Proof: $(a) \Longrightarrow (b)$. Suppose K is a regular closed subset of Y. $Cl_{\beta Y}K$ is a regular closed subset of βY. Hence $Cl_{\beta Y}K$ has a countable neighborhood basis $\{V_i{:}i < w\}$ consisting of regular open subsets of βY, since βY is a compact Oz-space, for each $i < w, let\, U_i = V_i \cap Y$. Then, we shall show that the sequence $\{U_i{:}i < w\}$ has the properties (i) and (ii) above. (i) is obviously satisfied. (ii) Let U be a regular open subset of Ycontaining K, then K and $Y \setminus U$ are completely separated provided that$K$ and $Y \setminus U$ are regular closed subsets of an Oz-space Y. Hence $Cl_{\beta Y}A \subset U^{\beta}$. Therefore, for some i, $Cl_{\beta Y}A \subset V_i \subset U^{\beta}$. Thus $U_i \subset U$ for some$i$. Hence **(ii)** is satisfied.

$(b) \Longrightarrow (a)$. Suppose N is a regular closed subset of βY. Then $K = N \cap Y$ is a regular closed subset of Y. Hence $\exists$ a sequence $\{U_i{:}i < w\}$ of regular open subsets of Y which satisfies the properties (i) and (ii) above. Then it is obvious that $Cl_{\beta Y}K = N = \bigcap\{U_i^{\beta}{:}i < w\}$. Hence N is a zero-set of βY, since βY is normal.

Corrollary 4.2: For a normal space Y, the following are equivalent;

i. βY is Oz

ii. Every regular closed subset of Y has a countable neighborhood basis.

Theorem 4.11: Suppose βY is Oz, then for any regular closed subset $X$of Y, $\partial_Y X$ is relatively pseudocompact.

Proof: Let X be a regular closed subset of Y. Suppose that $\partial_Y X$ is not relatively pseudocompact. Then, we shall show that the condition (b) in the **theorem 4.10** above is not satisfied. To do this, let $\{U_i{:}i < w\}$ be a sequence of regular open subsets of Ycontaining$X$. Since $\partial_Y X$ is not relatively pseudocompact, then $Cl_{\beta Y}(\partial_Y X) \cap (\beta Y \setminus \upsilon Y)$ is nonempty. Suppose x is a point of $Cl_{\beta Y}(\partial_Y X) \cap (\beta Y \setminus \upsilon Y)$, it is obvious that $x \in Cl_{\beta Y}(U_i \setminus X)$ for each $i < w$. Since $x \notin \upsilon Y$, there exist a discrete sequence $f_i \subset U_i \setminus X$ for each $i < w$. Now, let $U = Y \setminus \cup \{F_i{:}i < w\}$Y\∪{F_i:i<w}. Then, U is a regular open subsets of Ycontaining$X$. But U does not contain any family of $\{U_i{:}i < w\}$ by the construction. Therefore $\partial_Y X$ is relatively pseudocompact.

Theorem 4.12: Suppose υY is of countable type, then the following are satisfies:

i. βY is Oz

ii. For any regular closed subset $X$of Y, $\partial_Y X$ is a relatively pseudocompact zero-set.

Proof: $(i) \Longrightarrow (ii)$, this imply that Y must be Oz, $\partial_Y X$ is a zero-set for any regular closed subset $X$of Y. Then by **theorem 4.11** above, this implication is obvious.

$(ii) \Longrightarrow (i)$. Let A be a regular closed subset of βY. Then $A \cap Y$ is a regular closed subset of Y. So, $\partial_Y(A \cap Y)$ is a relatively pseudocompact zero-set of Y. Provided that $\partial_{\beta Y} A = Cl_{\beta Y}(\partial_Y(A \cap Y))$, hence $\partial_{\beta Y} A$ is a compact zero-set of υY. Therefore, by the assumption that υY is of countable type, implying that $\partial_{\beta Y} A$ is a $G_\delta -$ set in βY. Hence A is G_δ in βY.

Corollary 4.3: Suppose Y is a real compact space and every closed subset of Y has a countable neighborhood basis, then Y is perfectly normal.

