Global Scientific JOURNALS

GSJ: Volume 9, Issue 3, March 2021, Online: ISSN 2320-9186

www.globalscientificjournal.com

ON COMPARATIVE PERFORMANCES OF ARIMA, HYBRID ARIMA-ARCH AND HYBRID ARIMA-GARCH MODELS IN MODELING THE VOLATILITY OF FOREIGN EXCHANGE

Adam, Shuaibu Idris[1], Yusuf Abbakar Mohammed[2] and Akeyede, Imam[3].

[1,2]*Department of Mathematical Science, University of Maiduguri, P.M.B 1069 Maiduguri, Nigeria.*
[1]*shabaddeen88@gmail.com*, [2]*yusufabbakarm@gmail.com.*

[3]*Department of Statistics, Federal University Lafia, P.M.B 146, Lafia, Nigeria.*
[3]*Akeyede.imam@science.fulafia.edu.ng.*

Abstract

Foreign exchange markets are important in the field of finance in order to measure the currency value of a country with respect to another. The foreign exchange rates data are volatile time series as they have huge price swings and jumps in a shortage or demanding periods. It is known that, most economic and financial data are either non-linear or non-stationary, which is a problem when inappropriate model is applied and the result of the prediction may be inaccurate. In this study, we compared the performances of ARIMA, hybrid ARIMA-ARCH and hybrid ARIMA-GARCH models in modeling the volatility of the foreign exchange (official exchange rates). The capabilities of the models were evaluated using some selected criteria. It was concluded that, the hybrid ARIMA-ARCH/GARCH models performed better compared to Box-Jenkins ARIMA model in terms of fitting and forecasting the official exchange rates.

Keywords

ARIMA, HYBRID ARIMA-ARCH AND HYBRID ARIMA-GARCH.

1.0 Introduction

Volatility measures the dispersion of asset price returns. Recognizing the importance of foreign exchange volatility for risk management and policy evaluation; academics, policy makers, regulators, and market practitioners have long studied and estimated models of foreign exchange volatility and jumps. Financial economists have long sought to understand and characterize foreign exchange volatility, because the volatility process tells us about how news affects asset prices, what information is important and how markets process that information.

Policy makers are interested in measuring asset prices volatility to learn about market expectation and uncertainty about policy. For examples, one might think that a clear understanding of policy objectives and tools would tend to reduce market volatility. More practically, understanding and estimating asset price volatility is important for asset pricing, portfolio allocation and risk management. Traders and regulators must be considering not only to the expected return from their trading activity but also the trading strategy's exposure to risk during period of high volatility. Traders risk's-adjusted performance depends upon the accuracy of their volatility predictions. Therefore, both traders and regulators use volatility prediction as inputs to models of risk management, such as: value-at-Risk (VAR). The goal for volatility modelers has been to simultaneously account for the most prominent features of foreign exchange volatility.

To account for these characteristic, researchers started modeling weekly and daily volatility with parametric ARCH/GARCH models in the 1980s. Practitioners often use the Risk-metrics statistically models, which is a member of large ARCH/GARCH family. These models effectively describe the auto-correlation in daily and weekly volatility. At intraday horizons, however, institutional features- that is, market opening/closings and news announcement- create strong intraday patterns, including discontinuities in prices. Many researchers on intraday data sorted out the factors behind these periodic patterns and discontinuity. The use of intraday data enabled the next big advance in volatility modeling ''realized volatility'', which is the use of very high frequency returns to calculate the volatility at every instant. A few years later, researchers began to develop increasingly sophisticated models that estimate jumps and that combined auto regressive volatility jumps. In short, academic researchers have improved volatility estimation remarkably quickly in this last thirty years and policy-makers, traders and regulators have benefitted from these advances.

One of the problems of forecasting lies in the use of appropriate methods to fit the time series data depending on the nature of the data .It is known that, most economic and financial data are either non-stationary or non-linear, which is a problem when inappropriate model is applied and the result of the prediction may be in-accurate and may not give the appropriate picture of what could be the future events. Therefore, it is necessary to look for other methods which are more appropriate and produce more accurate forecasts when the data is non-linear or non-stationary .In this study, we will compare the performances of the methods of traditional Box-Jenkins ARIMA, hybrid ARIMA-ARCH and hybrid ARIMA-GARCH models.

