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Stability Analysis Of Rectangular CCSS and CCCS Isotropic Plates using 3rd Order Energy Functional

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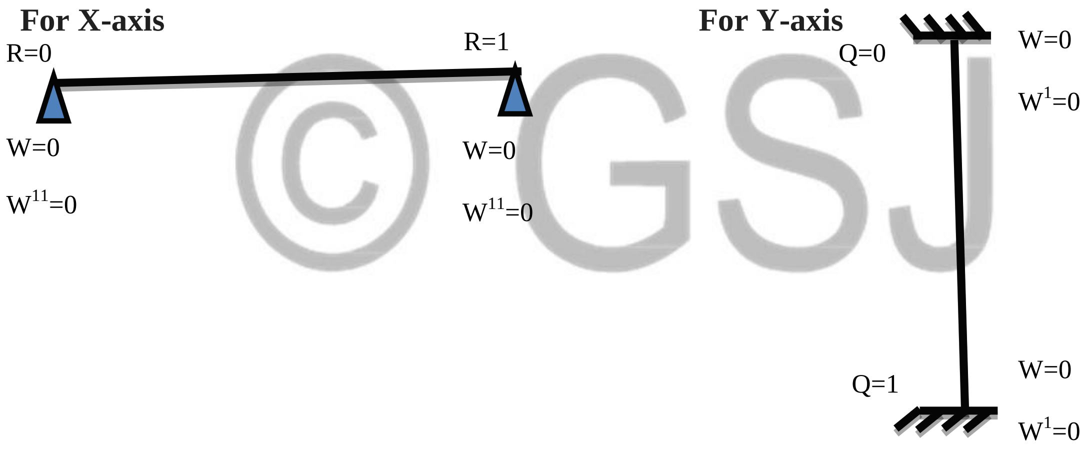
. **Key words:** Stability Analysis, Critical Buckling Load, Third Order Strain Energy,, Stiffness coefficients, Third Order Total Potential Energy Functional

ABSTRACT

This paper covers the buckling tendency of rectangular Isotropic CCSS and CCCS plates using (odd number)Third Order Energy Functional. The third order strain energy was formulated from the summation of the product of the stress and strain at every point in the plate within the plate domain. This strain energy was further added to the external load to obtain the Third order total potential energy functional. This was achieved by adopting the direct variation method by making buckling load the subject of the expression. Their shape functions were considered at the major X and Y axis . For the derivation of shape function, the deflection and 1st derivative of deflection were considered for clamped support, while the deflection and the second derivative of deflection where considered. Upon the derivation of the shape functions of the various boundary conditions , their differential values were integrated , to obtain the needed stiffness coefficients . These values were finally substituted in to the critical buckling load equation to obtain their various coefficients. Graphically, it was shown that as the of the aspect ratio increases , critical buckling load coefficients.

INTRODUCTION; The main purpose of buckling analysis of a rectangular plate is to determine the critical buckling loads and the corresponding buckled configuration of equilibrium and the smallest value of the load producing the buckling is called the critical buckling load. Stability analysis of plate deals with buckling tendency of plates, and buckling of plate on the other hand is the transition of the plate from the stable state of equilibrium to the unstable state. Buckling of a rectangular plate is of great importance in the initiation of a deflection pattern, which if loads are further increased above their critical values, rapidly leads to a very large lateral deflection. The plate leading from the stable to unstable configuration of equilibrium always passes through the neutral state of equilibrium. It also leads to large bending stresses which may cause complete failure of the plate. Rectangular plate has width size comparable to size of their lengths and so is modeled as two dimensional planes unlike beams and columns. Under compression, rectangular plate tends to buckle out of their plane. The buckled shape depends on the loading and support conditions. Generally the supports are considered along the X and Y axis, as shown the diagrams below.



Other researchers before now have gotten solution to the stability analysis of CCSS and CCCS plates using both the Second and Fourth the Order energy functional for Buckling of plate. None of the researchers have any work on buckling of plate using Third order energy functional. This is exactly the vacuum this thesis tends fill.

