

Slow Roll Hybrid Inflation Theories

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Abstract. In this work, I have considered slow roll inflation in theories of two scalar fields, minimal and quadratic non-minimal cases has been considered separately. It is found that the inflationary observables, the spectral indices and tensor to scalar ratio at Hubble exit, depends on theory parameters in complicated manner due to the presence of two scalar fields, for the case of non-minimal couplings to gravity, the dependence complexity further increases because the scalar fields also influences the gravity sector, this is actually a special case of scalar-tensor gravity.

Keywords: Slow Roll Inflation, Hybrid Inflation, Non-Minimal Couplings to Gravity, Scalar Field Theory

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1 Introduction

As we all know that in order to solve the flatness and horizon problems, along with other problems, of the Big Bang theory, we need an inflationary phase in the very early Universe. In inflation, it's assumed that the source which driven the inflation is a field called the inflaton field with a suitable potential. The differences between different versions of the inflationary models comes from the choices of the nature of the inflaton field and its potential. The standard philosophy, known as chaotic inflation scenarios[1][2], is that inflation started with a minimally coupled classical inflaton slowly rolling to a stable minimum, due to particles production, reheating took place, forming the standard model particles (and also any undetected particles). Slow roll inflation formalism has also been used in theories where the potential is a more complicated function, in particular, in the Axion inflation scenario [3] in which the potential is a trigonometric function. In principle, the slow roll inflation formalism can be applied to any scalar field theory in which slow roll regime exist. Many different models of inflationary scenario are consistent with cosmological data [4], the question is if our Universe is an inflationary Universe then which one scenario has been realized? In elementary particle theories, more than one scalar fields can exist, for example, the mentioned Higgs field and an explicit inflaton field. More than one scalar degree of freedom is a natural possibility but this increases complexity in our system. If the scalar fields are interacting then complexity increases further. Two scalar field inflation theories has been discussed in [2] [5] with some specific assumptions about the parameters of the theories. Making some specific assumptions about the parameters of such theories simplify the system drastically, different sets of assumptions can lead to widely different inflationary Universes. Non-minimal coupling of inflaton has been used in quartic potential chaotic inflation scenario described in the Einstein's frame in the Higgs inflation scenario[6] and in the Jordan frame [7]. This coupling has also been used to resurrect the quadratic potential chaotic inflation scenario

described in the Einstein frame in [8](also see [9]). Slow roll inflation formalism has also been used in theories where the potential is a more complicated function, in particular, in the Axion inflation scenario [3] in which the potential is a trigonometric function and in the R^2 gravity inflation scenario in the Einstein's frame [10] in which the potential is an exponential function. It is known that the non-minimal coupling between a scalar field and the Ricci scalar arises due to renormalization, hence such a coupling is natural. The classical equivalence of the Einstein frame and the Jordan frame and the corresponding quantum field parametrization ambiguity are also known. In this paper, I have considered slow roll inflation in theories of two scalar fields such as massive fields and self-interacting fields, minimal and non-minimal couplings to gravity cases has been considered separately.

2 Slow Roll Hybrid Inflation Theories with Minimal Couplings to Gravity

In realistic particle theories, more than one type of scalar field can exist. In this section, two scalar fields are considered. The formulas for the relevant quantities are given below [11]. The first slow roll parameter is defined as

$$\epsilon = \frac{|\dot{H}|}{H^2} \approx \frac{M_p^2}{2} \frac{\left(\frac{dV(\phi)}{d\phi}\right)^2 + \left(\frac{dV(\Phi)}{d\Phi}\right)^2}{(V(\phi, \Phi))^2} = \Sigma_\alpha \epsilon^\alpha \quad (2.1)$$

Where,

$$\epsilon^\alpha = \frac{M_p^2}{2} \frac{\left(\frac{dV(\alpha)}{d\alpha}\right)^2}{(V(\phi, \Phi))^2} \quad (2.2)$$

The last N exponential folds is given by

$$N = -M_p^2 \Sigma_\alpha \int_\alpha^{\alpha_{end}} \frac{V(\alpha)}{V'(\alpha)} d\alpha \quad (2.3)$$

The second slow roll parameters are

$$\epsilon_2^\alpha = \frac{\ddot{\alpha}}{H\dot{\alpha}} \approx \epsilon - M_p^2 \frac{d^2V(\alpha)}{d\alpha^2} \frac{1}{V(\phi, \Phi)} \quad (2.4)$$

The spectral indices are given by

$$n_{\mathcal{R}} - 1 \approx -\frac{10}{3}\epsilon - 2\epsilon^{-1}\Sigma_\alpha \epsilon^\alpha \epsilon_2^\alpha \quad (2.5)$$

$$n_T \approx -2\epsilon \quad (2.6)$$

The tensor to scalar ratio is given by

$$r \approx 16\epsilon \quad (2.7)$$

The curvature power spectrum is given by

$$\mathcal{P}_{\mathcal{R}} \approx \frac{1}{2M_p^2\epsilon} \left(\frac{H}{2\pi}\right)^2 \quad (2.8)$$

The tensor power spectrum is given by

$$\mathcal{P}_T \approx \frac{8}{M_p^2} \left(\frac{H}{2\pi}\right)^2 \quad (2.9)$$

2.1 Self-Interacting Fields

Consider the theory of non-interacting fields

$$V(\phi, \Phi) = m_1^2 \phi^2 / 2 + m_2^2 \Phi^2 / 2 + \lambda_1 \phi^4 + \lambda_2 \Phi^4 \quad (2.10)$$

Assuming $m_1^2, m_2^2 \ll \lambda_1 M_p^2, \lambda_2 M_p^2$ and $\lambda_2 > \lambda_1$. Since inflation takes place at large field values greater than the planck mass, we can write

$$V(\phi, \Phi) \approx \lambda_1 \phi^4 + \lambda_2 \Phi^4 \quad (2.11)$$

The initial constraints are

$$V(\phi_{0i}) \approx \lambda_1 \phi_{0i}^4 \sim M_p^4; V(\Phi_{0i}) \approx \lambda_2 \Phi_{0i}^4 \sim M_p^4 \quad (2.12)$$

This gives initial constraints on field values

$$\phi_{0i} \sim \lambda_1^{-1/4} M_p; \Phi_{0i} \sim \lambda_2^{-1/4} M_p \quad (2.13)$$

During inflation, the field equations are

$$3H\dot{\phi} + 4\lambda_1 \phi^3 \approx 0 \quad (2.14)$$

$$3H\dot{\Phi} + 4\lambda_2 \Phi^3 \approx 0 \quad (2.15)$$

These equations can be written as

$$3H + \frac{4\lambda_1 \phi^3}{\dot{\phi}} \approx 0 \quad (2.16)$$

$$3H + \frac{4\lambda_2 \Phi^3}{\dot{\Phi}} \approx 0 \quad (2.17)$$

Subtracting first equation with second equation, we get

$$\frac{\lambda_1 \phi^3}{\dot{\phi}} \approx \frac{\lambda_2 \Phi^3}{\dot{\Phi}} \quad (2.18)$$

