Global Scientific JOURNALS

GSJ: Volume 6, Issue 2, February 2018, Online: ISSN 2320-9186
www.globalscientificjournal.com

THE EFFECT OF REPLACING THE JOUKOWSKI MAP WITH THE GENERALIZED KARMAN-TREFFTZ MAP IN THE METHOD OF ZEDAN

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ABSTRACT

In this research paper, the conformal mapping aspect of the method of Zedan (1990) for computation of the lift coefficient (c_l) and surface pressure coefficient distribution (c_p) on arbitrary airfoils in potential flows is generalized by replacing the inverse Joukowski transformation in the method by an inverse of the Karman-Trefftz transformation in order to asses its performance.The generalization which ensures that the airfoil contour at and around the trailing edge region is now more properly accounted for in the mapping process is applied to the NACA 4412 airfoil and its performance measured by comparing its c_l and c_p values with those of the original version of the method by Zedan (1990) using NACA experimental data as yardstick. The modified method has given reasonable predictions of these aerodynamic coefficients and has outperformed the original version of the method by Zedan (1990)

Keywords and phrases: Airfoil, NACA airfoil, Angle of attack, Lift coefficient, Pressure coefficient , Joukowski map, Karman-Trefftz map

1.0 INTRODUCTION

The method of Zedan (1990) for computation of the lift coefficient (c_l) and surface pressure coefficient distribution (c_p) on arbitrary airfoils in potential flows uses the inverse Joukowski transformation to conformally map the flow around and on the boundary of an airfoil (except at the trailing edge) in the z plane onto the flow exterior to and on the boundary of a pseudo circle, respectively, in the w plane where the c_l and c_p on the arbitrary airfoil are now computed. The Joukowski map used in the mapping process permanently fixes the trailing edge angle τ of the airfoil at zero degree. The value $\tau = 0^0$ is not a realistic one since real airfoils have blunt trailing edges or can be modified to have finite none zero angles at their trailing edges for computational purposes (Gómez and Álvaro, 2006, Nico, 2012). Consquently, the Joukowski map may not be able to properly account for the airfoil contour at and around the trailing edge point (Kapania *et al.* 2008). It is therefore expected that if a mapping function can be found with the ability to correct the defect in the Joukowski map, the accuracy of the method may likely improve since the pseudo circles generated in this case will be more truly the images of these airfoils. This suggests replacement of the Joukowski map in the method of Zedan (1990) with the generalized Karman-Trefftz map that has a parameter that controls the trailing edge angle and includes the Joukowski map as a special case. This research paper therefore intends to explore this modified approach by applying it to the NACA 4412 airfoil at two different flow angles of attack and measuring its performance by comparing its c_l and c_p values with those of the original method of Zedan (1990) using NACA experimental data as yardstick. We shall also be interested in the results on the modified NACA 4412 which is done by adjusting the coefficient -0.1015 in the formula for thickness to have the value -0.1036. This modification results in the least overall change in the airfoil shape when compared with similar ones (Nico, 2012).

2.0 THE GENERALIZED CONFORMAL MAPPING ASPECT OF THE METHOD OF ZEDAN FOR COMPUTATION OF THE LIFT AND PRESSURE DISTRIBUTION ON ARBITRARY AIRFOILS

In this modified approach, an inverse of the Karman-Trefftz transformation defined by Mateescu and Abdo (2005) as

$$\frac{z-nc}{z+nc} = \left(\frac{w-c}{w+c}\right)^n \ , \ \left(n = 2 - \frac{\tau}{\pi}\right) \tag{1}$$