STRAIN ENERGY; The strain energy forms the bases for the formation of the total potential energy, needed for the functional. The strain energy is as a result of the combination of three different relationship namely, strain–displacement, strain–deflection and stress- strain-poison ratio equations. The various substitutions of the relationships mentioned above gave rise to the relationship that exist between the in-plain stresses and the out of plane displacement known as the stress-deflection relationships. These includes

$$\sigma_x = \frac{-Ez}{1-\mu^2}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)$$

also

$$\sigma_y = \frac{-Ez}{1-\mu^2}\left(\mu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)$$

and

$$\tau_{xy} = -\frac{Ez(1-\mu)}{(1-\mu^2)}\frac{\partial^2 w}{\partial x \partial y}$$

where $$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}$$

But strain energy is defined as indefinite summation of the product of stress and strain at every point in the plate within the plate domain (0≤ x ≤ a : 0≤ y ≤ b; -t/2 ≤ z ≤ t/2). Where a, b and t are the lenght of the plate in x, y and z directions respectively. And so mathematically this can be expressed as:

$$U = \frac{1}{2}\iiint_{xyz} \sigma\varepsilon \, dxdydz \, ,$$

which can in turn can be expressed as

$$U = \frac{1}{2}\iint_{xy}\left[\int_{-t/2}^{\frac{t}{2}} \sigma\varepsilon \, dz\right] dxdy$$

where $$\overline{U} = \int_{-t/2}^{t/2} \sigma\varepsilon \, dz$$

This means that

$$U = \frac{1}{2}\iint_{xy} \overline{U} \, dxdy$$

recalling that $\sigma\varepsilon = \sigma_x\varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy}$

Where σ , ε, τ, γ are strain , stress, shear stress and shear strain respectively. Substituting the values into the equation and minimizing gives

$$U = \frac{1}{2}\int_{-t/2}^{t/2} \frac{-Ez}{(1-\mu^2)} \int \left[\left(\frac{\partial^2 w}{\partial x^2} + u\frac{\partial^2 w}{\partial y^2}\right) \times -z\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial^2 w}{\partial y^2} + u\frac{\partial^2 w}{\partial x^2}\right) \times -z\frac{\partial^2 w}{\partial y^2} + (1-u)\frac{\partial^2 w}{\partial x \partial y} \times -2Z\left(\frac{\partial^2 w}{\partial x \partial y}\right)\right] dz$$

Considering the flexural rigidity, D as $\frac{Ez^3}{12(1-\mu^2)}$ leaves the strain energy as

$$U = \frac{D}{2}\int_0^a \int_0^b \left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) dxdy$$

TOTAL POTENTIAL ENERGY; The minimization of the strain energy gave rise to 3rd order strain energy equation. Mathematically, the equation can be expressed as

$$U = \frac{D}{2}\int_0^a \int_0^b \left(\frac{\partial^3 w}{\partial x^3}.\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x \partial y^2}.\frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3}.\frac{\partial w}{\partial y}\right) dxdy \qquad \text{or}$$

$$U = \frac{D}{2}\int_0^a \int_0^b \left(\frac{\partial^3 w}{\partial x^3}.\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x^2 \partial y}.\frac{\partial w}{\partial y} + \frac{\partial^3 w}{\partial y^3}.\frac{\partial w}{\partial y}\right) dxdy$$

But the summation of the 3rd order strain energy and the external load will give the Third order total potential energy functional. That is

$$\Pi = \frac{D}{2}\int\int \left(\frac{\partial^3 w}{\partial x^3}.\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x^2 \partial y}.\frac{\partial w}{\partial y} + \frac{\partial^3 w}{\partial y^3}.\frac{\partial w}{\partial y}\right) dx - \frac{1}{2}\int\int (Nx\frac{\partial^2 w}{\partial x^2} + 2Nxy\frac{\partial w}{\partial x}.\frac{\partial w}{\partial y}dxdy + Ny\frac{\partial^2 w}{\partial y^2})dxdy$$

Considering Nxy = Nx = 0, the equation can reduced to

$$\Pi = \frac{D}{2}\int\int \left(\frac{\partial^3 w}{\partial x^3}.\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x^2 \partial y}.\frac{\partial w}{\partial y} + \frac{\partial^3 w}{\partial y^3}.\frac{\partial w}{\partial y}\right) dxdy - \frac{1}{2}\int\int (Nx\frac{\partial^2 w}{\partial x^2}.w)\, dxdy$$