This gives the approximate relation

$$\Phi = \phi \sqrt{\frac{\lambda_1}{\lambda_2(1 + A_1 \phi^2 / M_p^2)}} \quad (2.19)$$

Here A_1 is a constant which would be fixed by a boundary condition. Using the initial conditions, A_1 can be calculated to be

$$A_1 = \sqrt{\lambda_1} \left(\sqrt{\frac{\lambda_1}{\lambda_2}} - 1 \right) \quad (2.20)$$

This theory has been investigated in [2] with the assumption $\lambda_2 \gg \lambda_1$ (which imply $1 + A_1 \phi^2 / M_p^2 \approx 1$ during inflation), I have relaxed this assumption and considered $\lambda_2 > \lambda_1$. Number of exponential folding can be calculated and set to the typical Hubble exit value:

$$\int_{\phi_{end}}^{\phi_*} \frac{(\lambda_2 \Phi^4 + \lambda_1 \phi^4) d\phi}{4\lambda_1 \phi^3 M_p^2} = \frac{1}{4M_p^2} \left(\phi^2 / 2 + \frac{\lambda_1 \phi^2}{2\lambda_2(1 + A_1 \phi^2 / M_p^2)} \right) \Big|_{\phi_{end}}^{\phi_*} = 60 \quad (2.21)$$

The first slow roll parameter is

$$\epsilon = \frac{8y^4 A_1 + 8y \frac{\lambda_1 A_1}{\lambda_2}}{(y-1)(y^2 + (\frac{\lambda_1}{\lambda_2}))^2} \quad (2.22)$$

Where, $y = 1 + A_1 \phi^2 / M_p^2$. $\epsilon(\phi_{end}) = 1$ imply

$$y^5(\phi_{end}) - y^4(1 + 8A_1) + 2y^3 \frac{\lambda_1}{\lambda_2} - 2 \frac{\lambda_1}{\lambda_2} y^2 + y((\frac{\lambda_1}{\lambda_2})^2 - 8 \frac{\lambda_1 A_1}{\lambda_2}) - (\frac{\lambda_1}{\lambda_2})^2 = 0 \quad (2.23)$$

The other relevant quantities can be calculated, the results are given below.

$$\epsilon^\phi = \frac{8A_1 y^4 \lambda_2^2}{(y-1)(\lambda_2 y^2 + \lambda_1)^2} \quad (2.24)$$

$$\epsilon^\Phi = \frac{\lambda_1}{\lambda_2 y^3} \epsilon^\phi \quad (2.25)$$

The second slow roll parameters are given by

$$\epsilon_2^\phi = \epsilon - \frac{12\lambda_2 A_1 y^2}{(\lambda_2 y^2 + \lambda_1)(y-1)} \quad (2.26)$$

$$\epsilon_2^\Phi = \epsilon - \frac{12\lambda_2 A_1 y}{(\lambda_2 y^2 + \lambda_1)(y-1)} \quad (2.27)$$

2.2 A Massive and a Self-Interacting Fields

Consider the theory of non-interacting fields

$$V(\phi, \Phi) = m_1^2 \phi^2 / 2 + m_2^2 \Phi^2 / 2 + \lambda_2 \Phi^4 \quad (2.28)$$

Assuming $m_2^2 \ll \lambda_2 M_p^2$. Since inflation takes place at large field values greater than the planck mass, we can write

$$V(\phi, \Phi) \approx m_1^2 \phi^2 / 2 + \lambda_2 \Phi^4 \quad (2.29)$$

The initial constraints are

$$V(\phi_{0i}) = m_1^2 \phi_{0i}^2 / 2 \sim M_p^4; V(\Phi_{0i}) \approx \lambda_2 \Phi_{0i}^4 \sim M_p^4 \quad (2.30)$$

This gives initial constraints on field values

$$\phi_{0i} \sim m_1^{-1} M_p^2; \Phi_{0i} \sim \lambda_2^{-1/4} M_p \quad (2.31)$$

During inflation, the field equations are

$$3H\dot{\phi} + m_1^2 \phi \approx 0 \quad (2.32)$$

$$3H\dot{\Phi} + 4\lambda_2 \Phi^3 \approx 0 \quad (2.33)$$

These equations can be written as

$$3H + \frac{m_1^2 \phi}{\dot{\phi}} \approx 0 \quad (2.34)$$

$$3H + \frac{4\lambda_2 \Phi^3}{\dot{\Phi}} \approx 0 \quad (2.35)$$

Subtracting first equation with second equation, we get

$$\frac{m_1^2 \phi}{\dot{\phi}} \approx \frac{4\lambda_2 \Phi^3}{\dot{\Phi}} \quad (2.36)$$

This can easily be integrated to give

$$\phi = \phi_0 \exp\left(-\frac{m_1^2}{8\lambda_2 \Phi^2}\right) \quad (2.37)$$

ϕ_0 can be calculated using the boundary conditions

$$\phi_0 = \frac{M_p^2}{m} \exp\left(-\frac{m^2}{8M_p^2 \sqrt{\lambda_0}}\right) \quad (2.38)$$

Number of exponential folds can be calculated and set to the typical Hubble exit value:

$$\int_{\Phi_{end}}^{\Phi^*} \frac{(\lambda_2 \Phi^4 + m_1^2 \phi^2/2) d\Phi}{4\lambda_2 \Phi^3 M_p^2} = \frac{1}{4M_p^2} (\Phi^2/2 + \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2}))|_{\Phi_{end}}^{\Phi^*} = 60 \quad (2.39)$$

The first slow roll parameter is

$$\epsilon = 2M_p^2 \frac{m_1^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2}) + 16\lambda_2^2 \Phi^6}{(m_1^4 \phi_0^4 \exp(-\frac{m_1^2}{2\lambda_2 \Phi^2}) + \lambda_2^2 \Phi^8 + m_1^2 \lambda_2 \Phi^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2}))} \quad (2.40)$$

Inflation ends when $\epsilon(\Phi_{end}) = 1$, therefore, we get

$$2M_p^2 (m_1^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi_{end}^2}) + 16\lambda_2^2 \Phi_{end}^6) = m_1^4 \phi_0^4 \exp(-\frac{m_1^2}{2\lambda_2 \Phi_{end}^2}) + \lambda_2^2 \Phi_{end}^8 + m_1^2 \lambda_2 \Phi_{end}^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi_{end}^2}) \quad (2.41)$$