Theorem 4.13: Suppose Y is a real compact space. Then, the following are equivalent;

i. βY is Oz
ii. Any regular closed subset X of Y has a countable neighborhood basis in Y.
iii. For any regular closed subset X of Y,then $\partial_Y X$ is a compact subset which has a countable neighbourhood basis in Y

Proof: Before we provide the proof for this, we need to state the following Lemma below;

LEMMA 4.3: (Skljarenko E.G 1953) For any open subset X of a space Y, the equality $\partial_{\beta Y}(X^\beta) = Cl_{\beta Y}(\partial_Y X)$ holds.

Now, from this Lemma, for any regular closed subset X of Y, implies that $Cl_{\beta Y} X = Cl_{\beta Y}(\partial_Y X) \cup (Int_Y X)^\beta$ and $Cl_{\beta Y} Y \setminus X = Cl_{\beta Y}(Y \setminus X) \cup (Y \setminus X)^\beta = Cl_{\beta Y}(\partial_Y X) \cup (Y \setminus X)^\beta$. Thus, $Cl_{\beta Y} X \cap Cl_{\beta Y}(Y \setminus X) = Cl_{\beta Y}(\partial_Y X)$. Thus, $Cl_{\beta Y} X \cap Cl_{\beta Y}(Y \setminus X) = Cl_{\beta Y}(\partial_Y X)$. Therefore $Cl_{\beta Y}(\partial_Y X)$ is G_δ in βY, since βY is Oz. by **Theorem 4.12 above**, $\partial_Y X$ is relatively pseudocompact in Y. Since Y is realcompact, $\partial_Y X$ must be compact. Hence $\partial_Y X$ has a countable neighbourhood basis in Y.

$(iii) \Longrightarrow (ii)$ from **lemma 4.3** above,

$(ii) \Longrightarrow (i)$ we observe from the **lemma 4.4** specified below

LEMMA 4.4: Suppose Y is a real compact space and X is a closed subset of Y. If X has a countable neighbourhood basis in Y, then $Cl_{\beta Y}X$ is a zero-set of βY. We note if Y is extremely disconnected or pseudocompact Oz, then βY is Oz. Otherwise, we have the following theorem:

Theorem 4.14: If βY is Oz, then for each discrete sequence $\{U_i{:}i < w\}$ of open subsets of Y, $\exists\ M_o \ni U_i$ extremely disconnected for each $i \geq M_o$.

Proof: We prove this theorem from the contradictory point of view. Suppose βY is not Oz. then $\exists$ a sequence $\{U_i{:}k < w\}\ of\ \{U_i{:}i < w\} \ni U_{ik}$ is not extremely disconnected for each k. For each k, let T_k be an open subset of U_{ik} such that $Cl_{U_{ik}}T_k$ is not open. Let $A = \cup\{Cl_Y T_k{:}k < w\}$, obviously A is regular and closed. Then we shall show that the condition **(b) of theorem 4.10 above** is not satisfied. Therefore let $\{W_i{:}i < w\}$ be a sequence of regular open subsets of Y containing A. then, for each k, $\exists$ a regular closed subset B_k of Y such that $B_k \subset (W_k \cap U_{ik}) \setminus A$. Let us designate $U = X \setminus \cup \{B_k{:}k < w\}$. Then U is a regular open subset of Y which has no member of sequence $\{W_i{:}i < w$, which ends the proof.

Corollary 4.4: If every open subset of a space Y is not extremely disconnected, then the following condition are satisfied

i. βYYis Oz
ii. Y is pseudocompact and Oz

Theorem 4.15: Let Y be an Oz-space whose Hewitt realcompactification vY is of countable type. Then, the following conditions are equivalent;

i. βY is Oz
ii. Y can be expressed as the union of an extremely disconnected open subset and a closed relatively pseudocompact subset.

Proof: To prove **theorem 4.15** we will need the support of **corollary 4.4** above.

$(i) \Longrightarrow (ii)$ Suppose M is a family of all extremely disconnected open subsets of Y, then M is partially ordered by the inclusion relation $\subset$. Suppose M_i is a linearly ordered subset of M. Then, it is easy to set that $\cup\{U{:}U \in M_i\} \in M$, hence, using Zorn's lemma, there exist a maximal member E of M. So that $A = Y \setminus E$. Let us assume that A is not relatively pseudocompact. Then

$\exists$ a discrete sequence $\{U_i{:}i < w\}$ of open subsets of Y such that $U_i \cap A \neq \emptyset$ for each i. If U_i is extremely disconnected, then $U_i \cup E$ is also extremely disconnected. But this contradicts the maximality of E. Hence, each U_i is not extremely disconnected, therefore by **theorem 4.14** above shows a contradiction. Thus A is relatively pseudocompact.