2.0 Methodology

The first step in developing Box-Jenkins ARIMA model is to determine if the series of data is stationary or not and if there is any significant seasonality that needs to be modeled. Stationarity can be accessed from a run sequence plot. The run sequence plot should show a constant location and scale. It can also be detected from an auto-correlation plot. Specifically, non-stationarity is often indicated by an auto-correlation plot with very slow decay. Moreover, unit root tests provide a more formal approach to determining the degree of differencing such as Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Phillips-Perron unit root test. The KPSS test has null hypothesis (H_0) of level stationary against an alternative (H_1) of unit root (non-stationary). The decision rule for KPSS test is that, if the P-value of its test statistic is greater than the critical value, say 0.05, then reject the null hypothesis (H_0) of having a level stationary and conclude the alternative hypothesis that it has a unit root (non-stationary). While for Phillips-Perron unit root test reverse is the case. Stationary of the data is important because it describe the future behaviour of the process.

If the data are not stationary, we must transform them by using first difference. First differences are the data changes from one period to the next. Plotting the data of the first difference can reveal whether the data has been transformed to a stationary series or not. If it is still not stationary, the second difference is taken. Model fitting can be carried out once the stationary of the series has been achieved.

i) *Box-Jenkins ARIMA Model*

Box-Jenkins ARIMA model has been used widely in many areas of time series analysis. Since ARIMA is among the earliest models, the capability of this model always being tested and widely used as a benchmark with other time series models. Box-Jenkins ARIMA is known as ARIMA (p,d,q) model where p is the number of autoregressive (AR) terms, d is the number of difference taken and q is the number of moving average (MA) terms. ARIMA models always assume the variance of the data to be constant. The ARIMA (p,d,q) model can be represented by the following equation:

$$y_t = \emptyset_1 y_{t-1} + \ldots + \emptyset_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \quad (1)$$

Where $\varepsilon_t \sim N(0,\sigma_t^2)$, p and q are the number of autoregressive terms and the number of lagged forecast errors, respectively.

The identification of modelling the conditional mean value is based on the analysis of estimated autocorrelation and partial autocorrelation (ACF, PACF). These estimators may be strongly inter-correlated. It is therefore recommended not to insist on unambiguous determination of the model order, but to try more models. We must not forget to carry out the verification, which is based on retrospective review of the assumptions imposed on the random errors.

ii) ***ARCH Model***

ARCH (Auto-regressive conditional Heteroskedastic model) is the first and the basic model in stochastic variance modelling and is proposed by Engle (1982). The key point of this mode is that it already changes the assumption of the variation in the error terms from constant var(ε_t) = σ^2 to be a random sequence which depended on the past residuals ({ε_1,...,ε_{t-1}}). That is to say, this model has changed the restriction from homoskedastic to be heteroskedasticity. This breakthrough is explained by Baillie and Bollerslev (1989). And this is an accurate change to reflect the volatility data's features. Let ε_t as a random variable that has a mean and a variance conditionally on the information set I_{t-1}, the ARCH model of ε_t has the following properties, come from Terasvirta (2006). First, conditional mean

$$E(\varepsilon_t | I_{t-1}) = 0$$

And second, conditional variance

$$\sigma^2 = E(\varepsilon_t^2 | I_{t-1})$$

Is a positive valued parametric function of I_{t-1}, the sequence {ε_t } may be observed directly, or it may be gotten from the following formula. In the letter case, it gives;

$$\varepsilon_t = y_t - \mu_t(y_t)$$

Where y_t is observed value, and μ_t (y_t) = E(y_t|I_{t-1},) is the conditional mean of y_t given I_{t-1}, Engle's (1982) application was this type. In what follows, the ε_t could be expressed as another way on parametric forms of σ^2_t.So, here ε_t is assumed as follows:

$$\varepsilon_t = Z_t \sigma_t$$

Where {Z_t } is a sequence of independent, identically distributed (iid) random variables with Zero mean and unit variance. This implied:

$$\varepsilon_t \sim D(0, \sigma^2_t),$$

So the ARCH model of order q is like this

$$\sigma^2_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \ldots + \alpha_q \varepsilon^2_{t-q} = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i} \quad (2)$$

Where $\propto_0 > 0$, and $\propto_i >= 0$, i >0. To assure $\{\sigma^2_t\}$ is asymptotically stationary random sequence, we can assume that $\propto_1 + \propto_2 + \ldots \propto_q < 1$. This is the ARCH model.