The other slow roll parameters are given by

$$\epsilon_2^\phi = \epsilon - M_p^2 \frac{m_1^2}{m_1^2 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2})/2 + \lambda_2 \Phi^4} \quad (2.42)$$

$$\epsilon_2^\Phi = \epsilon - M_p^2 \frac{12\lambda_2 \Phi^2}{m_1^2 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2})/2 + \lambda_2 \Phi^4} \quad (2.43)$$

$$\epsilon^\phi = 2M_p^2 \frac{m_1^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2})}{(m_1^4 \phi_0^4 \exp(-\frac{m_1^2}{2\lambda_2 \Phi^2}) + \lambda_2^2 \Phi^8 + m_1^2 \lambda_2 \Phi^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2}))} \quad (2.44)$$

$$\epsilon^\Phi = 2M_p^2 \frac{16\lambda_2^2 \Phi^6}{(m_1^4 \phi_0^4 \exp(-\frac{m_1^2}{2\lambda_2 \Phi^2}) + \lambda_2^2 \Phi^8 + m_1^2 \lambda_2 \Phi^4 \phi_0^2 \exp(-\frac{m_1^2}{4\lambda_2 \Phi^2}))} \quad (2.45)$$

2.3 Two Massive Fields

Consider the theory of non-interacting fields

$$V(\phi, \Phi) = m_1^2 \phi^2/2 + m_2^2 \Phi^2/2 \quad (2.46)$$

The initial constraints are

$$V(\phi_{0i}) = m_1^2 \phi_{0i}^2/2 \sim M_p^4; V(\Phi_{0i}) \approx m_2^2 \Phi_{0i}^2/2 \sim M_p^4 \quad (2.47)$$

This gives initial constraints on field values

$$\phi_{0i} \sim m_1^{-1} M_p^2; \Phi_{0i} \sim m_2^{-1} M_p^2 \quad (2.48)$$

During inflation, the field equations are

$$3H\dot{\phi} + m_1^2\phi \approx 0 \quad (2.49)$$

$$3H\dot{\Phi} + m_2^2\Phi \approx 0 \quad (2.50)$$

These equations can be written as

$$3H + \frac{m_1^2\phi}{\dot{\phi}} \approx 0 \quad (2.51)$$

$$3H + \frac{m_2^2\Phi}{\dot{\Phi}} \approx 0 \quad (2.52)$$

Subtracting first equation with second equation, we get

$$\frac{m_1^2\phi}{\dot{\phi}} \approx \frac{m_2^2\Phi}{\dot{\Phi}} \quad (2.53)$$

This can easily be integrated to give

$$\Phi = A_1\phi^{\left(\frac{m_2}{m_1}\right)^2} \quad (2.54)$$

A_1 can be calculated using the boundary condition

$$A_1 = \frac{M_p^2}{m_2} \left(\frac{M_p^2}{m_1} \right)^{-\left(\frac{m_2}{m_1}\right)^2} \quad (2.55)$$

Number of exponential folding can be calculated and set to the typical Hubble exit value:

$$\int_{\phi_{end}}^{\phi_*} \frac{(A_1^2 m_2^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2} / 2 + m_1^2 \phi^2 / 2) d\phi}{m_1^2 \phi M_p^2} = \frac{1}{4M_p^2} (\phi^2 + A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2}) \Big|_{\phi_{end}}^{\phi_*} = 60 \quad (2.56)$$

The first slow roll parameter is

$$\epsilon = 2M_p^2 \frac{m_1^4 \phi^2 + m_2^4 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2}}{(m_1^2 \phi^2 + m_2^2 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2})^2} \quad (2.57)$$

Inflation ends when $\epsilon(\phi_{end}) = 1$, therefore, we get

$$2M_p^2 (m_1^4 + m_2^4 A_1^2 \phi_{end}^{2\left(\frac{m_2}{m_1}\right)^2 - 2}) = m_1^4 \phi_{end}^2 + m_2^4 A_1^4 \phi_{end}^{4\left(\frac{m_2}{m_1}\right)^2 - 2} + 2m_1^2 m_2^2 A_1^2 \phi_{end}^{2\left(\frac{m_2}{m_1}\right)^2} \quad (2.58)$$

The other slow roll parameters are given by

$$\epsilon_2^\phi = \epsilon - M_p^2 \frac{m_1^2}{m_1^2 \phi^2 / 2 + m_2^2 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2}} \quad (2.59)$$

$$\epsilon_2^\Phi = \epsilon - M_p^2 \frac{m_2^2}{m_1^2 \phi^2 / 2 + m_2^2 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2}} \quad (2.60)$$

$$\epsilon^\phi = 2M_p^2 \frac{m_1^4 \phi^2}{(m_1^2 \phi^2 + m_2^2 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2})^2} \quad (2.61)$$

$$\epsilon^\Phi = 2M_p^2 \frac{m_2^4 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2}}{(m_1^2 \phi^2 + m_2^2 A_1^2 \phi^{2\left(\frac{m_2}{m_1}\right)^2})^2} \quad (2.62)$$

2.4 An Axion and a Massive Fields

Consider an axion field and an explicit inflaton field with the theory

$$V(\phi_a, \phi) = m^2\phi^2/2 + \Lambda_a^4(1 - \cos(N\phi_a/f_a)) \quad (2.63)$$

Assuming the initial constraints

$$V(\phi_{0i}) = m^2\phi_{0i}^2/2 \approx M_p^4; V(\phi_{0i}^a) \approx M_p^4 \quad (2.64)$$

This gives initial constraints on field values

$$\phi_{0i} \approx \sqrt{2}m^{-1}M_p^2; \phi_{0i}^a \approx \frac{f_a}{N} \text{acos}(1 - M_p^4\Lambda_a^{-4}) \quad (2.65)$$

During inflation, the field equations are

$$3H\dot{\phi} + m^2\phi \approx 0 \quad (2.66)$$

$$3H\dot{\phi}_a + \frac{\Lambda_a^4 N}{f_a} \sin(N\phi_a/f_a) \approx 0 \quad (2.67)$$

These equations can be written as

$$3H + \frac{m^2\phi}{\dot{\phi}} \approx 0 \quad (2.68)$$

$$3H + \frac{\Lambda_a^4 N}{\dot{\phi}_a f_a} \sin(N\phi_a/f_a) \approx 0 \quad (2.69)$$

Subtracting first equation with second equation, we get

$$\frac{m^2\phi}{\dot{\phi}} \approx \frac{\Lambda_a^4 N}{\dot{\phi}_a f_a} \sin(N\phi_a/f_a) \quad (2.70)$$

This can easily be integrated to give

$$\phi = A_1 \left(\text{cosec}\left(\frac{N\phi_a}{f_a}\right) + \cot\left(\frac{N\phi_a}{f_a}\right) \right)^{-f} \quad (2.71)$$

Here $f = \frac{f_a^2 \Lambda_a^{-4} m^2}{N^2}$ and A_1 can be calculated using the initial conditions

$$A_1 = \sqrt{2}M_p^2 m^{-1} \left(\text{cosec}(\text{acos}(1 - M_p^4\Lambda_a^{-4})) + \cot(\text{acos}(1 - M_p^4\Lambda_a^{-4})) \right) \frac{f_a^2 \Lambda_a^{-4} m^2}{N^2} \quad (2.72)$$