Upon substitution of the Ah for w in the third order total potential energy equation,

$$\Pi \text{ becomes } \frac{A^2 D}{2}\int_0^a \int_0^b \left(\frac{\partial^3 h}{\partial x^3}.\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x \partial y^2}.\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial y^3}.\frac{\partial h}{\partial y}\right) dxdy - \frac{A^2 Nx}{2}\int\int \frac{\partial^2 h}{\partial x^2} dxdy \quad \text{or}$$

$$\Pi = \frac{A^2 D}{2}\int_0^a \int_0^b \left(\frac{\partial^3 h}{\partial x^3}.\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x^2 \partial y}.\frac{\partial h}{\partial y} + \frac{\partial^3 h}{\partial y^3}.\frac{\partial h}{\partial y} - \frac{Nx}{2}\int\int \frac{\partial^2 h}{\partial x^2}\right) dxdy$$

and this is therefore called Third Order Total Energy Functional which is finally minimized by making the Nx the subject of the equation. Mathematically expressed as

$$Nx = \frac{\frac{2AD}{2}\int_0^1 \int_0^1 \left(\left(\left[\frac{\partial^3 h}{\partial x^3}\right].\frac{\partial h}{\partial x} + 2\left[\frac{\partial^3 h}{\partial x^2 \partial y}\right].\frac{\partial h}{\partial y} + \left[\frac{\partial^3 h}{\partial y^3}\right].\frac{\partial h}{\partial y}\right) dxdy\right.}{\frac{2A}{2}\int_0^a \int_0^b \left(\frac{\partial h}{\partial x}\right)^2 dxdy} \quad \text{or}$$

$$Nx = \frac{\frac{2AD}{2}\int_0^1\int_0^1 \left(\left(\left[\frac{\partial^3 h}{\partial x^3}\right]\cdot\frac{\partial h}{\partial x} + 2\left[\frac{\partial^3 h}{\partial x\,\partial y^2}\right]\cdot\frac{\partial h}{\partial x} + \left[\frac{\partial^3 h}{\partial y^3}\right]\cdot\frac{\partial h}{\partial y}\right)dxdy\right.}{\frac{2A}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dxdy}$$

after the integration of the differential values, the values of the stiffness coefficients were obtained and substituted to get $Nx = \frac{D(k_1+\frac{2}{p^2}k_2)+\frac{1}{p^4}k_3))}{k_6 a^2}$ for b/a or

and $Nx = \frac{Dk_1 + 2p^2k_2 + p^4k_3}{k_6 a^2}$ for a/b

where k_1, k_2, k_3 and k_6 are the stiffness coefficients while the shape functions, h for CCSS and CCCS panels are $(1.5R^2-2.5R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$ and $(1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$ respectively. (Ibearugbulem O., 2014)

DETERMINATION OF NON-DIMENSIONAL BUCKLING LOAD PARAMETERS; The shape functions derived were differentiated to different degrees and that gave rise to the differential values ($\overline{\boldsymbol{K}} - \boldsymbol{values}$). By Integrating the differential values of the shape functions, h of the two plates we obtain their stiffness coefficients k_i as mention above

For the case of CCSS panel, the ki-values are

$$k_1 = \int_0^1\int_0^1(-45R + 185.5R^2 - 240R^3 + 96R^4) \text{ x } (2.25Q^4 - 7.5Q^5 + 9.25Q^6 - 5Q^7 + Q^8)\, dRdQ$$

which gives

$$k_1 = (-1\tfrac{4}{5}) \text{ x } (\tfrac{19}{2520}) = \tfrac{-19}{1400} k1 = (-1\tfrac{4}{5}) \text{ x } (\tfrac{19}{2520}) = \tfrac{-19}{1400}$$

also

$$k_2 = \int_0^1\int_0^1(9R^2 - 45R^3 + 80.25R^4 - 60R^5 + 16R^6) \text{ x } (4.5Q^2 - 30Q^3 + 58.5Q^4 - 45Q^5 + 12Q^6) dRdQ$$

$$= [[(\frac{9R^3}{3} - \frac{45R^4}{4} + \frac{80.25R^5}{5} - \frac{60R^6}{6} + \frac{16R^7}{7}) \text{ x } (\frac{4.5Q^3}{3} - \frac{30Q^4}{4} + \frac{58.5Q^5}{5} - \frac{45Q^6}{6} + \frac{12^7}{7})]^1]^1$$