Where τ is the trailing edge angle of the airfoil, is now used to transform an arbitrary airfoil with its tail at the point $z = nc$ in the z plane on to a pseudo circle in the w plane. The constant c in equation (1) is estimated as $1/2n$ of the distance between the trailing edge and a point mid way between the leading edge and the centre of curvature of the nose. The leading edge radius of the the NACA 4412 airfoil is given by Abbot and Von Doenhoff (1959) as $1.1019t^2$, where t is the maximum thickness of the airfoil.The centroid of the pseudo circle w_* in the w plane is then determined using the approximation given by Björn (2006) and the axes of the w plane are translated to w_* and rotated by angle α so that the real axis is in the direction of the free stream. The coordinate plane obtained following axes translation and rotation is called the ζ plane. The translation of axes and rotation by angle α is equivalent to the transformation

$$\zeta = (w - w_*)e^{-i\alpha} \tag{2}$$

The relationship between the velocities at points in the plane of the airfoil v_z to the corresponding points in the plane of the pseudo circle v_ζ is derived as

$$v_z = v_\zeta \left|\frac{dw}{dz}\right| \left|\frac{d\zeta}{dw}\right| = v_\zeta \left|\frac{dw}{dz}\right| \quad (3)$$

where

$$\frac{dw}{dz} = \frac{1}{\left(\frac{4n^2c^2(w+c)^{n-1}(w-c)^{n-1}}{[(w+c)^n-(w-c)^n]^2}\right)} \quad (4)$$

and

$$\left|\frac{d\zeta}{dw}\right| = 1 \quad (5)$$

from equations (1) and (2). Notice from equation (3) that the singularity $w = +c$ is a source of error for the computation of the velocity field and hence pressure distribution on the airfoil.

To compute v_ζ, the method assumes a solution for the complex potential $\Omega(\zeta)$ of the flow past the pseudo circle as

$$\Omega(\zeta) = \mathrm{v}_\infty\zeta + \sum_{k=1}^{\infty}\frac{c_k}{\zeta^k} + i\frac{\Gamma}{2\pi}\ln\zeta \quad (6)$$

where the coefficients of the series in the second term $c_k = a_k + ib_k \quad (k = 1,2,3,\ldots)$. The first term in equation (6) represents a uniform flow with free stream velocity of magnitude v_∞, the infinite series in the middle represents a doublet at the origin and the higher order terms to account for the deviation from an exact circle. The last term represents a vortex flow with circulation Γ taken clockwise.

The complex velocity

$$\frac{d\Omega}{d\varsigma} = v_\infty + \sum_{k=1}^{\infty}\frac{-kc_k}{\zeta^{k+1}} + i\frac{\Gamma}{2\pi\zeta} \quad (7)$$

is analytic everywhere except at the origin; the point $\zeta = 0$. This singularity is within the contour of the pseudo circle and therefore posses no problem to the method since the flow under consideration is that which is external to the pseudo circle.The velocity field in the plane of the pseudo circle also satisfies the infinity boundary condition in equation (7); that is,

$$\frac{d\Omega}{d\varsigma} \to v_\infty \quad as \quad |\zeta| \to \infty.$$

If $\Omega(\zeta) = \phi + i\psi$ and $\zeta = re^{i\theta}$ in equaton (6) , then

$$\psi = v_\infty r \sin\theta + \sum_{k=1}^{\infty} \left\{ a_k \left(-\frac{\sin k\theta}{r^k}\right) + b_k \left(\frac{\cos k\theta}{r^k}\right) \right\} + \Gamma\left(\frac{\ln r}{2\pi}\right) \quad (8)$$

The function ψ is the stream function of the flow. Setting the stream function to a constant generates the streamlines of the flow. Let ψ_0 denote the stream line corresponding to the flow on the boundary of the pseudo circle. On applying the condition of constant stream line to equation (8) and noting that since ψ_0 is finite, the infinite series on the right hand side of the equation must converge, the equation then takes the form

$$\sum_{k=1}^{m} a_k \left(\frac{\sin k\theta}{r^k}\right) + \sum_{k=1}^{m} b_k \left(-\frac{\cos k\theta}{r^k}\right) + \Gamma\left(-\frac{\ln r}{2\pi}\right) + \psi_0 = v_\infty r \sin\theta \quad (9)$$

on retaining a limited number of terms , say m, in the series.Since the derivative of the complex potential yields the conjugate of the velocity field, if we let

