time to the consulting section of the (n+1) th patient from the nth patient was registered and the time from when the patient goes in to see a consultant until the time that patient leaves the consulting room (having been examined by the consultant). This was taken for the two consultants simultaneously. The data was collected over a period of six days, from Monday to Saturday, taking four (4) hours daily (8:00am to 12:00 noon). The data was recorded on the following table.

Table 1: data collection table proformer

			1st consultant		2nd consultant		
Patient no.	Clock time Time at arrival	Clock time at previous arrival	Clock time at beginning of service	Clock time at end of service	Clock time at beginning of service	Clock time at end of service	Waiting time
1							
2							
3							
-							
-							
-							

METHOD OF ANALYSIS

The hospital consultancy unit is made up of a single queue with two consultants and was analysed using the double channel (M/M/2) queuein model. We assumed that the inter arrival and service times are independent and identically distributed. All arriving customers enter the queuing system and remain there until service has been completed, the queuing discipline is first – come, first – served. A test of randomness was carried out on the time between arrivals and the service time to confirm if the sequences are a result of a random process.

To determine if the time between arrivals and service times generated from the hospital are actually from a random process, we employ the following procedure:

Hypothesis

1. H_o: The occurrence of pattern for the Time Between Arrivals is generated by a random process
2. H_o: The occurrence of pattern for the Service Times is generated by a random process

Table 2: Time Between Arrivals and Service Times and their codes

S/N	Service Times	Code	Time Btw Arrivals	Code
1	3	0	5	0
2	7	1	4	0
3	3	0	4	0
4	6	1	0	0
5	6	1	7	1
6	7	1	5	0
7	5	0	6	0
8	7	1	8	1
9	8	1	0	0
10	7	1	3	0
11	4	0	7	1
12	3	0	5	0
13	3	0	6	0
14	7	1	15	1
15	5	0	6	0
16	11	1	8	1
17	5	0	15	1
18	6	1	0	0
19	4	0	14	1
20	9	1	7	1
21	6	1	11	1
22	11	1	15	1
23	9	1	3	0
24	3	0	11	1
25	7	1	4	0
26	4	0	9	1
Mean	6		6.846	
N1	11		14	
N2	15		12	
R	17		16	

Given that:
r is the number of runs
μ_r is the expected number of runs; and
σ_r is the standard deviation of the number of runs.
Then:

$$Z_{cal} = \frac{r - \mu_r}{\sigma_r}$$

The values of μ_r and σ_r are computed as follows:

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1$$

$$\sigma_r = \sqrt{\frac{(2n_1n_2)(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}$$

For the Service Times,
$n_1 = 14$
$n_2 = 12$
$r = 16$

$$\mu_r = \frac{2(14)(12)}{14+12} + 1 = 13.923$$

$$\sigma_r = \sqrt{\frac{(336)(336-14-12)}{(14+12)^2(14+12-1)}} = 2.4826$$

$$Z_{cal} = \frac{r - \mu_r}{\sigma_r}$$

$$Z_{cal} = \frac{15 - 13.923}{2.4826} = 0.434$$

$$Z_{tab} = Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

Since $Z_{cal}\,(= 0.434) < Z_{tab}\,(= 1.96)$, we accept H_o and conclude that the Service Times come from a random process.

For the Time Between Arrivals,
$n_1 = 11$
$n_2 = 15$
$r = 17$

$$\mu_r = \frac{2(11)(15)}{11+15} + 1 = 13.692$$

$$\sigma_r = \sqrt{\frac{(330)(330-11-15)}{(11+15)^2(11+15-1)}} = 2.436$$

$$Z_{cal} = \frac{r - \mu_r}{\sigma_r}$$

$$Z_{cal} = \frac{17 - 13.692}{2.436} = 1.358$$

$$Z_{tab} = Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

Since $Z_{cal}\,(= 1.358) < Z_{tab}\,(= 1.96)$, we accept H_o and conclude that the Time Between Arrivals come from a random process.

