This also includes

$$\propto = \frac{1}{2}\sqrt{c_2\tau} + \frac{c_4}{3c_2} \quad (3.5(e))$$

For which the probabilities i. p_l, *1*=1. .... *5.* Are positive.

Comparing the results in provision with the cumulants of the process X. this explains why the circumstances scale concerning the time-step- τ. Particularly, when the limit as $\tau \to 0$, which is our area of concern.

Limitations of the Lattice-Model.

In the below segment the restrictions of the lattice model in continuous time as $\tau \to 0$ are taken into account. Besides, there is an assumption that the third and fourth cumulants $C_3$and $C_4$are real positive numbers. If the cumulants were zero then it would be valid to deduce that the lattice will merge to a geometric Brownian motion because of the condition of positivity.

Concerning the discreet model having a continuous time limit, the condition of positivity must be workable or seen as$\tau \to 0$. In the limit, (6) deduces to

$$c_4 c_{2\geq} 3c_3^2 \text{and} c_{4\geq} 0 \quad (3.5(f))$$

The other assumption is that (3.5(f)) holds. Note that the requirement of $c_4 \geq O$ corresponds to the expectation of a positive excess of kurtosis.

We aspire to have the lattice containing an explained maximum as the size-step approaches zero. It follows that we assume that the model has a limit as$\tau \to 0$. Given this, let us presuppose that the limit is given by α=$lim_{t\to 0}$α where it is within a specified spread.

For continuation purposes, new probabilities, (q_1, q_2, $q4$,q_5) are included to refer to the branch probabilities:

$$\lim_{\tau} \to 0(1/\tau p_1(\tau) = \frac{-4x_0^2\, c_2 - 4x_0 c_3 + c_4}{384 x_0^4} = \lambda q_1$$

$$\lim_{\tau} \to 0(1/\tau p_2(\tau) = \frac{16x_0^2\, c_2 - 2x_0 c_3 - c_4}{96 x_0^4} = \lambda q_2$$

$$\lim_{\tau} \to 0(1/\tau p_3(\tau) - 1 = \frac{(-20x_0^2 c_2 + c_4)}{64 x_0^4} = -\lambda$$

$$\lim_{\tau} \to 0(1/\tau p_4(\tau) = \frac{16x_0^2\, c_2 - 2x_0 c_3 - c_4}{96 x_0^4} = \lambda q_4$$

$$\lim_{\tau} \to 0(1/\tau p_5(\tau) = \frac{-4x_0^2\, c_2 + 4x_0 c_3 + c_4}{384 x_0^4} = \lambda q_5$$

Therefore,

$q_1 + q_2 + q_4 + q_5 = 1$

If we use α to simplify to

$\lambda=\frac{3x_2^2}{2C_4}$

$q_1=\frac{1}{6}\,(1-c_3\sqrt{\frac{3}{C_2C_4}}\,)$

$q_2=\frac{1}{3}\,(1+c_3\sqrt{\frac{3}{C_2C_4}}\,)$

$q_4=\frac{1}{3}\,(1-c_3\sqrt{\frac{3}{C_2C_4}}$

$q_5=\frac{1}{6}\,(1+c_3\sqrt{\frac{3}{C_2C_4}}\,)$

In the limit, $Z(\tau)$ approaches an increasing compound Poisson process with the following description:

$$C_1t+\sum_{k=0}^{N_t}W_k$$

N_t, = Poisson process with force $\lambda$and the W_k which are iid r.v with the below given dispersal.

$$W_k = \begin{cases} -4x_0 \text{ with probability } q_1 \\ -4x_0 \text{ with probability } q_2 \\ 2x_0 \text{ with probability } q_4 \\ 4x_0 \text{ with probability } q_5 \end{cases} \qquad (3.5(h))$$

Theorem 3.

Reflect a permanent range given by; [0, *T]* of time where the count of steps *n* is expanding. Then the size of the steps,$\tau = T/n$. Our area of interest is in the r.v at time *T* described below as:

$$X_n = \sum_{k=1}^{n} Z_k(\tau)$$

Implying as *n* -∞, $\boldsymbol{X_n}$, yields to the below distribution

$$\boldsymbol{C_1 T} + \sum_{k=0}^{N_T} W_k \quad (3.5(i))$$

Given that N_T is a Poisson distribution with an average ofλT and the W_k, which are iid r.v described in (3.5(h)).