$$=(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7}) \text{ x } (\frac{4.5}{3} - \frac{30}{4} + \frac{58.5}{5} - \frac{45}{6} + \frac{12}{7})$$

which gives $k_2 = (\frac{3}{35}) \text{ x } (\frac{-3}{35}) = \frac{-9}{1225}$

also

$$k_3 = \int_0^1 \int_0^1 (2.25R^4 - 7.5R^5 + 9.5R^6 - 5R^7 + R^8) \ \mathrm{x}\ (-45Q + 184.5Q^3 - 240Q^3 + 96Q^4)\ dRdQ$$

$$= [[(\frac{2.25R^5}{5} - \frac{7.5R^6}{6} + \frac{9.5R^7}{7} - \frac{5R^8}{8} + \frac{R^9}{9})\ \mathrm{x}\ (\frac{-45Q^2}{2} + \frac{184.5Q^3}{3} - \frac{240Q^4}{4} + \frac{96Q^5}{5})]^1]^1$$

$$= (\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9})\ \mathrm{x}\ (\frac{-45}{2} + \frac{184.5}{2} - \frac{240}{3} + \frac{96}{5})$$

which gives

$$k_3 = (\frac{19}{2520})\ \mathrm{x}\ (-1\frac{4}{5}) = \frac{-19}{1400}$$

and finally for CCSS panel

$$k_6 = \int_0^1 \int_0^1 (9R^2 - 45R^3 + 80.25R^4 - 60R^5 + 16R^6)$$

$$\mathrm{x}\ (2.25Q^4 - 7.5Q^5 + 9.25Q^6 - 5Q^7 + Q^8) dRdQ$$

$$= [[(\frac{9R^3}{3} - \frac{45R^4}{4} + \frac{80.25R^5}{5} - \frac{60R^6}{6} + \frac{16R^7}{7})\ \mathrm{x}\ (\frac{2.25Q^5}{5} - \frac{7.5Q^6}{6} + \frac{9.25Q^7}{7} - \frac{5Q^8}{8} + \frac{Q^9}{9})]^1]^1$$

$$= [[(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7})\ \mathrm{x}\ (\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9})]^1]^1$$

which gives

$$k_6 = (\frac{3}{35})\ \mathrm{x}\ (\frac{19}{2520}) = \frac{19}{29400}$$

For the case of CCCS panel, the k_i-values are

$$k_1 = \int_0^1 \int_0^1 (-45R + 184.5R^2 - 240R^3 + 96R^4)\ \mathrm{x}\ (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\ dRdQ$$

$$= [[(\frac{-45R^2}{2} + \frac{184.5R^3}{3} - \frac{240R^4}{4} + \frac{96R^5}{5})\ \mathrm{x}\ (\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{6Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9})]^1]^1$$

$$= [[(\frac{-45}{2} + \frac{184.5}{3} - \frac{240}{4} + \frac{96}{5})\ \mathrm{x}\ (\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9})$$

which gives

$k_1 = (-1\frac{4}{5})\ x\ (\frac{1}{630})\ =\ \frac{-9}{3150}$

also

$$k_2 = \int_0^1 \int_0^1 (9R^2 - 45R^3 + 80.25R^4 - 60R^5 + 16R^6)\ x\ (2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6)dRdQ$$

$$= [[(\frac{9R^3}{3} + \frac{45R^4}{4} + \frac{80.25R^5}{5} - \frac{60R^6}{6} + \frac{16R^7}{7})\ x\ (\frac{2Q^3}{3} - \frac{16Q^4}{4} + \frac{38Q^5}{5} - \frac{36Q^6}{6} + \frac{12^7}{7})]^1]^1$$

$$= (\frac{9}{3} + \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7})\ x\ (\frac{2}{3} - \frac{16}{4} + \frac{38}{5} - \frac{36}{6} + \frac{12}{7})$$

which gives

$k_2 = (\frac{2}{35})\ x\ (\frac{-2}{105})\ =\ \frac{-6}{3675}$

also

$$k_3 = \int\int (2.25R^4 - 7.5R^5 + 9.25R^6 - 5R^7 + R^8)\ x\ (-24Q + 120Q^2 - 192Q^3 + 96Q^4)$$

which gives

$$= [[(\frac{2.25R^5}{5} - \frac{7.5R^6}{6} + \frac{9.25R^7}{7} - \frac{5R^8}{8} + \frac{R^9}{9})\ x\ (\frac{-24Q^2}{2} + \frac{120Q^3}{3} - \frac{192Q^4}{4} + \frac{96Q^5}{5})]^1]^1$$

$$= (\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9})\ x\ (\frac{-12}{1} + \frac{24}{2} + \frac{72}{3} - \frac{192}{4} + \frac{96}{5})$$