$(ii) \Longrightarrow (i)$. If $Y = E \cup A$, where E is an extremely disconnected open subset and p is a closed relatively pseudocompact subset, we shall show that for each regular closed subset X of Y, $\partial_Y X$ is Oz. Now, we can show that $\partial_Y X \subset A$. This follows from the following that:

$$\begin{aligned}\partial_Y X = Cl_Y(Int_Y X) \setminus Int_Y X &= Cl_Y(((Int_Y X) \cap E) \cup ((Int_Y X) \cap A)) \setminus Int_Y X \\ &= (Cl_Y(Cl_E((Int_Y X) \cap E)) \setminus Int_Y X) \cup (Cl_Y((Int_Y X) \cap A) \setminus Int_Y X) \subset A \cup A = A\end{aligned}$$

Theorem 4.16: A topological space Y is said to be a quasi Oz-space iff $R(Y) = Z(Y)^*$

To prove this theorem we need the following definition

Proof: Suppose that $G(Y) = \{M \in Z(Y)^*{:}Cl_Y(Y \setminus M) \in Z(Y)^*\}$. Thus $G(Y) \subseteq Z(Y)^* \subseteq R(Y)$. Suppose $R(Y) = G(Y)$, then Y is clearly a quasi Oz-space.

Conversely, suppose $A \subset Y$ is an open subset of Y, then $Cl_Y(A), Cl_Y(Y \setminus Cl_Y(A)) \in R(Y) = Z(Y)^*$. So, $Cl_Y(A) \in G(Y)$. Thus, $R(Y) = G(Y)$. Therefore, Y is quasi Oz-space.

Theorem 4.17: For a Oz topological space Y, the following must be satisfied:

i. Y is quasi Oz-space
ii. Every open subset $M \subset Y$ is Z^*-embedded in Y
iii. Every dense open subset $M \subset Y$ of Y is Z^*-embedded in Y

Proof: $(i) \Longrightarrow (ii)$, suppose $Z \in Z(M)$, $\exists$ a closed subset $K \in Y$ *and* $Z = K \cap M$. Since M is open, then $Cl_M(int_M(Z)) = Cl_Y(int_Y(K)) \cap M$ and $Cl_Y(int_Y(K)) \in \Re(Y) = Z(Y)^*$. Hence, M is Z^*-embedded in Y

$(ii) \Longrightarrow (iii)$ is trivial

$(iii) \Longrightarrow (i)$, suppose $M \subset Y$ is any open subset of Y. Let $A = M \cup (Y \setminus Cl_Y(M))$ then, A is open and dense in Y.We define a map $g: A \longrightarrow R$ by $g(y) = 0$ if $y \in M$ and $g(y) = 1$ if $y \in Y \setminus Cl_Y(M)$, then g is continuous and $g^{-1}(0) = M,$ such that M is a zero-set in A. Since $A \subset Y$ is open and dense in Y, $\exists$ a cozero-set $Z \in Y \ni Cl_A(Int_A(M)) = Cl_Y(M) \cap A = Cl_Y(Int_Y(Z)) \cap A$. since A is dense in Y and $Cl_Y(M), Cl_Y(Int_Y(Z))$ are regular closed sets in Y, so $Cl_Y(M) = Cl_Y(Int_Y(Z))$. Hence, $R(Y) \subseteq Z(Y)^*$, therefore, Y is a quasi Oz-space.

Definition 4.2: A topological space Y is said to be basically disconnected if every cozero-set $X \in Y$ is C*-embedded in Y.

Definition 4.3: Let Y be a topological space and $X \subset Y Y X$ is said to be Z*- embedded in Y if for any $A \in Z(X)^* \exists\, U \in Z(Y)^* \ni A = U \cap X$.

Definition 4.4: Suppose $U(Y)$ is the set of clopen sets in a space Y. Then Y is said to be basically disconnected iff $B(Y) = Z(Y)^*$.

Definition 4.5: A completely regular space Y is said to be 3 a quasi Oz-space if for any regular closed set $K \in Y$. $\exists$ a zero-set $X \in Y \ni K = Cl_Y(Int_Y(X))$.

LEMMA 4.5: $A \subseteq Y$ is said to be dense in Y iff $A \cap K \neq \emptyset$ whenever $\emptyset \neq K \in Y$

Theorem 4.18: Suppose βY is a Stone-Čech compactification of a topological space Y. Then βY is a Baire space.