The above hypothesis leads us to a lattice model that is applied in the European pricing options. Secondly, we utilize Fourier transform methods as in Carr &Madan (1999) to generate the costing of European options. Note that the characteristic it is vital to know the function of the distribution here. For the r.v described by (12), the attribute result of Fourier transform (Breiman, 1992) is deduced to:

$$\emptyset_T(\mu) = e^{iuc_1 T} \exp\left(\lambda T \sum_{l \in \{1,2,4,5\}} q_1 \left(e^{iu(2l-6)x} - 1\right)\right) (3.5(j))$$

The above will be applied when it comes to evaluating European in the below sections

Applying the Fourier Transform to price European Options (Calls & Puts)

This sample engenders reference to the pioneer assignment of Carr & Madan (1999) on cost of options that utilize the Fourier transform. It is evident that when it is a significantly useful model, especially when the user knows the risk-neutral probabilities for Fourier Transform. The case scenario is a slight reversal if what Can and Madan (1999) proposed since a non-continuous spread is utilized and therefore anon-continuous

Fourier transform.

Let q_T(n) be the discrete risk-neutral probability distribution of the r.v described in (12) above. Employing the limiting distribution in (12) as the risk-neutral probabilities, a call options amount is deduced as an allowable payoff expectation as stated below:

$$CT\,(\check{k}, K) = e^{-rt} \sum_{n=\check{k}}^{\infty} \left(e^{C_{1T+2x_0}n} - K\right) q_T\,(n) \qquad (3.5(k))$$

Where $\check{k} > \left(\frac{\ln\left(\frac{K}{S}\right) - C_{1T}}{2x_0}\right)$

As said earlier, the model corresponds to Carr & Madan (1994).

Fourier Transform pricing Equation & Formula

The cost of a European call option that does not pay money from its dividends on the repressed holding represented as $S_{0e}{}^{x_t}$ with the original cost S_0, the dissemination of X_t at the lapse of time T of (12), and selling cost $\boldsymbol{K}$ is given or derived as follows:

$$C_T\,(\check{k}, K) = \frac{e^{-\beta \check{k}}}{2\pi} \int_{-\pi}^{\pi} \Psi\,(u, K) e^{-iu\check{k}}\, du \qquad (3.5(g))$$

where β>0 is a specification to describe Fourier Transform.,

$$\Psi(u, K) e^{-rT} \left(\frac{1}{1-e^{-(\beta+iu)}}\right) \left[S_0 e^{C_1 T} \emptyset \left(-i\left(\frac{\beta+2\alpha_0+iu}{2\alpha_0}\right) - K\emptyset\left(-i\left(\frac{\beta+iu}{2\alpha_0}\right)\right)\right]\right. \qquad (3.5(h))$$

And

$$\emptyset(\boldsymbol{\mu}) = E\left[e^{iuX}\right] = \sum_{n=-\infty}^{\infty} e^{iu2\alpha_0 n} qT\,(n) = exp\left(\lambda T \sum_{l\in\{1,2,4,5\}} q_1 \left(e^{(2l-6)\propto_0} - 1\right)\right) \qquad (3.5(i))$$

Is the mgf of the limiting distribution in (3.5(i)).

After factoring in the formula described above, a Fast-Fourier transform algorithm that is widely applied in the comprehensive computation of the asset price yields. This utilization of fast Fourier transform was established by Madan (1999). In a given procedure aloft, incorporation of the rate of return *(c1)* provided by

Levy process is applied in risk-neutral options to establish the value of the underlying non-dividend paying asset. **NOTE** The*c1(drift)*should be inconsiderate with the below risk-neutral condition.

e^{r^T}=[e^{X_T}]= $\emptyset$(-i) = exp (C_1T)exp$\left(\lambda T \sum_{l=\{1,2,4,5\}} q_l \left(e^{(2l-6)\alpha_0} - 1\right)\right)$

Solving for $C_i$yields

C_i= r-λ$\sum_{l\in\{1,2,4,5\}} q_1 \left(e^{(2l-6)\propto_0} - 1\right)$

Conclusion

We have examined a pentanomial lattice representation that assimilates kurtosis and skewness .We have also controlled the states on kurtosis and skewness under continuous time. We came up with the restricting distribution which is ideal compound Poisson distribution. In the end, we came up with a formula involving Fourier transform techniques that systematically employed to compute European option prices. Thus, this explains a compatible representation for estimating American and European option prices under kurtosis and skewness.

This project analyzed through a recombining pentanomial tree for coasting European options with unchanging volatilities. The above was attained by letting the time steps regular and risk neutral probabilities remaining the same for the entire contract time. This recombining trinomial tree is more flexible/ easy to use than just a tree with a lot of nodes which are more the same and thus suitable for predicting prices of options.

I recommend the use of Pentanomial lattice model in option pricing

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