$k_3 = (\frac{19}{2520})\ x\ (-\frac{4}{5})\ =\ \frac{-19}{3150}$

and finally for CCCS panel

$$k_6 = \int_0^1 \int_0^1 (9R^2 - 45R^3 + 80.25R^4 - 60R^5 + 16R^6)$$

$$x\ (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)dRdQ$$

$$= [[(\frac{9R^3}{3} - \frac{45R^4}{4} + \frac{80.25R^5}{5} - \frac{60R^6}{6} + \frac{16R^7}{7})\ x\ (\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{6Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9})]^1]^1$$

$$= [[(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7}) \text{ x } (\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9})]^1]^1$$

which gives

$$k_6 = (\frac{3}{35}) \text{ x } (\frac{1}{630}) = \frac{3}{22050}$$

The substitution of these coefficients into the buckling equation gives the results discussed in the next section.

RESULTS AND DISCUSSION; From the graphs plotted, the polynomial functions obtained were of second order of degree.. In each case as the aspect ratio increases the corresponding critical buckling load parameters decreases thereby making the plate most stiff plate.

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For Clamped-Clamped-Simple-Simple isotropic rectangular plate

The critical buckling load coefficients for **CCSS** plate for aspect ratio of b/a are presented in Table a, while the present study of the plate was compared with the previous works in Table c.

Table a; Critical Buckling load parameters for CCSS plate for aspect ratio of b/a

$\frac{b}{a}$	2	1.9	1.8	1.7	1.6
$N_x = N\frac{D}{a^2}$	27.99763	28.91064	30.01896	31.38274	33.08694

1.5	1.4	1.3	1.2	1.1	1
35.25447	38.06802	41.80765	46.91812	54.13558	64.73865

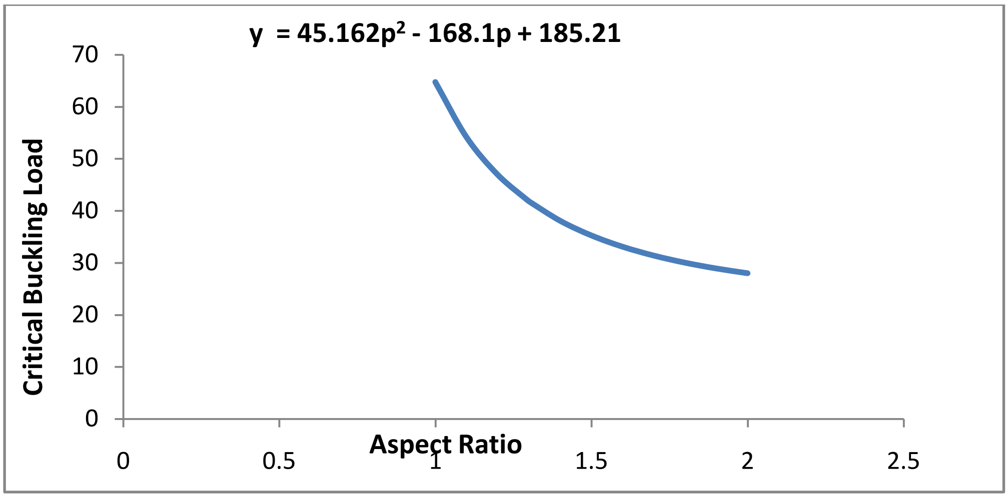

Figure 1 Graph of CCSS Plate

For Clamped-Clamped-Clamped-Simple isotropic rectangular plate.

The critical buckling load parameters for **CCCS** plate of aspect ratio of b/a were presented in Table b while Figure 2, shows the graph of the critical buckling load coefficient against the aspect ratio.