Number of exponential folding can be calculated and set to the typical Hubble exit value:

$$-M_p^2 \left(A_1^2 \left(\text{cosec}\left(\frac{N\phi_a}{f_a}\right) + \cot\left(\frac{N\phi_a}{f_a}\right) \right)^{-2f} / 4 - (f_a/N)^2 \log(1 + \cos(N\phi_a/f_a)) \right) \Big|_{\phi_a^*}^{\phi_a^{end}} = 60 \quad (2.73)$$

The first slow roll parameter is

$$\epsilon = \frac{M_p^2}{2} \frac{m^4\phi^4 + (\Lambda_a^4 \frac{N}{f_a} \sin(\frac{N\phi_a}{f_a}))^2}{(m^2\phi^2/2 + \Lambda_a^4(1 - \cos(\frac{N\phi_a}{f_a})))^2} \quad (2.74)$$

Inflation ends when $\epsilon(\phi_{end}^a) = 1$, therefore, we get

$$m^4\phi_{end}^2 + (\Lambda_a^4 \frac{N}{f_a} \sin(\frac{N\phi_{end}^a}{f_a}))^2 = 2M_p^{-2} (m^2\phi_{end}^2/2 + \Lambda_a^4(1 - \cos(\frac{N\phi_{end}^a}{f_a})))^2 \quad (2.75)$$

The other slow roll parameters are given by

$$\epsilon_2^\phi = \epsilon - M_p^2 \frac{m^2}{m^2 A_1^2 (\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a}))^{-2f/2} + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a}))} \quad (2.76)$$

$$\epsilon_2^{\phi_a} = \epsilon - M_p^2 \frac{\Lambda_a^4 (\frac{N}{f_a})^2 \cos(N\phi_a/f_a)}{m^2 A_1^2 (\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a}))^{-2f/2} + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a}))} \quad (2.77)$$

$$\epsilon^\phi = \frac{M_p^2}{2} \frac{m^4 A_1^4 (\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a}))^{-4f}}{(m^2 A_1^2 (\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a}))^{-2f/2} + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a})))^2} \quad (2.78)$$

$$\epsilon^{\phi_a} = \frac{M_p^2}{2} \frac{(\Lambda_a^4 \frac{N}{f_a} \sin(\frac{N\phi_a}{f_a}))^2}{(m^2 \phi^2/2 + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a})))^2} \quad (2.79)$$

2.5 An Axion and a Self-Interacting Fields

Consider an axion field and an explicit inflaton field with the theory

$$V(\phi_a, \phi) = m^2 \phi^2/2 + \lambda \phi^4 + \Lambda_a^4 (1 - \cos(N\phi_a/f_a)) \quad (2.80)$$

Assuming $m^2 \ll \lambda M_p^2$. During inflation, we have

$$V(\phi_a, \phi) \approx \lambda \phi^4 + \Lambda_a^4 (1 - \cos(N\phi_a/f_a)) \quad (2.81)$$

Assuming the initial constraints

$$V(\phi_{0i}) \approx \lambda \phi_{0i}^4 \approx M_p^4; V(\phi_{0i}^a) \approx M_p^4 \quad (2.82)$$

This gives initial constraints on field values

$$\phi_{0i}^2 \approx \sqrt{\lambda}^{-1} M_p^2; \phi_{0i}^a \approx \frac{f_a}{N} \arccos(1 - M_p^4 \Lambda_a^{-4}) \quad (2.83)$$

During inflation, the field equations are

$$3H\dot{\phi} + 4\lambda\phi^3 \approx 0 \quad (2.84)$$

$$3H\dot{\phi}_a + \frac{\Lambda_a^4 N}{f_a} \sin(N\phi_a/f_a) \approx 0 \quad (2.85)$$

These equations can be written as

$$3H + \frac{m^2 \phi}{\dot{\phi}} \approx 0 \quad (2.86)$$

$$3H + \frac{\Lambda_a^4 N}{\dot{\phi}_a f_a} \sin(N\phi_a/f_a) \approx 0 \quad (2.87)$$

Subtracting first equation with second equation, we get

$$\frac{4\lambda\phi^3}{\dot{\phi}} \approx \frac{\Lambda_a^4 N}{\dot{\phi}_a f_a} \sin(N\phi_a/f_a) \quad (2.88)$$

This can easily be integrated to give

$$\phi^2 = (A_1 + \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a}))^f)^{-1} \quad (2.89)$$

Here $f = 8 \frac{f_a^2 \Lambda_a^{-4} \lambda}{N^2}$ and A_1 can be calculated using the initial conditions

$$A_1 = \sqrt{\lambda} M_p^{-2} - \log\left(\frac{\sqrt{2\Lambda_a^4 - M_p^4}}{M_p^2}\right) \quad (2.90)$$

Number of exponential folding can be calculated and set to the typical Hubble exit value:

$$-M_p^2 \left((A_1 + \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a})))^f \right)^{-1} / 8 - (f_a/N)^2 \log(1 + \cos(N\phi_a/f_a)) \Big|_{\phi_a^*}^{\phi_a^{end}} = 60 \quad (2.91)$$

The first slow roll parameter is

$$\epsilon = \frac{M_p^2}{2} \frac{16\lambda^2 \phi^6 + (\Lambda_a^4 \frac{N}{f_a} \sin(\frac{N\phi_a}{f_a}))^2}{(\lambda \phi^4 + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a})))^2} \quad (2.92)$$

Inflation ends when $\epsilon(\phi_{end}^a) = 1$, therefore, we get

$$16\lambda^2 \phi_{end}^6 + (\Lambda_a^4 \frac{N}{f_a} \sin(\frac{N\phi_{end}^a}{f_a}))^2 = 2M_p^{-2} (\lambda \phi_{end}^4 + \Lambda_a^4 (1 - \cos(\frac{N\phi_{end}^a}{f_a})))^2 \quad (2.93)$$

The other slow roll parameters are given by

$$\epsilon_2^\phi = \epsilon - M_p^2 \frac{12\lambda\phi^2}{\lambda(A_1 + \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a})))^2 + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a}))} \quad (2.94)$$

$$\epsilon_2^{\phi_a} = \epsilon - M_p^2 \frac{\Lambda_a^4 (N/f_a)^2 \cos(N\phi_a/f_a)}{\lambda(A_1 + \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a})))^2 + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a}))} \quad (2.95)$$

$$\epsilon^\phi = \frac{M_p^2}{2} \frac{16\lambda^2 (A_1 + f \times \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a})))^{-3}}{(\lambda(A_1 + f \times \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a})))^{-2} + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a})))^2} \quad (2.96)$$

$$\epsilon^{\phi_a} = \frac{M_p^2}{2} \frac{(\Lambda_a^4 \frac{N}{f_a} \sin(\frac{N\phi_a}{f_a}))^2}{(\lambda(A_1 + f \times \log(\operatorname{cosec}(\frac{N\phi_a}{f_a}) + \cot(\frac{N\phi_a}{f_a})))^{-2} + \Lambda_a^4 (1 - \cos(\frac{N\phi_a}{f_a})))^2} \quad (2.97)$$

