (left symmetric) (hence we have unique left division with $x\backslash y = x*y$)

We observed that left distributive quasigroups are idempotent: $x*(x*x) = (x*x)*(x*x)$ by left distributivity and we can cancel from the right.

4.1 Remark. Every left distributive quasigroup that is idempotent is a Latin quandle.

Non – idempotent medial quasigroups exist indeed, abelian groups are examples. Also it is observed that idempotent trimedial binary algebras are distributive given a, b, c ϵ A, the sub algebra (a, b, c) is medial, hence $(a*b)*(a*c) = (a*a)*(b*c) = a*(b*c)$ and dually for right distributivity;

A binary algebra is called a left quandle, if it is idempotent, left distributive and has unique left division (remarkably, the three conditions correspond to the three Reidemeister moves in Knot Theory) [18]

Quandles that also have unique right division are called ***Latin quandles.*** Indeed Latin quandles and left distributive quasigroups and the very same things.

Alhamdadi [17] hence discussed that Latin quandles are right distributive quasigroups and left – distributive Latin Quandles are distributive quasigroups. Given a binary algebra (A, $*$), it is natural to consider *left translations La(x)* = a$*$ x, and R_a (x) = x$*$ a, and the semigroups they generate, the left multiplication semi-group LMlt (A, $*$) = (L_a : a ϵ A), the right multiplication semigroup Mlt (A, $*$) = (L_a, R_a : a ϵ A). Unique left division turns left translations into permutations, thus the left multiplication semi group into a group (and usually for right translations). We observe that L_a^{-1} (x) = a\ x and R_a^{-1} (x) = x/a. Also note that (A, $*$) is left distributive if and only if L_a is an endomorphism for every a ϵ A. Hence, in quandles, LMlt (A, $*$) is a subgroup of the automorphism group.

4.2 Remark: All Latin quandles are quasigroup but all quasigroups are not Latin quandles. Therefore Latin quandles generally obey the following;

$$a * (a\backslash b) = b \quad (4)$$

$$a \backslash (a * b) = b \quad (5)$$

Since $$a * x = b \leftrightarrow x = a\backslash b \quad (6)$$

And $$x \backslash x = x = x/x \quad (7)$$

For universal consideration, (Q, *, \ ) is quandle. But (Q, *, \ ) is a Latin quandle.

Therefore, while quandle generally are equipped with two binary operations, Latin quandles aare equipped with three binary operations. This structure is completely devoid of the identity element e such that $ae = ea = a$ otherwise, (Q, *, e \, /) is a loop.

4. 3 Definition [3, 14] A finite quandle Q is therefore connected if for every $a, b \in Q$ there exists $x_{1, *}$ (x_2 * (… (… (x_n * a)))).

The above definition however means that the inner mappings group of Q act transitively on Q. **Remark 4.3** Therefore All Latin Quandles are connected quandles, (proposition 3.1) since,

$L_{x/y}$ (y) = x [15]. That is if (Q, *) is a Latin quandle, for all x, y ∈ Q, one can write (x/ y) * y = x, or x = y * (y\x).

5. 0 CONSTRUCTION OF LATIN QUANDLES:

5. 1Theorem. Let Q be a commutative quasigroup of order 2n + 1; n≥ 1 such that

$x + y = y^{-1} . xx \; for \; all \; x, y \in Q.$

Then (Q, +) is a Latin quandle.

Proof:

Since Q is already a quasigroup, then the left and right division laws holds. We only show that idempotent and distributive laws hold in Q. Let $x, \; y, \; z \in$ Q

i) Let $x \in Q$, then $x + x = x^{-1}. xx = (x^{-1} x) = ex = x$ (idempotent law).

ii) $(x + y) + z = (y^{-1}. xx) + z = z^{-1}. ((y^{-1}. xx)(y^{-1}. xx)$ (8)

Similarly

$(x + z) + (y + z) = (z^{-1}. xx) + (z^{-1}. yy) = z(yy^{-1}) (z^{-1}. xx) (z^{-1}. xx)$ (9)

Next we need to show that equation (8) = (9)

$(z^{-1}. xx) + (z^{-1}. yy) = z(yy^{-1}) (z^{-1}. xx)(z^{-1}. xx)$ then

$z(yy^{-1}) (z^{-1}. xx)(z^{-1}. yy) \quad ^{-1} . (z^{-1}. xx)$

$\left((z^{-1}))^{-1}(y)^{-1})(z^{-1}. xx)\right) . \left((z^{-1}))^{-1}(y)^{-1})(z^{-1}. xx)\right)$

$\left(1 . (y)^{-1})(z^{-1}. xx)\right). \left(1 . (y)^{-1})(z^{-1}. xx)\right)$

Recall that $(x + y) = (y^{-1}. xx)$ then

$(z^{-1}(x + y) . (z^{-1}(x + y)$

$(z^{-1}(x + y)(x + y))$ (10)

This concludes the prove for the right distributive law.