The simulation model is then developed as follows:

Table 3: Probability Distributions of Time Between Arrivals and Service Times and Their Mapping to Random Numbers

Time Between Arrivals	Service Times

ITA	Prob.	C/Prob.	Mapping		ST	Prob.	C/Prob.	Mapping
0	0.12	0.12	0.00 - 0.12		1	0.04	0.04	0.00-0.04
3	0.08	0.20	0.13 - 0.20		2	0.04	0.08	0.05-0.08
4	0.12	0.32	0.21 - 0.32		3	0.12	0.19	0.09-0.19
5	0.12	0.44	0.33 - 0.44		4	0.15	0.35	0.20-0.35
6	0.12	0.56	0.45 - 0.56		5	0.08	0.42	0.36-0.42
7	0.12	0.68	0.57 - 0.68		6	0.12	0.54	0.43-0.54
8	0.08	0.76	0.69 - 0.76		7	0.15	0.69	0.55-0.69
9	0.04	0.80	0.77 - 0.80		8	0.08	0.77	0.70-0.77
11	0.08	0.88	0.81 - 0.88		9	0.12	0.88	0.78-0.88
14	0.04	0.92	0.89 - 0.92		10	0.04	0.92	0.89-0.92
15	0.12	1.00	0.93 – 0.99		11	0.08	1.00	0.93-0.99

We then use the above mapping to generate random arrivals of patients and random service times for the simulation process. The main simulation table is shown in table 4.

Table 4: The Main Simulation Table

Opening:		7:00AM				Consultant 1		Consultant 2		
Patient S/N	RND	TBA	CTA	RND	ST	CTBS	CTES	CTBS	CTES	WT
1	0.62	7	7	0.49	6	7	13			0
2	0.20	3	10	0.06	2	0	0	10	12	0
3	0.44	5	15	0.64	7	15	22	FALSE	0	0
4	0.58	7	22	0.12	3	22	25	FALSE	0	0
5	0.16	3	25	0.47	6	25	31	FALSE	0	0
6	0.06	0	25	0.01	1	0	0	25	26	0
7	0.38	5	30	0.41	5	30	35	FALSE	0	0
8	0.11	0	30	0.89	9	0	0	30	39	0
9	0.49	6	36	0.69	7	36	43	FALSE	0	0
10	0.99	15	51	0.24	4	51	55	FALSE	0	0
11	0.47	6	57	0.53	6	57	63	FALSE	0	0
12	0.50	6	63	0.74	8	63	71	FALSE	0	0
13	0.76	8	71	0.65	7	71	78	FALSE	0	0
14	0.99	15	86	0.36	4	86	90	FALSE	0	0
15	0.28	4	90	0.80	9	90	99	FALSE	0	0
16	0.43	5	95	0.90	10	0	0	95	105	0
17	0.75	8	103	0.84	9	103	112	FALSE	0	0
18	0.36	5	108	0.61	7	0	0	108	115	0
19	0.28	4	112	0.70	8	112	120	FALSE	0	0
20	0.86	11	123	0.09	3	123	126	FALSE	0	0
21	0.14	3	126	0.43	5	126	131	FALSE	0	0
22	0.18	3	129	0.19	3	0	0	129	132	0
23	0.07	0	129	0.35	4	129	133	FALSE	0	0
24	0.56	6	135	0.01	1	135	136	FALSE	0	0
25	0.34	5	140	0.47	6	140	146	FALSE	0	0
26	0.25	4	144	0.95	11	0	0	144	155	0

144	151	0

From the main simulation table above,

The mean Time Between Arrivals of patients $\left(\frac{1}{\lambda}\right) = 5.539\ minutes$

Therefore, mean Patients Arrival Rate, $\lambda = 0.181\ patients\ per\ minute$ or 10.86 patients per hour

The mean Doctors' Service time $\left(\frac{1}{\mu}\right) = 5.808\ minutes$

Therefore, mean Doctors' Service Rate, $\mu = 0.172\ patients\ per\ minute$ or 10.32 patients per hour

The Doctors' utilization Factor (since this is an M/M/2 system) is:

$$\rho = \frac{\lambda}{2\mu} = \frac{0.181}{2(0.172)} = 0.526$$

We computed a 95% confidence interval for this value of the Doctor Utilization Factor (DUF) as follows:
The simulation model for the operations of the system for the two doctors for the 26 patients (Table 4) was replicated 30 times.
This generated the following 30 values for the doctor utilization factor:

0.66	0.48	0.46	0.62	0.48
0.36	0.40	0.49	0.49	0.62
0.53	0.59	0.39	0.43	0.57
0.46	0.55	0.39	0.51	0.46
0.40	0.43	0.59	0.43	0.47
0.65	0.49	0.44	0.40	0.43

From the above values,

Sample Size	30
Mean (DUF)	0.49
Std Dev.	0.08339

For 95% confidence interval for the value of the doctor utilization factor (DUF); we have

$$\bar{x} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right) = 0.49 \pm 1.960\left(\frac{0.08339}{\sqrt{30}}\right)$$

i.e

$$(0.49 - 0.029841) \leq \boldsymbol{\rho} \leq (0.49 + 0.029841)$$

or

$$0.459 \leq \boldsymbol{\rho} \leq 0.519$$

With 95% confidence interval established for our Doctor Utilization Factor, we can say that the facility has a utilization factor for her Doctors of 0.49.
With this value of the Doctor Utilization Factor, we compute the other system characteristics as follows:

Probability that there are no patients in the facility: $P_o = \frac{1}{M}$

Where $M = \sum_{j=0}^{2} \frac{(2\rho)^j}{j!} + \frac{(2\rho)^2}{2!}\left(\frac{\rho}{1-\rho}\right)$

$M = 2.9216$
Therefore:
$P_o = 0.342$

Mean number of patients in the Queue: $L_q = \frac{\rho(2\rho)^2\ P_o}{2!(1-\rho)^2} = \frac{0.49(2\times0.49)^2\ 0.342}{2!(1-0.49)^2} = 0.309\ Patients\ per\ hour$

Mean number of patients in the System: $L_s = \frac{\rho(2\rho)^2 P_o}{2!(1-\rho)^2} + 2\rho = \frac{0.49(2\times0.49)^2\ 0.342}{2!(1-0.49)^2} + 2\times0.49 = 1.289\ Patients\ per\ hour$

Mean time a patient spends in the Queue: $W_q = \frac{(2\rho)^2 P_o}{2!(1-\rho)^2 2\mu} = \frac{(2\times0.49)^2\times0.342}{2!(1-0.49)^2 2\times10.32} = 0.0612 hours\ \approx 3.79 minutes$

Mean time a patient spends in the System: $W_s = \frac{(2\rho)^2 P_o}{2!(1-\rho)^2 2\mu} + \frac{1}{\mu} = \frac{(2\times0.49)^2\times0.342}{2!(1-0.49)^2 2\times10.32} + \frac{1}{10.32} = 0.158 hours\ \approx 9.49 minutes$

From the results of the analysis, the system is fairly free most of the time. The doctors are free up to 34.2% of the time during peak hours. Recall that the system was observed at peak hours of 8:00 am to 12:00 noon.
The patients spend less than 4 minutes on average waiting to see a consultant and spend on average nine and a half minutes in the facility.
The average number of patients waiting to see a consultant is less than two signifying that one consultant is free most of the time.

CONCLUSION

From the system characteristics computed above, it can clearly be seen that the facility has excess capacity in terms of the consultant services. The number of consultants is in excess of the need of the polytechnic community.
This is however, common with health care systems. In health care, speciality is a necessary requirement for a facility to attain a required status. E.g. a hospital requires a medical doctor to be considered a secondary health care facility irrespective of the size of its patronage. The institutional facility is a special facility and may have this excess capacity because of its special nature.

RECOMMENDATION

For a hospital facility to have excess capacity in a place where medical care is still less than desired would be a great undoing.
This study believes that the following suggestions would have great implication for policy decisions regarding the health centre.

1. That the management of the facility considers extending its services to the neighbouring community especially as a referral for the neighbouring clinics and dispensaries.
2. That other special services be introduced to utilize the excess capacity.
3. In line with (1) above, publicity should be made of the available services to the neighbourhood, especially the neighbouring clinics
4. Service costs should be subsidized

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