Proof: Let $A_i \subseteq \beta Y$ be dense and open for each $i \in N$. Suppose $\emptyset \neq K \in \tau$. By **Lemma 4.5** above, $K \cap A_1 \neq \emptyset$∩A_1≠∅, so, we can choose $x_i \in K \cap A_i$. Due to the regularity of the space Y, we can find an open set $U_1 \ni x_1 \in U_1 \subseteq Cl\, U_1 \subseteq K \cap A_1$. Since A_2 is dense, $U_1 \cap A_2 \neq \emptyset$. Also, we find an open set $U_2 \neq \emptyset \ni U_2 \subseteq Cl\, U_2 \subseteq U_1 \cap A_2$. By iteration, we find non-empty open set $U_i \ni U_n \subseteq Cl\, U_n \subseteq U_{n-1} \cap U_n, \forall\, n$. Now, all $Cl\, U_n$ are non-empty and nested interval $Cl\, U_1 \supseteq Cl\, U_2 \supseteq ...$ holds. Then, the family $\{Cl\, U_i\}$ has the finite intersection property, so that compactness gives $\bigcap_i Cl\, U_i \neq \emptyset$. By construction, $\bigcap_i Cl\, U_i \subseteq K \cap \bigcap_i A_i$. Therefore, $K \cap \bigcap_i A_i \neq \emptyset$ which proved that βY is a Baire space.

The **theorem 4.19** below provides a result that the Borel set G_δ *in* βY as shown in **theorem 4.12 and 4.13** above is a Baire space

Definition 4.6: suppose $\beta\gamma$ is a Stone-$\breve{C}$ech compactification of a topological space .Then $\gamma \subset \beta\gamma$ is said to nowhere dense if the interior of its closure is empty.

Definition 4.7: The space $\gamma \subset \beta\gamma$ is of first category, if it is a countable union of nowhere dense subsets of $\beta\gamma$.

Definition 4.8: The space $\gamma \subset \beta\gamma$ is of second category, if it is not first category; that is if it cannot be express as a countable union of nowhere dense subsets of $\beta\gamma$.

Theorem 4.19: Every $G_\delta - set$ in $O_z - space\ \beta\gamma$ is a Baire space.

Proof : Let $\beta\gamma$ be a compact Housdorff space and $\gamma \subseteq \beta\gamma\ is\ G_\delta, thus, \gamma = \cap_i K_i\ with\ K_i \subseteq \beta\gamma$ open for every $i \in N$ and let $A_i \subseteq \gamma$ be dense open for every $i \in N$. Clearly, $(\beta\gamma)^I = Cl\gamma$ is Compact Housdorff and $\gamma \subseteq \beta\gamma$ is dense. Then ${K^I}_i = K_i \cap (\beta\gamma) \subseteq (\beta\gamma)^I$ are open and dense in $\beta\gamma$, since each containsY. Further, there are open set $B_i \subseteq (\beta\gamma)^I$ such that $A_i = \gamma \cap B_i$. Since γ is dense in $(\beta\gamma)^I$ and A_i is dense in γ, each A_i is dense in $(\beta\gamma)^I, thus\ the\ open\ sets A_i \subseteq B_i$ are dense in $\beta\gamma$. Therefore $N = \cap_i A_i = \cap_i(\gamma \cap B_i) = \cap_i K_i \cap\ \cap_i B_i$ is countable intersection of dense open sets in $(\beta\gamma)^I$ and dense in $(\beta\gamma)^I$, By theorem above. Hence, $Cl_\gamma N = Cl(N) \cap \gamma = (\beta\gamma)^I \cap \gamma = \gamma$. Therefore N is dense in γ.

C. *The characterizations of the Stone-Čech compactification of the linear strongly B-convergent sequence space of Maddox as GA compactification, using continuum hypothesis.*

From **theorem 4.8**, since Y is locally compact, then we have the **theorem 4.20** below:

Theorem 4.20: The Stone-Cech compactification of locally compact space Y such that $|\beta Y| \leq 2^{\chi_0}$ is a GA compactification.