Similarly, for the left distributive law $x + (y + z) = (x + y) + (x + z)$ holds.

Example 5.1 Quasigroup (Q, .) order 5 in a Cayley Table representation

.	1	2	3	4	5
1	1	2	3	4	5
2	2	3	4	5	1
3	3	4	5	1	2
4	4	5	1	2	3
5	5	1	2	3	4

Table 1

Example 5.2 Latin quandle (Q,+) order 5 generated from example 5.1 in a Cayley Table representation

+	1	2	3	4	5
1	1	5	4	3	2
2	3	2	1	5	4
3	5	4	3	2	1
4	2	1	5	4	3
5	4	3	2	1	5

Table 2

Theorem 5.2:

Let Q be a non- commutative self invertible Quasigroup of order 2n + 1; n $\geq$ 1

Such that $x + y = y\, x^{-1}\, y\ for\ all\ x, y\ \in Q.$

Then (Q, +) is a Latin Quandle of order 2n +1.

Proof:

Let $x,\ y,\ z \in$ Q then,

i) $x + x = x\, x^{-1}\, x = x$ (idempotent law)
ii) Left and right distributive laws
a) $x + (\ y +\ z) = (x + y) +\ \ (x + z)$
b) $(x + y) +\ z = (x + z) + (y + z)$
iii) Since Q is already a quasigroup, then the left and right division laws holds. We only show that idempotent and distributive laws hold in Q.

Next we show that Let $x \in\ Q$, then $x + x = x\, x^{-1}\, x\ = (x\, x^{-1}\, x) = ex = x$ (idempotent law).

For left distributive law we show that (ii a) holds.

Let $x,\ y,\ z \in$ Q then, $x + (\ y +\ z) = (x + y) +\ \ (x + z)$

Since Q is a quasigroup, $x + (\ y +\ z) = x + (z\, y^{-1}\, z)\ = z\, y^{-1}\, z\ \ x^{-1}\, z\, y^{-1}\, z$ (11)

Then $(x + y) +\ \ (x + z) =\ y\, x^{-1}\, y +\ z\, x^{-1}\, z)$

$= z\, x^{-1}\, z\, (y\, x^{-1}\, y)^{-1}\ +\ z\, x^{-1}\, z$ (12)

Next, we need to show that (11) = (12)

$z\,x^{-1}\,z\,(y\,x^{-1}\,y)^{-1}\;\;z\,x^{-1}\,z$

$z\,x^{-1}\,z\,(\,y(x^{-1}\,)^{-1}\,y\,)\,z\,x^{-1}\,z$

$(z\,x^{-1}\,zy^{-1}\,)\,(y^{-1}\;\;z\,x^{-1}\,z)$

$x^{-1}\,(\,(z\,y^{-1}\,z)\;\;(z\,y^{-1}\,z))$

$x^{-1}\,(\,(y+z)\;\;(\,y\;+z))$

Therefore,

$$x+\;(\,y+z)=\;(\,(z\,y^{-1}\,z)\,x^{-1}\;\;(z\,y^{-1}\,z)) \qquad (13)$$

Note equation (11) = (13) and we conclude that equation (11) = (12)

Similarly, for the right distributive law,

$$\big(\,(z\,y^{-1}\,z)\,x^{-1}\;\;(z\,y^{-1}\,z)\big)=\;z\,x^{-1}\,z\,(\,y(x^{-1}\,)^{-1}\,y\,)\,z\,x^{-1}\,z$$

This concludes the proof.

Example 5.3 Quasigroup (Q,.) of order 7 in a Cayley Table representation

.	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	7	1	2	3	4	5	6
3	6	7	1	2	3	4	5
4	5	6	7	1	2	3	4
5	4	5	6	7	1	2	3
6	3	4	5	6	7	1	2
7	2	3	4	5	6	7	1

Table 3

Example 5.4 Latin Quandle (Q, +) of order 7 generated from example 5.3 in a Cayley Table representation

+	1	2	3	4	5	6	7
1	1	3	5	7	2	4	6
2	7	2	4	6	1	3	5
3	6	1	3	5	7	2	4
4	5	7	2	4	6	1	3
5	4	6	1	3	5	7	2
6	3	5	7	2	4	6	1
7	2	4	6	1	3	6	7

Table 4

6. 0 CONCLUSION

Quasigroups in general has been established with Theorem 5.1 and 5.2 as the parent algebraic structure of Latin Quandles which is very fruitful for future research work especially on investigation on some inverse properties peculiar to Latin quandles

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