3 Slow Roll Hybrid Inflation Theories with Non-Minimal Couplings to Gravity

Considering the gravity plus non-minimally coupled non-interacting scalar fields

$$\mathcal{A} = \int d^4x \sqrt{|g|} \left(\frac{M_p^2}{2} R + \sum_\alpha \left(\frac{1}{2} \xi_\alpha R \phi_\alpha^2 + \frac{1}{2} \partial_\rho \phi_\alpha \partial^\rho \phi_\alpha - V(\phi_\alpha) \right) \right) \quad (3.1)$$

Varying the action with respect to the metric gives

$$R_{\sigma\rho} - \frac{R}{2} g_{\sigma\rho} = - \frac{\sum_\alpha (T_{\sigma\rho}(\phi_\alpha) - \xi_\alpha (g_{\sigma\rho} - \nabla_\sigma \nabla_\rho) \phi_\alpha^2)}{3M_p^2 (1 + \sum_\alpha \xi_\alpha \phi_\alpha^2 / M_p^2)} \quad (3.2)$$

In flat FRW metric, one can get the modified first Friedmann equation

$$H^2 = \frac{\sum_\alpha (\frac{1}{2} \dot{\phi}_\alpha^2 + V(\phi_\alpha) - 6\phi_\alpha \dot{\phi}_\alpha H \xi_\alpha)}{3M_p^2 (1 + \sum_\alpha \xi_\alpha \phi_\alpha^2 / M_p^2)} \quad (3.3)$$

The ϕ_α field equation is

$$\ddot{\phi}_\alpha + 3H\dot{\phi}_\alpha - \xi_\alpha(12H^2 + 6\dot{H})\phi_\alpha + V'(\phi_\alpha) = 0 \quad (3.4)$$

During inflation, we have $\epsilon_1, \epsilon_2^\alpha \ll 1$, therefore, one gets

$$3H\dot{\phi}_\alpha - \xi_\alpha(12H^2)\phi_\alpha + V'(\phi_\alpha) \approx 0 \quad (3.5)$$

Assuming

$$\dot{\phi}_\alpha^2 \ll \xi_\alpha \phi_\alpha V'(\phi_\alpha), \phi_\alpha |\ddot{\phi}_\alpha| \ll \dot{\phi}_\alpha^2, |\ddot{\phi}_\alpha| \ll \xi_\alpha V'(\phi_\alpha), |\ddot{\phi}_\alpha| \phi_\alpha^2 \ll |\dot{\phi}_\alpha^3|, |\ddot{\phi}_\alpha V'(\phi_\alpha)| \ll |\dot{\phi}_\alpha^2 V''(\phi_\alpha)|. \quad (3.6)$$

One can approximately get the Hubble parameter and its derivatives from the above quadratic equation:

$$H = \frac{\dot{\phi}_\alpha}{8\xi_\alpha \phi_\alpha} \mp \sqrt{\frac{V'(\phi_\alpha)}{12\xi_\alpha \phi_\alpha}} \quad (3.7)$$

$$\dot{H} = -\frac{\dot{\phi}_\alpha^2}{8\xi_\alpha \phi_\alpha^2} \pm \dot{\phi}_\alpha \sqrt{\frac{V'(\phi_\alpha)}{12\xi_\alpha \phi_\alpha^3}} \pm \frac{\dot{\phi}_\alpha V'(\phi_\alpha) + \phi_\alpha \dot{\phi}_\alpha V''(\phi_\alpha)}{4\phi_\alpha \sqrt{3\xi_\alpha \phi_\alpha V'(\phi_\alpha)}} \quad (3.8)$$

$$\begin{aligned} \ddot{H} = & -\frac{\dot{\phi}_\alpha^3}{4\xi_\alpha \phi_\alpha^3} \mp 3\dot{\phi}_\alpha^2 \sqrt{\frac{V'(\phi_\alpha)}{48\xi_\alpha \phi_\alpha^3}} \mp 3\dot{\phi}_\alpha^3 \frac{(V'(\phi_\alpha) + \phi_\alpha V''(\phi_\alpha))}{8\phi_\alpha^2 \sqrt{3\xi_\alpha \phi_\alpha V'(\phi_\alpha)}} \\ & \pm \frac{(3\dot{\phi}_\alpha^2 V''(\phi_\alpha) + \phi_\alpha \dot{\phi}_\alpha^2 V'''(\phi_\alpha))}{4\phi_\alpha \sqrt{3\xi_\alpha \phi_\alpha V'(\phi_\alpha)}} + \mp \frac{\dot{\phi}_\alpha^2 (V'(\phi_\alpha) + \phi_\alpha V''(\phi_\alpha)) V''(\phi_\alpha)}{8\phi_\alpha V'(\phi_\alpha) \sqrt{3\xi_\alpha \phi_\alpha V'(\phi_\alpha)}}. \end{aligned} \quad (3.9)$$

During inflation, the modified first Friedmann equation reduces to

$$H^2 \approx \frac{\Sigma_\alpha (V(\phi_\alpha) - 6\phi_\alpha \dot{\phi}_\alpha \xi_\alpha H)}{3\Sigma_\alpha \xi_\alpha \phi_\alpha^2} \quad (3.10)$$

Putting the Hubble parameter value, one gets

$$\left(\frac{\dot{\phi}_\alpha}{8\xi_\alpha \phi_\alpha} \mp \sqrt{\frac{V'(\phi_\alpha)}{12\xi_\alpha \phi_\alpha}}\right)^2 \approx \frac{\Sigma_\alpha (V(\phi_\alpha) \pm \dot{\phi}_\alpha \sqrt{3\xi_\alpha \phi_\alpha V'(\phi_\alpha)})}{3\Sigma_\alpha \xi_\alpha \phi_\alpha^2}. \quad (3.11)$$

Keeping in mind that the summation over fields is only on the right side of the above equation. Using equations (3.7) with $\alpha = 1, 2$, one can get an equation relating the two scalar fields:

$$\frac{-\sqrt{3}\dot{\phi}_1}{4\xi_1 \phi_1} \pm \sqrt{\frac{V'(\phi_1)}{\xi_1 \phi_1}} \approx \frac{-\sqrt{3}\dot{\phi}_2}{4\xi_2 \phi_2} \pm \sqrt{\frac{V'(\phi_2)}{\xi_2 \phi_2}} \quad (3.12)$$

In principle, the above two equations with known potentials gives the fields as a function of cosmic time. The first slow roll parameter is given by

$$\epsilon = \frac{|\dot{H}|}{H^2} \quad (3.13)$$

The second slow roll parameter is given by

$$\epsilon_2^\alpha = \frac{\ddot{\phi}_\alpha}{H\dot{\phi}_\alpha} \quad (3.14)$$

The third slow roll parameter is given by [10]

$$\epsilon_3 = \frac{\dot{F}}{2HF} \approx \frac{\Sigma_\alpha \xi_\alpha \phi_\alpha \dot{\phi}_\alpha}{H(\Sigma_\alpha \xi_\alpha \phi_\alpha^2)} \quad (3.15)$$