Proof: if $\mathcal{B}$ is an open basis for Stone-Cech compactification of locally compact space Y such that $|\ \beta Y\ | \leq 2^{\chi_0}$. Without loss of generality, if we assume that $\mathcal{B}$ is closed under finite intersections and finite unions. Then, we can define the set

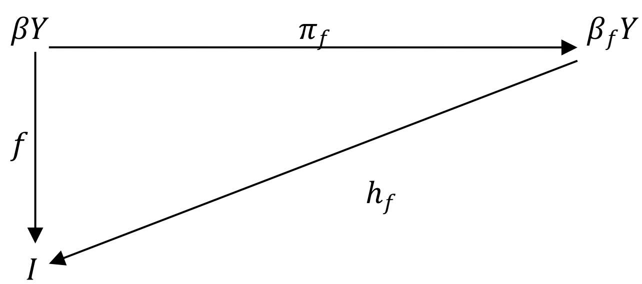
$$M = \{(Cl_{\beta Y}K_0),(Cl_{\beta Y}K_1)\setminus K_i \in \mathcal{B}, \text{for } (i = 0,1,2,...) \text{ and } Cl_{\beta Y}K_0 \cap Cl_{\beta Y}K_1 = \emptyset\} \quad \textbf{(4.2)}$$

From **(4.2)** above, for each pair $(Cl_{\beta Y}(K_0)),(Cl_{\beta Y}(K_1) \in M$, we can choose a function $f \in C(\beta Y, I) \ni f(Cl_{\beta Y}(K_0)) = 0$ and $f(Cl_{\beta Y}(K_1)) = 1$, by Uryshon function

Suppose $\mathcal{F}$ denotes the set of mappings by Uryshon function above such that $|\mathcal{F}| \leq 2^{\mathcal{X}_0}$. Then, by transfinite induction, we construct for each $f \in F$ a

$$\zeta_f \in (0,1) \ni Cl_{\beta Y}(f^{-1}[0,\zeta_f) \cap Cl_{\beta Y}(g^{-1}[0,\zeta_f)) \neq 0 \Rightarrow Cl_{\beta Y}(f^{-1}[0,\zeta_f) \cap Cl_{\beta Y}(g^{-1}[0,\zeta_g)) \cap Y \neq 0 \qquad \textbf{(4.3)}$$

Suppose $f \in F$ and $N = \{f^{-1}(t):Y\backslash t \in f(\beta Y\backslash Y)\} \cup \{\{x\}:x \in Y\}$ is an upper semi-continuous decomposition of βY, locally compact space of Y such that the decomposition space $\beta_f Y$ is a Stone-Cech compactification of locally compact space Y for which $f(\beta Y\backslash Y)$ is the remainder. Next, if π_f respresent the identity projection map on the space Y, and $h_f:\beta_f Y \to I$ defined as $h_f:f o\pi_{f^{-1}}$. Then, $h_f$is continuous and the diagram below is commutes;



As we see that $h_f\backslash_{\beta_f Y\backslash Y}$ is a homeomorphism. Then, we shall identify $\beta_f Y\backslash Y$ and $h_f(\beta_f Y\backslash Y)$ as follows:

Let $f_0 \in \mathcal{F}$ be the first point of $\mathcal{F}$and $\zeta_f = \frac{1}{2}$. Also, if every ζ_g have been constructed $\forall f > g$, where $g \in F$ such that **(4.3)** above is satisfied. Then, we follow the following step for our identifications:

STEP I: Let $X \in \beta_f Y$ be an open subset of $\beta_f Y$ and

$$A = \{\zeta \in (0,1):Cl(h_{f^{-1}}[0,\zeta)) \cap ClX \cap (\beta_f Y\backslash Y) \neq Cl(h_{f^{-1}}[0,\zeta) \cap X) \cap (\beta_f Y\backslash Y)\} \qquad \textbf{(4.4)}$$

If we let $B \subset \beta_f Y$, then $Cl(B)$ represent closure of $B \subset \beta_f Y$. Then (A) in **(4.4)** above will be a subset of $f(\beta_f Y\backslash Y)$, i.e $A \subset \beta Y\backslash Y$, while A is countable. Also, if we choose $\zeta \in I\backslash$ f$(\beta Y\backslash Y)$, then,

$Cl(h_{f^{-1}}[0,\zeta)) \cap ClX \cap (\beta_f Y \backslash Y) \subset h_{f^{-1}}[0,\zeta) \cap ClX \cap (\beta_f Y \backslash Y) = h_{f^{-1}}[0,\zeta) \cap ClX \cap (\beta_f Y \backslash Y) \subset Cl(h_{f^{-1}}[0,\zeta)) \cap ClX \cap (\beta_f Y \backslash Y)$, provided that $f^{-1}(\zeta) \cap\subset \beta Y \backslash Y = \emptyset$