Table b; Critical Buckling load parameters for cccs plate for aspect ratio of b/a

$\frac{b}{a}$	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1
$N_x = N\frac{D}{a^2}$	29.73555	31.01314	32.59168	34.57114	37.09519	40.37528	44.73133	50.66246	58.97557	71.02976	89.22687

Two graphs were plotted for the rectangular plates with Clamped – Clamped –Simple-Simple- edge support and also for Clamped –Clamped –Clamped – Simple. The graph contains the Critical Buckling Load against Aspect Ratio for the aspect of **b/a.** The aspect Ratio is of the range 1.0 to 2.0 with arithmetic increase of 0.1 in each case, From the graph of critical buckling load against the aspect ratios plotted, as the aspect ratio, the critical buckling load also decreases. The graph is of the 2nd order (degree) polynomial.

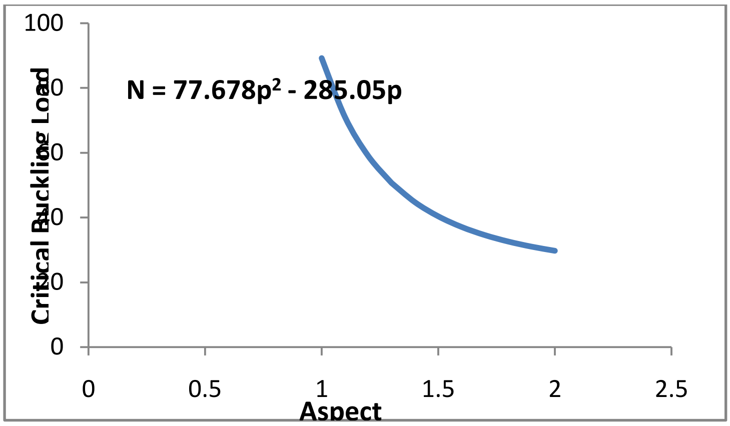

Figure 1.2 Graph of CCCS Plate.

Table c: N- values from present study compared with previous works for CCSS rectangular plate buckling .

Aspect Ratios (p = b/a)	N-Values from Present Study (i)	N-Values from Ibearugbulem et al. (2014) (ii)	N-Values from Ventsel& Krauthermmer (2001), Megson (2010),Chajes (1974) (iii)	Percentage Difference Between (i) and (ii)
1	64.7387	64.7359	64.723	0.004325
1.1	54.1356	54.1333	54.1166	0.004249
1.2	46.9181	46.9162	46.9017	0.00405
1.3	41.8077	41.8059	41.7894	0.004305
1.4	38.068	38.0664	38.0515	0.004203
1.5	35.2545	35.253	35.2384	0.004255
1.6	33.0869	33.0739	33.0574	0.03929
1.7	31.3827	31.3815	31.3647	0.003824

1.8	30.019	30.018	30.005	0.003331
1.9	28.9106	28.9097	28.8951	0.003113
2	27.9976	27.9967	27.9799	0.003215

Table d: N- values from present study compared with previous works for CCCS rectangular plate buckling

Aspect Ratios (p = b/a)	**N-Values from Present Study (i)**	**N-Values from Ibearugbulem et al. (2014) (ii)**	**N-Values from Ventsel& Krauthermmer (2001), Megson (2010),Chajes (1974) (iii)**	**Percentage Difference Between (i) and (ii)**
1	89.2269	89.3539	89.3372	-0.14233
1.1	71.0298	71.1311	71.1185	-0.14262
1.2	58.9756	59.0597	59.0463	-0.1426
1.3	50.6625	50.7347	50.7218	-0.14251
1.4	44.7313	44.7949	44.7769	-0.14218
1.5	40.3753	40.4325	40.4154	-0.14167
1.6	37.0952	37.1476	37.1315	-0.14126
1.7	34.5711	34.6198	34.6048	-0.14087

1.8	32.5917	32.6373	32.6225	-0.13991
1.9	31.0131	31.0564	31.0385	-0.13962
2	29.7356	29.7769	29.7638	-0.13889

CONCLUSIONS

From the research work done so far, one can confidently conclude that:

i. The third order energy functional has been formulated .

ii. General critical buckling load equation for CCCs and CCSS plates where derived.

iii The various stiffness coefficients for all the plates for CCCs and CCSS plates were also generated.

iv Finally the non-dimensional buckling load parameters where determined and found to be very close to those from previous works with the highest percentage difference as recorded being less than 0.2%.

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