$$\frac{d\Omega}{d\varsigma} = v_1 - iv_2$$

then we have on retaining the first m terms of the infinite series in equation (7), that

$$v_1 = v_\infty + \Gamma\left(\frac{\sin\theta}{2\pi r}\right) + \sum_{k=1}^{m} a_k \left(\frac{-k\cos(k+1)\theta}{r^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{-k\sin(k+1)\theta}{r^{k+1}}\right) \quad (10)$$

$$v_2 = \Gamma\left(-\frac{\cos\theta}{2\pi r}\right) + \sum_{k=1}^{m} a_k \left(\frac{-k\sin(k+1)\theta}{r^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{k\cos(k+1)\theta}{r^{k+1}}\right) \quad (11)$$

The Kutta condition requires that the trailing edge T is a stagnation point where the fluid velocity vanishes identically (Anderson, 1991); that is, $v_{1T} = v_{2T} = 0$. Thus,

$$\sum_{k=1}^{m} a_k \left(\frac{k\cos(k+1)\theta_T}{{r_T}^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{k\sin(k+1)\theta_T}{{r_T}^{k+1}}\right) + \Gamma\left(\frac{-\sin\theta_T}{2\pi r_T}\right) = v_\infty \quad (12)$$

$$\sum_{k=1}^{m} a_k \left(\frac{-k\sin(k+1)\theta_T}{{r_T}^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{k\cos(k+1)\theta_T}{{r_T}^{k+1}}\right) + \Gamma\left(\frac{-\cos\theta_T}{2\pi r_T}\right) = 0 \quad (13)$$

Determination of the series coefficients a_k, b_k $(k = 1,2,3, \dots m)$, circulation Γ, and the pseudo circle streamline ψ_0, is done by taking $2m$ control points on the contour of the pseudo circle and applying the condition of constant streamline given by equation (9) and the Kutta condition represented by equations (12) and (13). These control points are selected by the cosine spacing of the closed interval $0 \leq x \leq 1$ to obtain the airfoil coordinates which are then projected onto the pseudo circle by the inverse Karman-Trefftz map. Alternatively, the coefficients $a_k, b_k (k = 1,2,3, \dots, m)$, circulation Γ, and constant stream line ψ_0 are determined by applying equation (9) to a number of control points $n > 2m$ on the boundary of the pseudo circle, where m is low and n is relatively large, to obtain an over determined system of equations. This system is then solved using a least square error minimization scheme. The determined values of the coefficients a_k, b_k $(k = 1,2,3, \dots m)$ and circulation Γ are substituted in equations (10) and (11) to obtain the components v_1 *and* v_2 of the velocity vector on the surface of the pseudo circle. The total velocity is then evaluated as

$$v_\zeta = |u - iv| = \sqrt{u^2 + v^2} \tag{14}$$

The velocity on the surface of the airfoil v_z can now be computed in terms of v_ζ using equation (3). Finally the pressure coefficient distribution c_p is obtained using the formula given by Deglaire (2008) and Anderson (1991) as

$$c_p = 1 - \left(\frac{v}{v_\infty}\right)^2 \tag{15}$$

The lift coefficient is computed using the formula given by Anderson (1991) and Karamcheti (1966) as

$$c_l = \frac{2\Gamma}{v_\infty l} \tag{16}$$

where l is the airfoil length and Γ is the value of circulation computed from the system of equations.

3.0 COMPARISON OF THE LIFT COEFFICIENT DATA FOR THE NACA 4412 AIRFOIL BETWEEN THE MODIFIED ZEDAN AND ZEDAN'S METHOD

Tables 1 show comparisons between the lift coefficient as a function of the angle of attack on the NACA 4412 airfoil for the modified method of Zedan, Zedan's method, and experiment (Pinkerton, 1936), respectivey, while Table 2 are results on the modified airfoil.The c_l presented in the tables and indeed the entire paper were generated taking $m = 12$ terms in the assumed complex series solution and $n = 30$ points on the boundary of the pseudo circle.