The fourth slow roll parameter is given by [10]

$$\epsilon_4 = \frac{\dot{E}}{2HE} \approx \epsilon_3 + \frac{1}{F\Sigma_\alpha \dot{\phi}_\alpha^2 + 3\dot{F}^2 M_p^2/2} (2\dot{F}\ddot{F} - \dot{F}^3/F - \frac{2\dot{F}^2 \Sigma_\alpha \dot{\phi}_\alpha \ddot{\phi}_\alpha}{\Sigma_\alpha \dot{\phi}_\alpha^2}) \quad (3.16)$$

Where $E = F(1 + \frac{3\dot{F}^2 M_p^2}{2F\Sigma_\alpha \dot{\phi}_\alpha^2})$ and $F = 1 + \Sigma_\alpha \xi_\alpha \phi_\alpha^2/M_p^2 \approx \Sigma_\alpha \xi_\alpha \phi_\alpha^2/M_p^2$, $\dot{F} = 2\Sigma_\alpha \xi_\alpha \phi_\alpha \dot{\phi}_\alpha/M_p^2$, $\ddot{F} = 2(\Sigma_\alpha \xi_\alpha \dot{\phi}_\alpha^2/M_p^2 + \Sigma_\alpha \xi_\alpha \phi_\alpha \ddot{\phi}_\alpha/M_p^2)$.

The tensor to scalar ratio and the scalar and tensor spectral indices can be calculated using the above information[11]. The spectral indices are given by

$$n_{\mathcal{R}} = -4\epsilon - 2\frac{\Sigma_\alpha \dot{\phi}_\alpha^2 \epsilon_2^\alpha}{\Sigma_\alpha \dot{\phi}_\alpha^2} + 2\epsilon_3 - 2\epsilon_4 \quad (3.17)$$

$$n_T = -2\epsilon - 2\epsilon_3 \quad (3.18)$$

The tensor to scalar ratio is given by

$$r = \frac{8}{M^2 F} Q \quad (3.19)$$

At the end of inflation, $\dot{H}(\phi_{end}) \approx -H^2(\phi_{end})$, therefore, the Hubble parameter is

$$H(\phi_{end}) = \frac{\dot{\phi}}{4\xi\phi} \mp \sqrt{\frac{V'(\phi)}{6\xi\phi}} \Big|_{\phi=\phi_{end}} \quad (3.20)$$

Other quantities can be calculated just by replacing ξ with $\xi/2$ in the previously calculated quantities. ϕ_{end} can be calculated from

$$\left(-\frac{\dot{\phi}^2}{4\xi\phi^2} \pm \dot{\phi} \sqrt{\frac{V'(\phi)}{6\xi\phi^3}} \pm \frac{\dot{\phi}V'(\phi) + \phi\dot{\phi}V''(\phi)}{4\phi\sqrt{1.5\xi\phi V'(\phi)}}\right) \Big|_{\phi=\phi_{end}} \approx -\left(\frac{\dot{\phi}}{4\xi\phi} \mp \sqrt{\frac{V'(\phi)}{6\xi\phi}}\right)^2 \Big|_{\phi=\phi_{end}} \quad (3.21)$$

The field at typical Hubble exit time can be calculated from

$$\int_{\phi_*}^{\phi_{end}} H d\phi / \dot{\phi} \approx \int_{\phi_*}^{\phi_{end}} \left(\frac{1}{8\xi\phi} \mp \frac{1}{\phi} \sqrt{\frac{V'(\phi)}{12\xi\phi}}\right) d\phi \approx 60 \quad (3.22)$$

3.1 Self-Interacting Fields

Consider the theory of non-interacting fields

$$V(\phi, \Phi) = m_1^2 \phi^2/2 + m_2^2 \Phi^2/2 + \lambda_1 \phi^4 + \lambda_2 \Phi^4 \quad (3.23)$$

Assuming $m_1^2, m_2^2 \ll \lambda_1 M_p^2, \lambda_2 M_p^2$ and $\lambda_2 > \lambda_1$. Since inflation takes place at large field values greater than the planck mass, we can write

$$\rho \approx V(\phi, \Phi) \approx \lambda_1 \phi^4 + \lambda_2 \Phi^4 \quad (3.24)$$

The initial constraints are

$$V(\phi_{0i}) \approx \lambda_1 \phi_{0i}^4 \sim M_p^4; V(\Phi_{0i}) \approx \lambda_2 \Phi_{0i}^4 \sim M_p^4 \quad (3.25)$$

This gives initial constraints on field values

$$\phi_{0i} \sim \lambda_1^{-1/4} M_p; \Phi_{0i} \sim \lambda_2^{-1/4} M_p \quad (3.26)$$

$$H = \frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{\phi\sqrt{\lambda_1}}{\sqrt{3\xi_1}} \quad (3.27)$$

$$\dot{H} = -\frac{\dot{\phi}^2}{8\xi_1\phi^2} \pm \dot{\phi}\sqrt{\frac{3\lambda_1}{\xi_1}} \quad (3.28)$$

$$\ddot{H} = \frac{\dot{\phi}^3}{4\xi_1\phi^3} \quad (3.29)$$

$$\begin{aligned} \dot{\phi}^2\left(\frac{1}{64\xi^2\phi^2}\right) + \dot{\phi}\left(\mp\sqrt{\frac{\lambda_1}{3\xi_1}}\frac{1}{4\xi_1}\right) + \frac{\phi^2\lambda_1}{3\xi_1} \approx \\ \frac{(\lambda_1\phi^4 + \lambda_2\Phi^4 \pm 2\phi^2\dot{\phi}\sqrt{3\xi_1\lambda_1} \pm 2\Phi^2\dot{\Phi}\sqrt{3\xi_2\lambda_2})}{3\Sigma_\alpha\xi_\alpha\phi_\alpha^2}. \end{aligned} \quad (3.30)$$

$$\frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{\phi\sqrt{\lambda_1}}{\sqrt{3\xi_1}} \approx \frac{\dot{\Phi}}{8\xi_2\Phi} \mp \frac{\Phi\sqrt{\lambda_2}}{\sqrt{3\xi_2}} \quad (3.31)$$

$$\frac{1}{8\xi} \log\left(\frac{\phi_{end}}{\phi_*}\right) \mp \sqrt{\frac{\lambda}{3\xi}} \int_{\phi_*}^{\phi_{end}} \phi \frac{d\phi}{\dot{\phi}} \approx 60 \quad (3.32)$$

$$\left.\frac{\dot{\phi}}{\phi^2}\right|_{\phi=\phi_{end}} = \frac{8\xi^2}{1-4\xi} \left(\mp\left(\sqrt{\frac{6\lambda}{\xi}} - \sqrt{\frac{\lambda}{6\xi^3}}\right) \pm \sqrt{\frac{\lambda(36\xi^2+2-16\xi)}{6\xi^3}}\right) \quad (3.33)$$