With the generation of the ARCH model, it already can explain many problems in many fields, for instance, interest rates, exchange rates and trade options and stock index returns.

iii) ***Generalized –*** *ARCH model (GARCH).*

Because of some drawbacks and limitation on ARCH model, it has been extended by the so-called generalized ARCH (GARCH) model that Bollerslev (1986) and Taylor (1986) proposed independently of each other. Based on the ARCH model has been raised, it adds the lagged conditional variance term (σ^2_{t-j}) as a new term in the GARCH model. The improved ARCH (GARCH model) also reduces the number of estimated parameters. In this model, the conditional variance is still a linear function of its own lags and error terms, it has the following form:

$$\sigma^2_t = \propto_0 + \sum_{i=1}^{q} \propto_i \varepsilon^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j}$$

Where, σ^2_t will be replaced by h_t, now we have

$$h_t = \propto_0 + \sum_{i=1}^{q} \propto_i \varepsilon^2_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j} \quad (3)$$

$$h_t = \propto_0 + \propto_1 \varepsilon^2_{t-1} + \ldots + \propto_q \varepsilon^2_{t-q} + \beta_j h_{t-1} + \ldots + \beta_p h_{t-p}$$

where h_t is the conditional variance

h_{t-j} is the past conditional variance

ε^2_{t-i} past squared residual return.

$\propto_0 > 0$, $\propto_i >= 0$, $\beta_j >= 0$. Above is GARCH (q,p) model.

3.0 Data Analysis and Results

For the purpose of the flow of the analysis, the time series data on monthly exchange rates from government for the periods of twenty years was used for the analysis. However, the time series plot which display the observations on *y axis against equally spaced time intervals on the x axis used* to evaluates patterns and behavior in data over time as displayed in figure 1 below.