Table 1: Lift Coefficients for the NACA 4412 Airfoil taking $m = 12$ terms in the Assumed complex Series Solution and $n = 30$ Points on the Boundary of the Airfoil.

S/no	Angle of Attack (α) in degrees	Modified Zedan's Method (c_l)	Zedan's Method (c_l)	Experimental Data (c_l)
1	10^0	1.984	1.990	1.289
2	13.5^0	2.386	2.393	1.579

Table 2: Lift Coefficients for the Modified NACA 4412 Airfoil taking $m = 12$ terms in the Assumed complex Series Solution and $n = 30$ Points on the Boundary of the Airfoil.

S/no	Angle of Attack (α) in degrees	Modified Zedan's Method (c_l)	Zedan's Method (c_l)	Experimental Data (c_l)
1	10^0	1.707	1.708	1.289
2	13.5^0	2.113	2.115	1.579

Observe from Tables 1 and 2 that in either form of the airfoil's shape the predicted c_l values by both methods do not accurately predict the experimental value with higher deviations occurring in the case of the NACA 4412 airfoil.This is due to the fact that the theory does not take into consideration the effects of the viscous boundary layer. However, the c_l values predicted by the modified method of Zedan (1990) are certainly a reasonable improvement over those of Zedan's method moreso that the portion of the airfoil under consideration is small.

4.0 COMPARISON OF THE PRESSURE COEFFICIENT DISTRIBUTION DATA FOR THE NACA 4412 AIRFOIL BETWEEN THE MODIFIED ZEDAN AND ZEDAN'S METHOD

Figures 1 and 2 show MATLAB plots of the pressure coefficient distribution as a function of chordwise position for the NACA 4412 at flow angles of attack $\alpha = 10^0$ and $\alpha = 13.5^0$, respectively, by the modified Zedan' method, Zedan's method, and experiment, while figures 3 and 4 are predictions on the modified NACA 4412 airfoil at $\alpha = 10^0$ and $\alpha = 13.5^0$, respectively. Figures 1 and 2 show that the prediction of pressure distribution by both methods on the lower surface of the NACA 4412 airfoil is fairly good except at and around the trailing edge where the method fails at the point of intersection of the curves.

Figure 1: Pressure Distribution for the NACA 4412 Airfoil at 10^0 Angle of Attack $(m = 12,\ n = 30)$