Now, we shall show that A is countable, suppose A is not countable. Then as $A \subset R$ is uncountable subset of real numbers, then it must contain one condensation point. Also, it is obvious that there exist a condensation point ζ_0 which is a limit point from the left. Now, if U is an open Neighbourhood of $\zeta_0 \in \beta_f Y$, $\exists \zeta_1 \in U \cap A \ni \zeta_1 < \zeta_0$ and consequently, $\zeta_1 \in h_{f^{-1}}[0,\zeta_0) \cap U \cap ClX \subset U \cap Cl\left(h_{f^{-1}}[0,\zeta_0)\right) \cap X)$.

Therefore, it follows that, $\zeta_0 \in Cl(h_{f^{-1}}[0,\zeta_0)) \cap X \cap (\beta_f Y \backslash Y)$, and is a contradiction.

STEP II:$\exists$ a $\zeta_0 \in (0,1) \ni Cl\left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \cap Cl\left(h_{f^{-1}}[0,\zeta_0)\right) \cap (\beta_f Y \backslash Y) =$

$Cl\left(g^{-1}\left[0,\zeta_g\right)\right) \cap h_{f^{-1}}[0,\zeta_0) \cap Y \cap ((\beta_f Y \backslash Y) \forall f > g$ and $g \in F$. Also, as $|g \in F{:}f > g| < 2^{\mathcal{X}_0}$, since Y is locally compact and also open in $\beta_f Y$, we can conclude from **step I** above that;

$\left|\bigcup_{f>g}\left\{\zeta \in (0,1){:}Cl\left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \cap Cl\left(h_{f^{-1}}[0,\zeta)\right) \cap (\beta_f Y \backslash Y) \neq Cl(g^{-1}[0,\zeta)) \cap h_{f^{-1}}([0,\zeta) \cap Y) \cap \beta_f Y \backslash Y\right\}\right| < \mathcal{X}_0.2^{\mathcal{X}_0} = 2^{\mathcal{X}_0}$, with this, the choice for ζ_0 is possible.

STEP III: Let $\zeta_f = \zeta_0$. Then, we need to claim that **(4.3)** above is satisfied. Let $g \in F \ni f > g$ and we suppose that; $Cl_{\beta Y}(f^{-1}\left[0,\zeta_f\right) \cap Cl_{\beta Y}\left(g^{-1}\left[0,\zeta_g\right)\right) \neq \emptyset$. Then $\pi_f(Cl_{\beta Y}(f^{-1}\left[0,\zeta_f\right)) \cap \pi_f(Cl_{\beta Y}\left(g^{-1}\left[0,\zeta_g\right)\right) \neq \emptyset$ and also, $Cl(\pi_f f^{-1}\left[0,\zeta_f\right)) \cap Cl\left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \neq \emptyset$, since it is easily observed that $\pi_f(Cl_{\beta Y}X) = Cl(X \cap Y)$ for each open subset $X \subset \beta Y$. Therefore, $Cl(h_{f^{-1}}\left[0,\zeta_f\right) \cap Cl\left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \neq \emptyset$ **(4.5)**

Now, if we assume that **(4.5)** $\cap\, Y$ is disjoint. We have $Cl(h_{f^{-1}}\left[0,\zeta_f\right) \cap Cl\left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \cap \left(\beta_f Y \backslash Y\right) = Cl(h_{f^{-1}}\left[0,\zeta_f\right) \cap \left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \cap \left(\beta_f Y \backslash Y\right) \neq \emptyset$, **STEP II** above $\Longrightarrow h_{f^{-1}}\left[0,\zeta_f\right) \cap g^{-1}\left[0,\zeta_g\right) \cap Y \neq \emptyset$ is a contradiction. Therefore, **(4.5)** $\cap\, Y \neq \emptyset$. Now, if we let $x \in Cl(h_{f^{-1}}\left[0,\zeta_f\right) \cap Cl\left(g^{-1}\left[0,\zeta_g\right) \cap Y\right) \cap Y$; this show that **(4.3)** above holds for ζ_f.