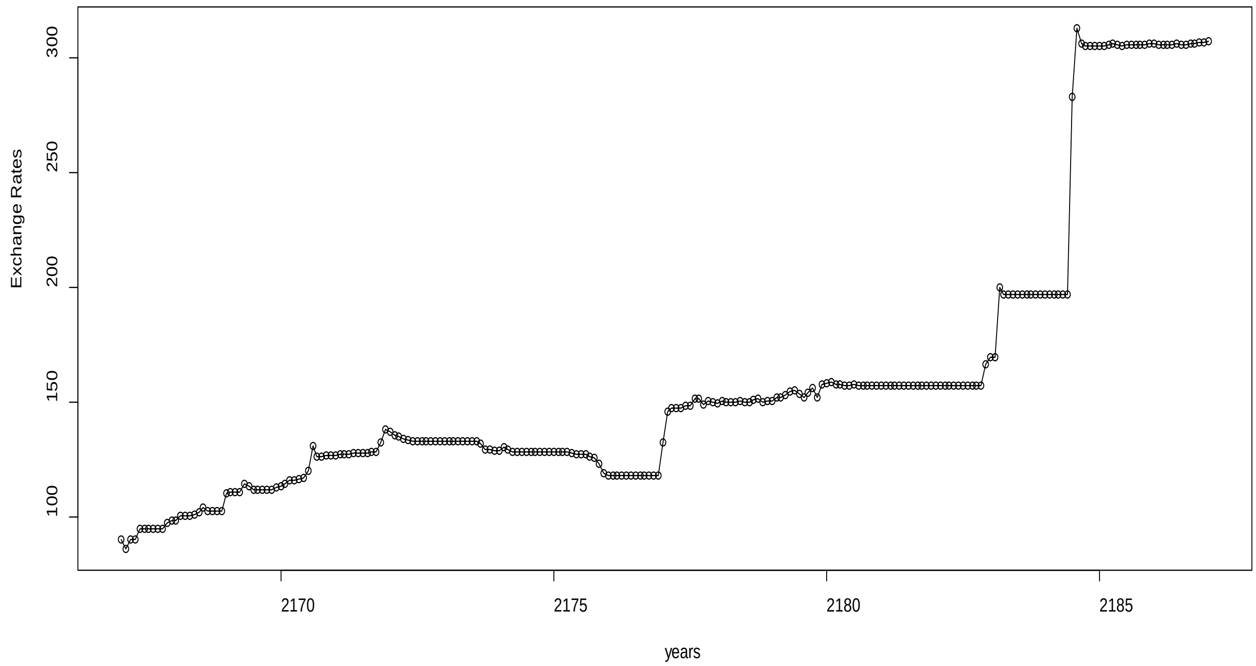

Figure 1 Official Exchange Rates

Table 1;Test For Unit Root Of Official Exchange Rates

Test statistic	Values	Lag order	P-value	Hypothesis (H_o)	Decision	Remark
ADF	-1.1283	6	0.9162	Unit root	Accept H_o	Not stationary
PP	-3.9548	4	0.8884	Unit root	Accept H_o	Not stationary
KPSS	4.1779	3	0.01	Stationary	Reject H_o	Not stationary

It is clear for the time series plot of the official exchange rate data series in figure 1 and the unit root test in table 1 suggests that the data need to be transformed or differenced since it is confirmed to have a unit root.

Table 2: Fitting the ARIMA Models after the first difference.

Model	Parameter				AIC	BIC	MSE
	AR(1)	AR(2)	MA(1)	MA(2)			
ARIMA(1,1,1)	0.2340 (±0.0641)	-	-0.9962 (±0.0297)	-	1564.67	1575.08	40.18
ARIMA(1,1,2)	-0.2640 (±0.1935)	-	-0.4547 (±0.1708)	-0.5337 (±0.1667)	1561.43	1575.02	39.36
ARIMA(2,1,1)	0.2604 (±0.0649)	-0.1304 (±0.0648)	-09904 (±0.0172)	-	1562.66	1576.55	39.59
ARIMA(2,1,2)	-0.2431 (±0.3298)	-0.0104 (±0.1219)	-0.4764 (±0.3239)	-0.5118 (±0.3232)	1563.42	1580.79	39.36

From table 2; ARIMA (1,1,2) has the smallest AIC, BIC and MSE and it is therefore regarded as the best model for fitting official exchange rate. Furthermore, the estimated coefficient values for all ARIMA (p,d,q) strictly conforms to the bounds of parameter, between -1 and 1. This has made the model to be stationary.

Table 3: Fitting hybrid ARIMA-ARCH on differenced official exchange rate

Model	ARIMA(1,1,1)+ ARCH(1)	ARIMA(1,1,2)+ ARCH(2)	ARIMA(2,1,1)+ ARCH(3)	ARIMA(2,1,2)+ ARCH(4)
AR(1)	-0.2362	-0.3034	-0.2606	0.7324
AR(2)	-	-	0.0116	0.2645
MA(1)	0.5207	0.5887	0.5459	-0.4526
MA(2)	-	0.0234	-	-0.5353
OMEGA	4.2292	4.2292	4.2292	4.2292
ALPHA(1)	0.1000	0.0500	0.0333	0.0250

ALPHA(2)	-	0.0500	0.0333	0.0250
ALPHA(3)	-	-	0.0333	0.0250
ALPHA(4)	-	-	-	0.0250
AIC	6.5599	6.6812	6.0013	6.3420
BIC	6.6812	6.3562	6.3215	6.4125
SIC	6.5981	6.6931	6.2123	6.3034

From table 3; hybrid ARIMA-ARCH (2,1,1;3) has the smallest AIC, BIC and SIC and it is therefore regarded as the best model for fitting official exchange rate. Furthermore, the estimated coefficient values for all ARIMA-ARCH (p,d,q) strictly conforms to the bounds of parameter, between -1 and 1. This has made the model to be stationary.

Table 4: Fitting hybrid ARIMA-GARCH model on differenced official exchange rates

Model	ARIMA(1,1,1)+ GARCH(1,1)	ARIMA(1,1,2)+ GARCH(1,2)	ARIMA(2,1,1)+ GARCH(2,1)	ARIMA(2,1,2)+ GARCH(2,2)
AR(1)	-0.4456	-0.3796	-0.3847	0.4650
AR(2)	-	-	-0.0053	0.6511
MA(1)	0.8911	1.0000	0.8815	-0.0046
MA(2)	-	0.2075	-	-0.8372
OMEGA	0.6628	0.8030	0.7663	1.4705
ALPHA(1)	0.3773	0.4512	0.1985	0.8270
ALPHA(2)	-	-	0.1277	0.00000001
BETA(1)	0.8373	0.1126	0.8171	0.00000001
BETA(2)	-	0.6840	-	0.6263
AIC	6.0710	6.0419	6.0703	5.9578
BIC	6.1438	6.1437	6.1721	6.0887
SIC	6.0702	6.0829	6.0686	5.9551