3.2 A Massive and a Self-Interacting Fields

Consider the theory of non-interacting fields

$$V(\phi, \Phi) = m_1^2\phi^2/2 + m_2^2\Phi^2/2 + \lambda_2\Phi^4 \quad (3.34)$$

Assuming $m_2^2 \ll \lambda_2 M_p^2$. Since inflation takes place at large field values greater than the Planck mass, we can write

$$\rho \approx V(\phi, \Phi) \approx m_1^2\phi^2/2 + \lambda_2\Phi^4 \quad (3.35)$$

The initial constraints are

$$V(\phi_{0i}) = m_1^2\phi_{0i}^2/2 \sim M_p^4; V(\Phi_{0i}) \approx \lambda_2\Phi_{0i}^4 \sim M_p^4 \quad (3.36)$$

This gives initial constraints on field values

$$\phi_{0i} \sim m_1^{-1} M_p^2; \Phi_{0i} \sim \lambda_2^{-1/4} M_p \quad (3.37)$$

$$H = \frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{m_1}{\sqrt{12\xi_1}} \quad (3.38)$$

$$\dot{H} = -\frac{\dot{\phi}^2}{8\xi_1\phi^2} \quad (3.39)$$

$$\ddot{H} = \frac{\dot{\phi}^3}{4\xi_1\phi^3} \quad (3.40)$$

$$\begin{aligned} & \dot{\phi}^2 \frac{1}{64\xi_1^2\phi^2} + \dot{\phi} \left(\mp \frac{m_1}{8\xi_1\phi\sqrt{3\xi_1}} \right) + \frac{m_1^2}{12\xi_1} \approx \\ & \frac{(m_1^2\phi^2/2 + \lambda_2\Phi^4 \pm m\phi\dot{\phi}\sqrt{3\xi_1} \pm 2\Phi^2\dot{\Phi}\sqrt{3\xi_2\lambda_2})}{3\Sigma_\alpha\xi_\alpha\phi_\alpha^2}. \end{aligned} \quad (3.41)$$

$$\frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{m_1}{\sqrt{12\xi_1}} \approx \frac{\dot{\Phi}}{8\xi_2\Phi} \mp \frac{\Phi\sqrt{\lambda_2}}{\sqrt{3\xi_2}} \quad (3.42)$$

$$\frac{1}{8\xi} \log\left(\frac{\phi_{end}}{\phi_*}\right) \mp \frac{m}{\sqrt{12\xi}} \int_{\phi_*}^{\phi_{end}} \frac{d\phi}{\dot{\phi}} \approx 60 \quad (3.43)$$

$$\frac{\dot{\phi}}{\phi} \Big|_{\phi=\phi_{end}} = \pm 4m\sqrt{\frac{\xi}{6}} \left(1 - 2\sqrt{\frac{\xi}{4\xi - 1}}\right) \quad (3.44)$$

3.3 Two Massive Fields

Consider the theory of non-interacting fields

$$V(\phi, \Phi) = m_1^2\phi^2/2 + m_2^2\Phi^2/2 \quad (3.45)$$

The initial constraints are

$$V(\phi_{0i}) = m_1^2\phi_{0i}^2/2 \sim M_p^4; V(\Phi_{0i}) \approx m_2^2\Phi_{0i}^2/2 \sim M_p^4 \quad (3.46)$$

This gives initial constraints on field values

$$\phi_{0i} \sim m_1^{-1}M_p^2; \Phi_{0i} \sim m_2^{-1}M_p^2 \quad (3.47)$$

$$H = \frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{m_1}{\sqrt{12\xi_1}} \quad (3.48)$$

$$\dot{H} = -\frac{\dot{\phi}^2}{8\xi_1\phi^2} \quad (3.49)$$

$$\ddot{H} = \frac{\dot{\phi}^3}{4\xi_1\phi^3} \quad (3.50)$$

$$\begin{aligned} & \dot{\phi}^2 \frac{1}{64\xi_1^2\phi^2} + \dot{\phi} \left(\mp \frac{m_1}{8\xi_1\phi\sqrt{3\xi_1}} \right) + \frac{m_1^2}{12\xi_1} \approx \\ & \frac{(m_1^2\phi^2/2 + m_2^2\Phi^2/2 \pm m_1\phi\dot{\phi}\sqrt{3\xi_1} \pm m_2\Phi\dot{\Phi}\sqrt{3\xi_2})}{3\Sigma_\alpha\xi_\alpha\phi_\alpha^2}. \end{aligned} \quad (3.51)$$

$$\frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{m_1}{\sqrt{12\xi_1}} \approx \frac{\dot{\Phi}}{8\xi_2\Phi} \mp \frac{m_2}{\sqrt{12\xi_2}} \quad (3.52)$$

$$\frac{1}{8\xi} \log\left(\frac{\phi_{end}}{\phi_*}\right) \mp \frac{m}{\sqrt{12\xi}} \int_{\phi_*}^{\phi_{end}} \frac{d\phi}{\dot{\phi}} \approx 60 \quad (3.53)$$

$$\frac{\dot{\phi}}{\phi} \Big|_{\phi=\phi_{end}} = \pm 4m\sqrt{\frac{\xi}{6}} \left(1 - 2\sqrt{\frac{\xi}{4\xi - 1}}\right) \quad (3.54)$$

3.4 An Axion and a Massive Fields

Consider an axion field and an explicit inflaton field with the theory

$$V(\phi_a, \phi) = m^2\phi^2/2 + \Lambda_a^4(1 - \cos(N\phi_a/f_a)) \quad (3.55)$$

Assuming the initial constraints

$$V(\phi_{0i}) = m^2\phi_{0i}^2/2 \approx M_p^4; V(\phi_{0i}^a) \approx M_p^4 \quad (3.56)$$

This gives initial constraints on field values

$$\phi_{0i} \approx \sqrt{2}m^{-1}M_p^2; \phi_{0i}^a \approx \frac{f_a}{N} \text{acos}(1 - M_p^4\Lambda_a^{-4}) \quad (3.57)$$

$$H = \frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{m}{\sqrt{12\xi_1}} \quad (3.58)$$

$$\dot{H} = -\frac{\dot{\phi}^2}{8\xi_1\phi^2} \quad (3.59)$$

$$\ddot{H} = \frac{\dot{\phi}^3}{4\xi_1\phi^3} \quad (3.60)$$

$$\frac{\dot{\phi}^2 \frac{1}{64\xi_1^2\phi^2} + \dot{\phi}(\mp \frac{m_1}{8\xi_1\phi\sqrt{3\xi_1}}) + \frac{m_1^2}{12\xi_1} \approx \frac{(m^2\phi^2/2 + \Lambda_a^4(1 - \cos(N\phi_a/f_a)) \pm m\phi\dot{\phi}\sqrt{3\xi_1} \pm \dot{\phi}_a\sqrt{3\xi_2}\Lambda_a^4\phi_a\sin(N\phi_a/f_a)(N/f_a))}{3\Sigma_\alpha\xi_\alpha\phi_\alpha^2}} \quad (3.61)$$

$$\frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{m}{\sqrt{12\xi_1}} \approx \frac{\dot{\phi}_a}{8\xi_2\phi_a} \mp \frac{\sqrt{\Lambda_a^4(N/f_a)\sin(N\phi_a/f_a)}}{\sqrt{12\xi_2\phi_a}} \quad (3.62)$$

$$\frac{1}{8\xi} \log\left(\frac{\phi_{end}}{\phi_*}\right) \mp \frac{m}{\sqrt{12\xi}} \int_{\phi_*}^{\phi_{end}} \frac{d\phi}{\dot{\phi}} \approx 60 \quad (3.63)$$

$$\frac{\dot{\phi}}{\phi}|_{\phi=\phi_{end}} = \pm 4m\sqrt{\frac{\xi}{6}}(1 - 2\sqrt{\frac{\xi}{4\xi - 1}}) \quad (3.64)$$