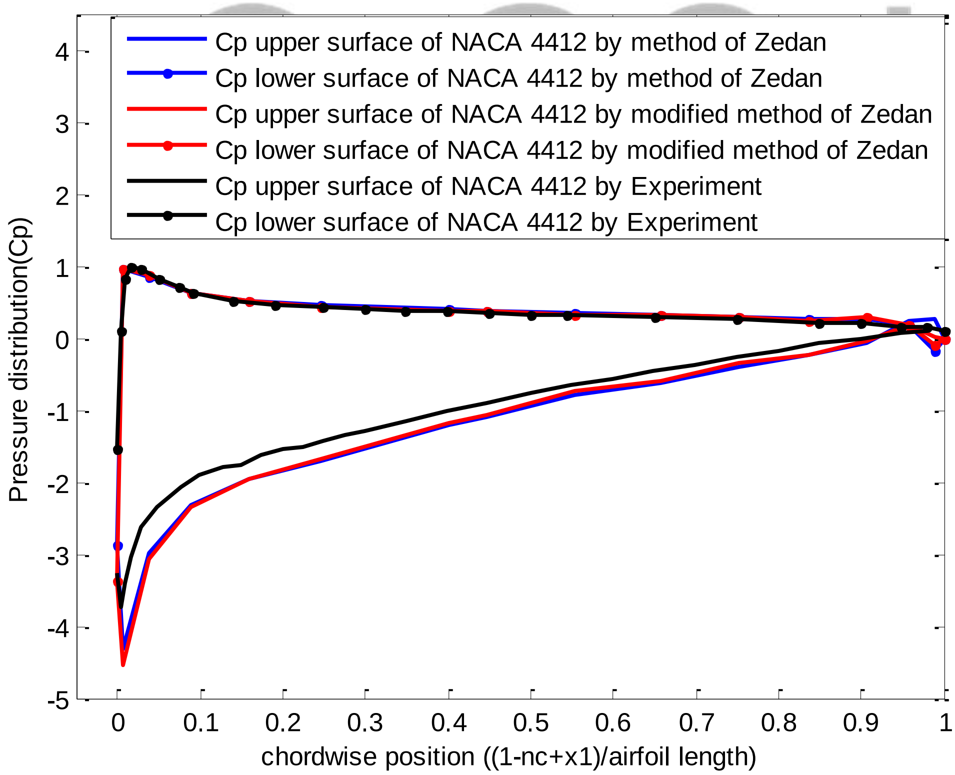


Figure 2: Pressure Distribution for the NACA 4412 Airfoil at 13.5^0 Angle of Attack $(m = 12,\ n = 30)$

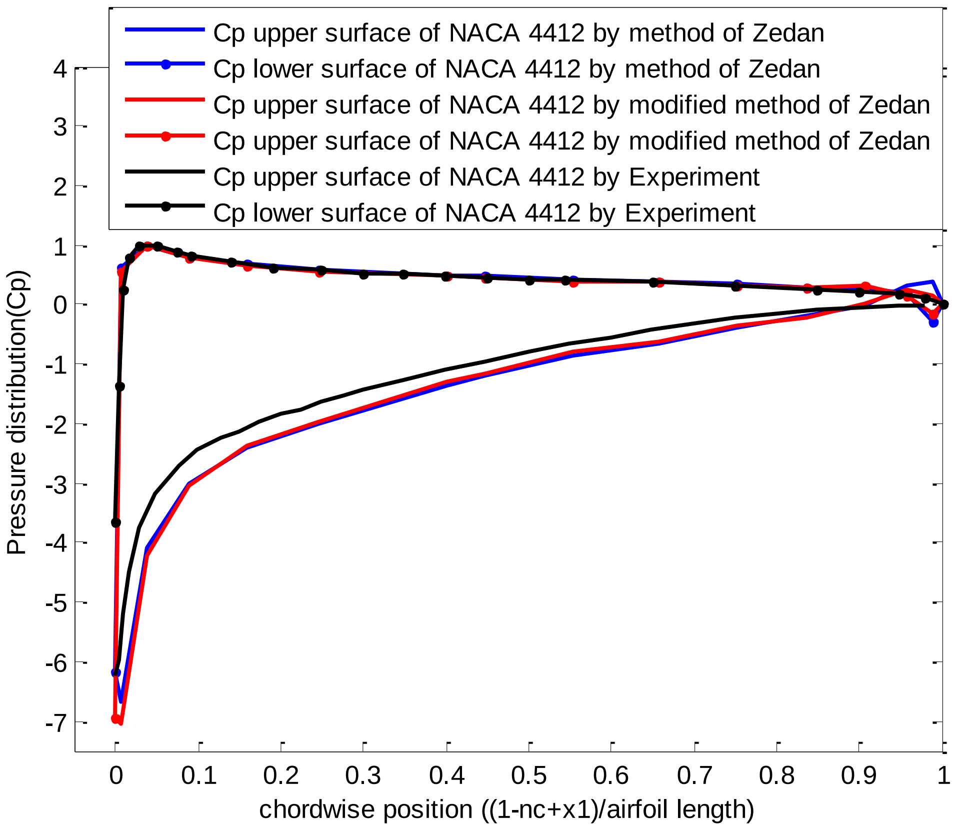


Figure 3: **Pressure Distribution for the Modified NACA 4412 Airfoil at 10^0 Angle of Attack ($m = 12,\ n = 30$)**