From Table 4, hybrid ARIMA-GARCH (2,1,2;2,2) has the smallest AIC, BIC and SIC and it is therefore regarded as the best model for fitting official exchange rate. Furthermore, the estimated coefficient values for all ARIMA-GARCH (p,d,q) strictly conforms to the bounds of parameter, between -1 and 1. This has made the model to be stationary.

Table 5: Best fitted models on official exchange rate data

Model	AIC	BIC
ARIMA(1,1,2)	1561.43	1575.02
ARIMA(2,1,1)+ARCH(3)	6.0013	6.3215
ARIMA(2,1,2)+GARCH(2,2)	5.9578	6.0887

From Table 5, are the three best models fitted on the official exchange rates data throughout this work. It can be seen that, the two hybrid models outperformed the ARIMA model, preferably, hybrid ARIMA-GARCH model performed best on the official exchange rate data.

4.0 Conclusion

In this paper, comparative performance of the traditional ARIMA model with the proposed hybrid ARIMA-ARCH and hybrid ARIMA-GARCH models was carried out on foreign exchange data (*official*). It can be seen that, because of the volatile nature of the foreign exchange data, ARIMA alone cannot capture the volatility of the data nor GARCH family alone. In the comparative performances of the three models, as hybrid ARIMA-ARCH and hybrid ARIMA-GARCH models outperformed ARIMA model. It can be concluded that, the hybrid ARIMA-ARCH and hybrid ARIMA-GARCH models captures volatility of the data better than the ARIMA model. Hence, hybrid ARIMA-GARCH model as the best model can be used to predict future values of Nigeria's official exchange rates.

References

Amon Mwenda, Dimitry Kuznetsov and Silas Mirau (2015), ''Analyzing the impact of historical data length in non-seasonal ARIMA models Forecasting'' Mathematical theory and modelling, Vol 5, No. 10, ISSN 2224-5804. PP.77 – 85

Box, G.E.P., Jenkins G.M (1976), Time series Analysis Forecasting and Control. Holden-Day Inc., San Francisco.

Central Bank of Nigeria, CBN (2013). ''Economic and Financial Review'' Vol. 34 No. 4

Chatayan W., and Songsak S. (2010). The comparison among ARMA-GARCH,- EGARCH,-GJR, and –PGARCH models on Thailand volatility index. The Thailand Econometrics Society, Vol. 2, No. 2, 140-148.

Engle R.F, and Patton, A.J. (2001), ''what is a good volatility model'', Quantitative Finance, 1, 237-245.

Mosayeb Pahlavani and reza Roshan (2015), The comparison among ARIMA and hybrid ARIMA –GARCH models in forecasting the exchanges Rate of Iran, International Journal of Business and Development studies, Vol. 7, No. 1, PP 31-50.

Nor Hamzah Miswan, Nor Azazi Ngatiman, Khairum Hamzah and Zaminor Zamzamir Zamzamin, "014), Comparative performance of ARIMA and GARCH Models in Modelling Forecasting Volatility of Malaysia Market Properties and Shares, Applied Mathematical Sciences, Vol. 8 No. 140, PP 7001-7012.

Nwankwo Steve C (2014), Autoregressive Integrated Moving Average (ARIMA) Model for exchange Rate (Naira to Dollar). Academic Journal of Interdisciplinary Studies, MCSER Publishing, Rome-Italy Vol. 3 No. 4 PP.429-433.

S.R. Yaziz, N.A. Azizan, R. Zakaria, M.H. Ahmad (2013), The performance of hybrid ARIMA-GARCH Modellinf in Forecasting gold price, 20th International Congress on Modelling and Simulation, Adeluide Australia, 16 December, 2013. www.mssanz.org.au/modsim2013

Shahla Ramzan, Shumila Ramzan, Faisal Maqbool Zahid (2012). Modelling and Forecasting Exchange Rates Dynamic in Pakistan Using ARCH Family of Models, Electronic Journal of Applied Statistical Analysis, Vol.5 Issue 1, 15-29.