3.5 An Axion and a Self-Interacting Fields

Consider an axion field and an explicit inflaton field with the theory

$$V(\phi_a, \phi) = m^2\phi^2/2 + \lambda\phi^4 + \Lambda_a^4(1 - \cos(N\phi_a/f_a)) \quad (3.65)$$

Assuming $m^2 \ll \lambda M_p^2$. During inflation, we have

$$V(\phi_a, \phi) \approx \lambda\phi^4 + \Lambda_a^4(1 - \cos(N\phi_a/f_a)) \quad (3.66)$$

Assuming the initial constraints

$$V(\phi_{0i}) \approx \lambda\phi_{0i}^4 \approx M_p^4; V(\phi_{0i}^a) \approx M_p^4 \quad (3.67)$$

This gives initial constraints on field values

$$\phi_{0i}^2 \approx \sqrt{\lambda}^{-1}M_p^2; \phi_{0i}^a \approx \frac{f_a}{N} \text{acos}(1 - M_p^4\Lambda_a^{-4}) \quad (3.68)$$

$$H = \frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{\phi\sqrt{\lambda}}{\sqrt{3\xi_1}} \quad (3.69)$$

$$\dot{H} = -\frac{\dot{\phi}^2}{8\xi_1\phi^2} \pm \dot{\phi}\sqrt{\frac{3\lambda}{\xi_1}} \quad (3.70)$$

$$\ddot{H} = \frac{\dot{\phi}^3}{4\xi_1\phi^3} \quad (3.71)$$

$$\frac{\dot{\phi}^2\left(\frac{1}{64\xi_1^2\phi^2}\right) + \dot{\phi}\left(\mp\sqrt{\frac{\lambda_1}{3\xi_1}}\frac{1}{4\xi_1}\right) + \frac{\phi^2\lambda_1}{3\xi_1}}{3\Sigma_\alpha\xi_\alpha\phi_\alpha^2} \approx \frac{(\lambda\phi^4 + \Lambda_a^4(1 - \cos(N\phi_a/f_a)) \pm 2\phi^2\dot{\phi}\sqrt{3\xi_1\lambda} \pm \dot{\phi}_a\sqrt{3\xi_2\Lambda_a^4\phi_a\sin(N\phi_a/f_a)(N/f_a)})}{3\Sigma_\alpha\xi_\alpha\phi_\alpha^2} \quad (3.72)$$

$$\frac{\dot{\phi}}{8\xi_1\phi} \mp \frac{\phi\sqrt{\lambda}}{\sqrt{3\xi_1}} \approx \frac{\dot{\phi}_a}{8\xi_2\phi_a} \mp \frac{\sqrt{\Lambda_a^4(N/f_a)\sin(N\phi_a/f_a)}}{\sqrt{12\xi_2\phi_a}} \quad (3.73)$$

$$\frac{1}{8\xi} \log\left(\frac{\phi_{end}}{\phi_*}\right) \mp \sqrt{\frac{\lambda}{3\xi}} \int_{\phi_*}^{\phi_{end}} \phi \frac{d\phi}{\dot{\phi}} \approx 60 \quad (3.74)$$

$$\frac{\dot{\phi}}{\phi^2}|_{\phi=\phi_{end}} = \frac{8\xi^2}{1-4\xi} \left(\mp \left(\sqrt{\frac{6\lambda}{\xi}} - \sqrt{\frac{\lambda}{6\xi^3}} \right) \pm \sqrt{\frac{\lambda(36\xi^2+2-16\xi)}{6\xi^3}} \right) \quad (3.75)$$

4 Results and Discussion

In this paper, I have considered non-interacting two scalar field theories of Inflationary Universe, minimal and quadratic non-minimal couplings to gravity has been considered separately. In these theories, the inflation is driven by both the fields, the relative contribution of each field to driven the inflation depends on the values of parameters of the theory under consideration, this leads to inflationary observables which depends on theory parameters in complicated manner. By making some specific assumptions about the theory parameters simplify the calculations significantly but such assumptions should be reasonable and a separate analysis should be conducted to see the effects beyond such assumptions. In the considered theories there is an integration constant which could only be fixed by a boundary condition. In the chaotic inflation theory [2], an initial condition is assumed for the potential energy density which states that the initial potential energy density is of the order of Planck density, this corresponds to an initial value for the inflaton field, this is chosen so that the spacetime at initial time can be treated classically. I have also considered the same initial condition and calculated the unknown integration constant.

References

- [1] B. A. Bassett, S. Tsujikawa, and D. Wands, *Inflation dynamics and reheating*, *Reviews of Modern Physics* **78** (May, 2006) 537–589.
- [2] A. Linde, *Particle physics and inflationary cosmology*, 2005.
- [3] K. Choi, S. H. Im, and C. S. Shin, *Recent progress in the physics of axions and axion-like particles*, *Annual Review of Nuclear and Particle Science* **71** (Sept., 2021) 225–252.
- [4] A. Linde, *Inflationary cosmology after planck 2013*, 2014.

- [5] A. Linde, *Hybrid inflation*, *Physical Review D* **49** (Jan., 1994) 748–754.
- [6] C. F. Steinwachs, *Higgs Field in Cosmology*, p. 253–287. Springer International Publishing, 2020.
- [7] E. Komatsu and T. Futamase, *Complete constraints on a nonminimally coupled chaotic inflationary scenario from the cosmic microwave background*, *Physical Review D* **59** (Feb., 1999).
- [8] T. Tenkanen, *Resurrecting quadratic inflation with a non-minimal coupling to gravity*, *Journal of Cosmology and Astroparticle Physics* **2017** (Dec., 2017) 001–001.
- [9] M. Eshaghi, M. Zarei, N. Riazi, and A. Kiasatpour, *A non-minimally coupled potential for inflation and dark energy after planck 2015: a comprehensive study*, *Journal of Cosmology and Astroparticle Physics* **2015** (Nov., 2015) 037–037.
- [10] A. De Felice and S. Tsujikawa, *$f(r)$ theories*, *Living Reviews in Relativity* **13** (June, 2010).
- [11] A. Mogiya, *An Introduction to Cosmology*. Academia.edu, 2024-2025.



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