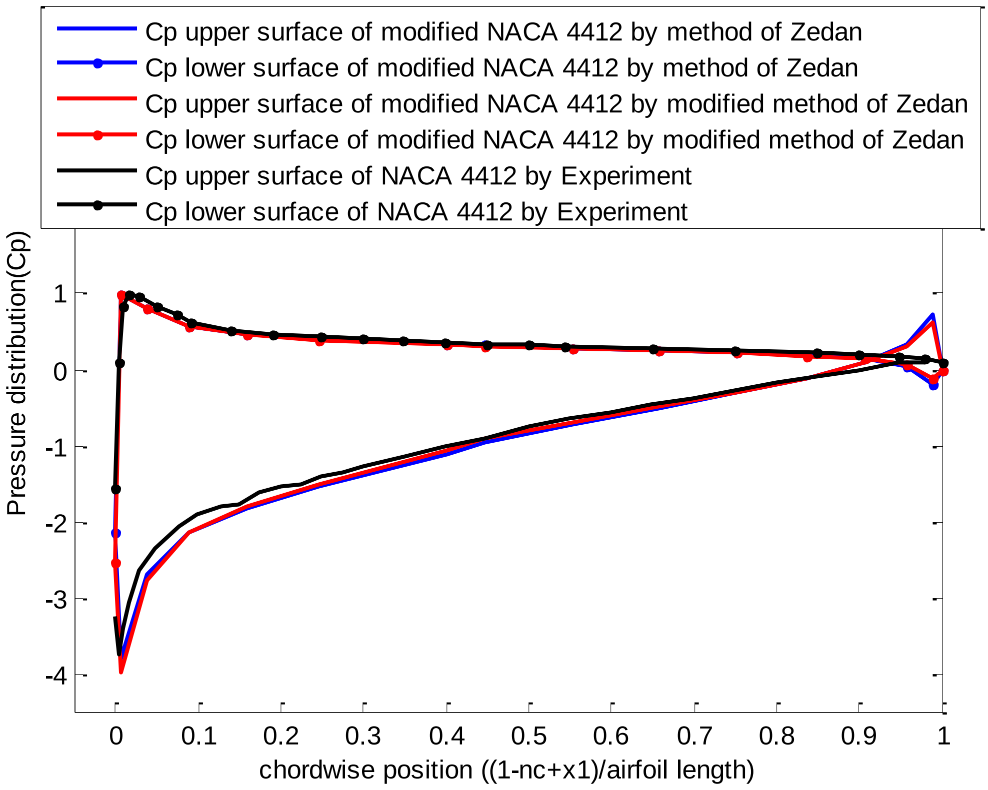
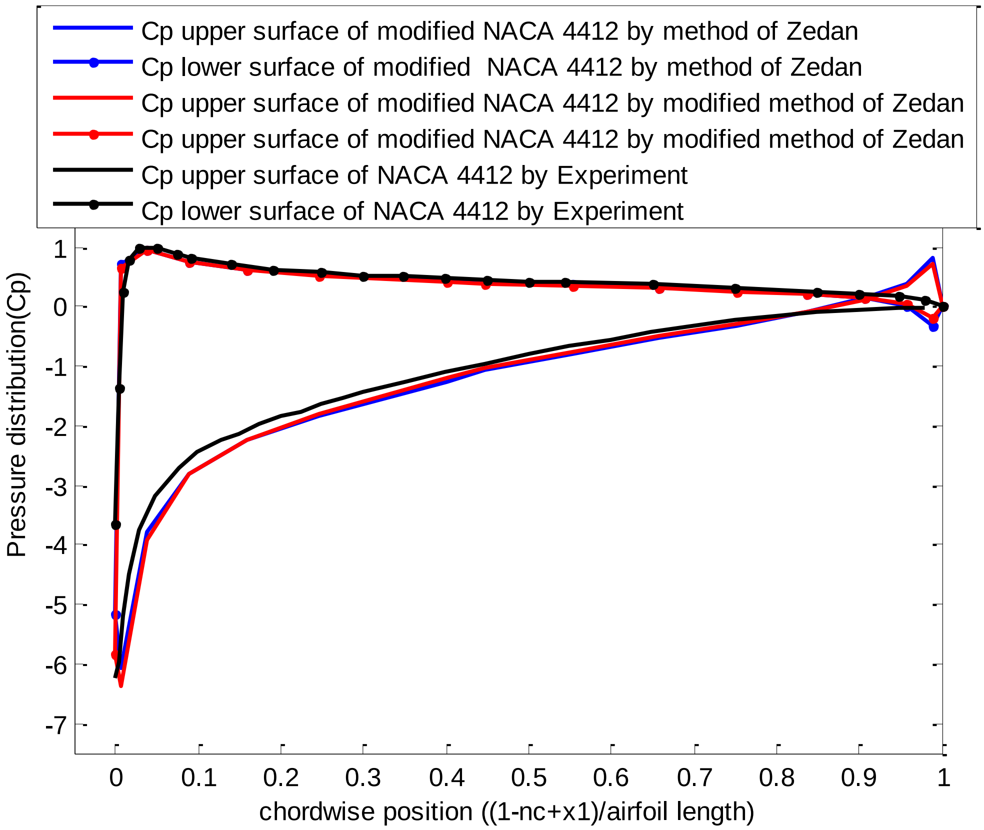


