GSJ: Volume 13, Issue 11, November 2025, Online: ISSN 2320-9186
www.globalscientificjournal.com

Using matrix algebra, how to show that the infinite power series [(1+2x)/3]^N is equal to (1+2 x)/(2-2 x), ∀x∈[0,1[

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Abstract.

In a previous article, we showed how to use techniques from Cairo statistical theory and its B-matrix chains to generate new mathematical expressions.

In this article, we focus on generating new algebraic and geometric formal series.

We present and provide statistical evidence for two important particular power series:

1- The infinite power series [(1+2x)/3]^N is equal to (1+2x)/(2-2x), ∀x∈[0,1[

2- The infinite power series [(1+x)/2]^N is equal to (1+x)/(1-x), ∀x∈[0,1[

Finally, we present a general solution to the following system of linear algebraic equations:

$A_{\square\square}X_{\square} + a_{\square\square}X_{\square} + \ldots + a_{\square n}X_n = b_{\square}$

………………………

$A_{n\square}X_{\square} + A_{n\square}X_{\square} + \ldots + A_{nn}X_n = bn$

For the specific cases n = 9 and n = 16.

The numerical results presented are remarkably precise.

I.INTRODUCTION

The essential question arises:

Is the Poisson matrix ill-conditioned by nature [1,2,3,5]?.

YES Poisson matrix is inherently ill-conditioned matrix [Quora Q/A 2022].

To our knowledge, the linear form of Poisson partial differential equation has no analytical solution in the general case (arbitrary boundary conditions and arbitrary source term.

On the other hand, we believe that Dirichlet's boundary conditions guarantee the uniqueness and convergence of its solution.

What if we want to go to picture this equation numerically using the method of finite differences FDM, and then apply either direct or iterative methods to solve the resulting system of simultaneous linear algebraic equations?

When converting a Dirichlet boundary-conditional PDE into an approximately equivalent system of algebraic difference equations using the finite difference method, the result is a non-homogeneous system of linear algebraic equations defined by:

Anxn **X** = **b(**n) . . . .. **(1)**

In regular conventions.

Unfortunately, almost in all cases, the matrix A is ill-conditioned and consequently its system of equations is ill-conditioned.

the solution does exist, but it is difficult to find.

For further clarification, the simplest conventional procedure of a solution is to get the inverse of the square matrix A^ - 1 and to connect the vectors **X** and **b** through the relationship

X = A^-1 . b . . . . . (2)

But since the matrix A itself is largely ill-conditioned, the reverse process does not work well or at all, especially if the number of equations n is large.

In some practical cases, n is at least G.E. 10, not to mention 100 or 1000 and beyond, values necessary to numerically solve the Poisson PDE with sufficient accuracy.

However, there are other methods of solving this linear algebraic system (2), rather than inverting the matrix such as the direct triangulation method and Jacobi and Gauss-Seidel iterative methods [2,4,5,9].

Recently, iterative methods have been slightly improved by introducing the Markov partitions matrix or similar as one of them reduces the matrix illness in part by multiplying another matrix called the pre-conditioner matrix [4].

Neither of these methods seem to work well due to complications of stability and convergence of the numerical solution as expected. #

Here we can only find tricky articles in the present literature while titled as PPDE general case they are treating the space charge as identically zero thus reducing the Poisson PDE to Laplace PDE.

One example among others is the resolution of the generalized Poisson equation using the finite difference method (FDM) by James R. Nagel et al., 2011[4], or even in the numerous Wikipedia articles [1].

Regarding PPDE matrix, why is it inherently ill-conditioned?

The answer is simple, because it's a sparse matrix full of zero entries. For large n, the matrix density can be 3-4% low.

The crucial discovery emerged when we consider that A is a physical statistical matrix and not a random set of numbers, as nature itself suggests: we find that it contains a large amount of informational data.

Consequently, we propose three axioms or mathematical principles:

[For positive symmetric physical power matrices such as B-matrix chains, the sum of their eigenvalues is equal to the eigenvalue of the sum of their matrix power series.]

We call this axiom or rule. . . . Golden Rule 1

This is the main topic of this article.

In this article, we again present the well-known appropriate stochastic transition matrix B defined and explained previously

[2,3,4,5,6] as used in the numerical statistical solution of spatio-temporal PDEs.

In reality, the B matrix and its chained power series are symmetric physical matrices and are not limited to a simple set of numbers. Their structure contains a wealth of information applicable to many physical and mathematical situations.

In this article, we show how the series of powers of appropriate transition matrices B naturally converges to the desired solution.

In other words, the mathematical or rather physical solution of the geometric series follows naturally from equations 1,2 in addition to the golden rules 1,2,3.

In conclusion, this new, unconventional approach directly addresses the problem of summing geometric power series using Axioms 1,2,3 and the mathematical physics properties of physical B matrices.