STEP IV: Let $\phi = \left\{Cl_{\beta Y}(f^{-1}\left[0,\zeta_f\right){:}f \in F\right\}$. Then, ϕ is a closed base for βY. Also we see that for k_0 , $k_1 \in \phi$, where $k_0 \cap k_1 \neq \emptyset \Longrightarrow k_0 \cap k_1 \cap Y \neq \emptyset$. Next, we need to show that ϕ is weakly normal an τ_1. To do these, we proceed as follows;

Let k_0 , $k_1 \in ϕ \ni k_0 \cap k_1 = \emptyset$. By the fact that βY is compact Hausdorff space, there exist closed set $B_0, B_1 \in ϕ \ni k_0 \cap B_1 = B_0 \cap k_1 = \emptyset$ and $\beta Y = B_0 \cup B_1$. From above, since $\mathcal{B}$ is closed under finite intersections and unions, $\exists\, L_0, L_2 \in \mathcal{B} \ni k_0 \subset L_0 \subset Cl_{\beta Y}(L_0)$ and $B_1 \subset L_1 \subset Cl_{\beta Y}(B_1)$ and $Cl_{\beta Y}(L_1) \cap Cl_{\beta Y}(L_1) = \emptyset$. Now, if we let ${k_1}^* \in ϕ \ni Cl_{\beta Y}(L_1) \subset {k_1}^*$ and ${k_1}^* \cap Cl_{\beta Y}(L_0) = \emptyset$. Similarly, we shall find ${k_0}^* \in ϕ \ni B_0 \subset {k_0}^*$ and ${k_0}^* \cap {k_1}^* = \emptyset$. Therefore, $k_0 \cap {k_1}^* = {k_0}^* \cap k_1 = \emptyset$ and ${k_0}^* \cup {k_1}^* = \beta Y$, which shows that, ϕ is a normal closed base.

Next, to show that ϕ is τ_1 –space, we suppose ϕ is τ_1 and $K_o \in ϕ$. We shall Show that cl $\{ K_o \} = \{ K_o \}$, to prove this, let $K_1 \in Cl\{ K_o \}$ such that $k_1 \notin \{ K_o \}$. As $K_o \neq K_1$ and ϕ is τ_1 –space, ∃ open set B_o, $B_1 \in \tau \ni K_o \in B_o$ but $K_1 \notin B_1$. $K_1 \in Cl\{ K_o \}$ imply that K_1 is a limit point of $\{ k_o \}$ imply that $[B_o ⩎ \{ K_o \}] \setminus \{ K_o \} \neq ø$ (where $K_1 \in B_o$ and $B_o \in \tau$) $\Longrightarrow \{ K_o \} \setminus \{K_1 \} \neq ø$ (provided $K_o \in B_o$) $\Longrightarrow ϕ \neq ø$; which is a contradiction. Hence $Cl\{ K_o \} \subset \{ K_o \}$. As usual $\{ K_o \} \subset cl \{ K_o \}$, then we have $Cl\{ K_o \} = \{ K_o \}$.

Conversely, let ϕ be a topological space such that $\{ K_o \}$ is closed set for each $K_o \in ϕ$. To prove that ϕ is a τ_1 –space, we let $K_1 \neq K_o$ in ϕ. As $Cl\{ K_o \}=\{ K_o \}$, we have $K_1 \notin \{ K_o \} = cl\{ K_o \}$. Hence, $K_1 \in ϕ \setminus \{ K_o \} = ϕ \setminus Cl\{ K_o\}$. Similarly, $K_o \notin \{ K_1 \} = Cl\{ K_1 \} \Longrightarrow K_o \in ϕ \setminus \{ K_1 \} = ϕ \setminus Cl\{K_1\}$. We define $B_o = ϕ \setminus \{ K_o \}$ and $B_1 = ϕ \setminus Cl\{ K_1 \}= ϕ \setminus \{K_1\}$. Then $B_o, B_1 \in \tau$ such that $K_1 \in B_O$ but $K_o \notin B_o$ and $K_o \in B_1$ but $K_1 \notin B_1$.Hence, ϕ is a τ_1 –space.

Conclusion

In this research work, some topological properties on Maddox topological space Y and its Stone-Čech compactification were compared. Then, the Stone-Čech compactification of linear strongly B – convergent topological space of Maddox were characterized as Oz space, Qausi-Oz space and Baire spaces. The study also characterizes this Stone-Čech compactification as a GA compactification by using continuum hypothesis.

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