Figure 4: Pressure Distribution for the Modified NACA 4412 Airfoil at 13.5^0 Angle of Attack ($m = 12,\ n = 30$)



Unfortunately, the predictions of the the pressure coefficient distribution on the lower airfoil surfaces is not so good. However, when the airfoil is modified the predictions of pressure coefficient distribution by both methods now agree better with experiment as is evident from figures 3 and 4. From the pressure distribution curves it is clear that the prediction of pressure coefficient distribution by both methods agree closely. This is not surprising since the lift coefficients predicted by both methods are approximately equal. A more careful observation of the pressure distribution curves, particularly on the lower airfoil surfaces in the figures, reveals that the prediction of the pressure coefficient distribution at and around the leading edge region by the method of Zedan (1990) is better than that of the modified Zedan's method. However, over a far wider range along chorwise position of the airfoil, the prediction of pressure distribution by the generalized method is better. This is further expressed in Table 1 which shows comparison between the c_p data along the lower surface of NACA 4412 airfoil at 10^0 angle of attack for the two methods.

Table 1: Pressure Coefficient Distribution along Lower surface of NACA 4412 Airfoil at 10^0 Angle of Attack

Chordwise Position	Modified Method of Zedan	Method of Zedan
0	-3.377828671202684	-2.878467245571699
0.007532945969321	-4.555036975032297	-4.314192417980244
0.037363732512958	-3.071090896910677	-2.977765109125950
0.088560234922329	-2.357507947096582	-2.330078446399043
0.159039450714188	-1.944521601827836	-1.967402495437064
0.245556548043059	-1.674901922023519	-1.694516917792395
0.400000000000000	-1.168538941673947	-1.211178083958283
0.448328960796576	-1.058644344873261	-1.084409698180603
0.553933373630800	-0.749003312449685	-0.798392618814597
0.656818988761430	-0.595962241456007	-0.614512132573968
0.752450614779983	-0.359983137050603	-0.402199376439186
0.836711038727167	-0.246215273606080	-0.228207807254721
0.906058761046486	-0.037444224146185	-0.062066754584059
0.957652848084493	0.187712089913129	0.231295858989577

0.989434997013202	0.016518421015997	0.257609866939174
1.000166526287315	0.219076290518619	0.522027014917768

5.0 CONCLUSION

In this research paper the conformal mapping aspect of the method of Zedan (1990) is modified by replacing the inverse Joukowski map in the method by an inverse of the Karman-Trefftz map to also account for the nonzero angles at the trailing edge of real airfoils. The modified method has given reasonable predictions on the c_l and c_p for the NACA 4412 and modified NACA 4412 airfoils and has outperformed the original version of the method by Zedan (1990).

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