To avoid dwelling on the details of the introduction, let us move directly to Section II, which deals with the theory.

II.THEORY

The physico-statistical matrix, such as the transition matrix B, is not a random set of numbers; it contains a lot of information data.

In fact, The physical statistical matrices can have important and unique applications in many physical situations such as the spatiotemporal solution of heat diffusion, Poisson and Laplace PDE [5,6] ..etc.

Once more, in this article we focus our analysis to finding adequate formulas for the summation of Finite and Infinite power series.

Note that this article follows the same approach as the previous ones, entitled i-control volume and ii-How to generate new mathematics [7,8,10,11].

We emphasize again that we continue to use the statistical theory of Cairo techniques and its B matrix chains here in two distinct cases [9,10,11]:

i- How to prove that the infinite power series [(1+x)/2]^N is equal to (1+x)/(1-x), ∀x∈[0,1[, as N tends to infinity.

ii- How to prove that the infinite power series [(1+2x)/3]^N is equal to (1+2x)/(2-2x), ∀x∈[0,1[as N tends to infinity.

iii- Finally, how to find a general formula for solving n systems of linear algebraic equations.

The theory of the present article is based on and proves the validity of the mathematical axioms or the PROPOSED statistical principle, namely:

[For positive symmetric physical power matrices, the sum of their eigenvalues is equal to the eigenvalue of the sum of their power series]. . . .Recall that principle (1) is called the golden rule 1.

It is clear that in the particular case posed by the question in the title of this article, it is essential to determine the stochastic transition matrix with basis B whose eigenvalue is equal to [(1 + 2x) / 3], and then to calculate the sum of the power series expansions of this matrix.

In other words, once this matrix B is determined, the following operations are merely a formality.

This is explained in more detail in Section III below, entitled **Applications and Numerical Results.**

In order not to worry too much about the theoretical details, let's move directly to full digital applications.

III.APPLICATIONS and NUMERICAL RESULTS

One of the most illustrative theoretical and experimental applications providing proof of the golden rules 1,2,3 in addition to the transition matrix B is achieved by placing a simple cube illustrated in Figure 1 in an infinite reservoir at constant temperature T [8,9].

Consider the cube of 8 free nodes subject to the Dirichlet boundary conditions illustrated in Figure 1.

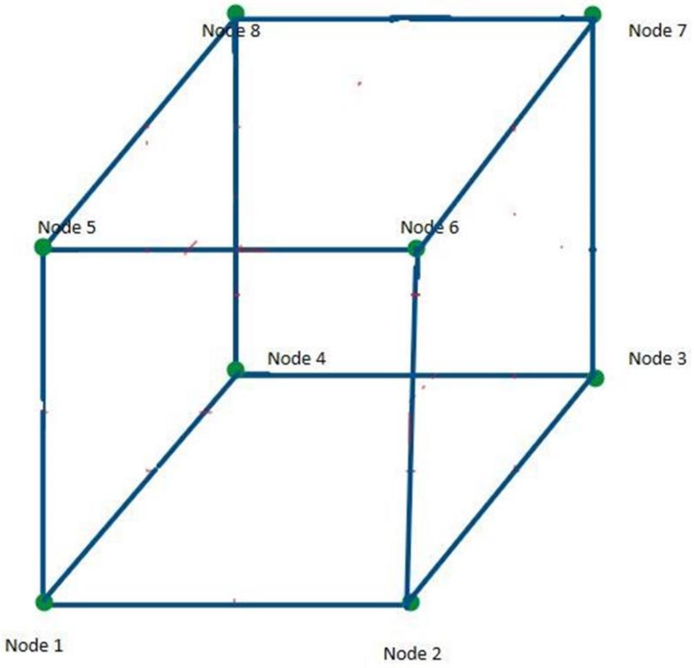

Fig.1. Cube of 8 free nodes subjected to the Dirichlet limit.

The statistical transition matrix for RO element of [0,1[is given by,

1-RO (1-RO) / 6 0 (1-RO) / 6 (1-RO) / 6 0 0 0

2- (1-RO) / 6 RO (1-R)/6 0 0 (1-RO) / 6 0 0

3-0 (1-RO) / 6 RO (1-RO) / 6 0 0 (1-RO) / 6 0

4- (1-RO) / 6 0 (1-RO) / 6 RO 0 0 0 (1-RO) / 6

5- (1-RO) / 6 0 0 0 RO (1-RO) / 6 0 (1-RO) / 6

6- 0 (1-RO) / 6 0 0 0 RO (1-RO) / 6 RO (1-RO) / 6 0

7 – 0 0 (1-RO) / 6 0 0 (1-RO) / 6 RO (1-RO) / 6

8- 0 0 0 0 (1-RO) / 6 (1-RO) / 6 (1-RO) / 6 RO

The 8x8 matrix B required for this situation in the case RO=0.4 is given by:

0.400000006 0.100000001 0.100000001 0.00000000
0.100000001 0.00000000 0.00000000 0.00000000

0.100000001 0.400000006 0.00000000 0.100000001
0.00000000 0.100000001 0.00000000 0.00000000

0.100000001 0.00000000 0.400000006 0.100000001
0.00000000 0.00000000 0.100000001 0.00000000

0.00000000 0.100000001 0.100000001 0.400000006
0.00000000 0.00000000 0.00000000 0.100000001

0.100000001 0.00000000 0.00000000 0.00000000
0.400000006 0.100000001 0.100000001 0.00000000

0.00000000 0.100000001 0.00000000 0.00000000
0.100000001 0.400000006 0.00000000 0.100000001

0.00000000 0.00000000 0.100000001 0.00000000
0.100000001 0.00000000 0.400000006 0.100000001

0.00000000 0.00000000 0.00000000 0.100000001
0.00000000 0.100000001 0.100000001 0.400000006

In the simplest case where RO=0, the statistical transition matrix B is given by:

```
0.00000000     0.166666672     0.166666672      0.00000000
0.166666672      0.00000000      0.00000000      0.00000000

  0.166666672      0.00000000      0.00000000     0.166666672
0.00000000     0.166666672      0.00000000      0.00000000

  0.166666672      0.00000000      0.00000000     0.166666672
0.00000000      0.00000000     0.166666672      0.00000000

   0.00000000     0.166666672     0.166666672      0.00000000
0.00000000      0.00000000      0.00000000     0.166666672

  0.166666672      0.00000000      0.00000000      0.00000000
0.00000000     0.166666672     0.166666672      0.00000000

   0.00000000     0.166666672      0.00000000      0.00000000
0.166666672      0.00000000      0.00000000     0.166666672

   0.00000000      0.00000000     0.166666672      0.00000000
0.166666672      0.00000000      0.00000000     0.166666672

   0.00000000      0.00000000      0.00000000     0.166666672
0.00000000     0.166666672     0.166666672      0.00000000
```

It is easy to show, even by inspection, that the eigenvector of B is composed of a single constant inputs (k, k, ..., k), k not equal to zero.

Moreover, if the eigenvalue of B is equal to ev1, the following equalities are satisfied:

the eigenvalue of B ^ 2 is ev2=ev1 ^ 2

the eigenvalue of B ^ 3 is ev1 ^ 3

. . . .

the eigenvalue of B ^ N is ev1 ^ N

We call this rule or axiom the golden rule 2.

In the case of a numerical example, where RO = 0.4 as indicated above, then the eigenvalue of $B = ev1 = 0.5$, that of $B^2 = 0.5^2$ and that of $B^3 = ev1^3$, and so on.

Let us now move on to the transfer matrix E defined as,

$$E = B^0 + B + B^2 + \dots\dots B^N \dots\dots (2)$$

$$B^0=I$$

We call this rule or axiom equation 2, golden rule 3.

Note that the proposed axioms or golden rules 1, 2, 3 are all compatible and integrate with each other.

Note also that matrix B and its power series are doubly symmetric physical matrices and are not simply a set of numbers, as much information data is inherent in their structure and can be applied to many physical and mathematical applications.

In this article, we also ensure that the appropriate chains of the transition matrix B and its particular geometric series naturally converge to the desired solution.

The unitary symmetric stochastic transition matrix B and the proof of its validity are generated mathematically, or rather physically, in a naturally rigorous way.

In conclusion, the new unconventional approach is a direct attack on the problem of summation of geometric series by the use of the 1,2,3 axioms (1,2,3 golden rules) and the mathematical properties of B-physical matrices [8,10].

This article is in fact a persistent continuation of the previous one, entitled How to In this article, we extend our analysis to the search for suitable formulas for the summation of series of finite and infinite powers [8,10].

1- This article follows on from the previous one and explains it: it shows how to prove that the infinite power series [(1 + x) / 2] ^ N is equal to (1 + x) / (1 - x), for all x ∈ [0, 1[, N ranging from 1 to infinity [9].

2- The theory presented in these two articles relies on the validity of proposed mathematical axioms or statistical principles [1, 2, 3] and proves it.

3- The 8x8 matrix B used for this application is given by:

1 - RO (1-RO) / 6 0 (1-RO) / 6 (1-RO) / 6 0 0 0 (1-RO) / 6 RO (1-RO) / 6 0 0 (1-RO) / 6 0 0

2- 0 (1-RO) / 6 RO (1-RO) / 6 0 0 (1-RO) / 6 0 0

3- (1-RO) / 6 0 (1-RO) / 6 RO 0 0 0 (1-RO) / 6

4- (1-RO) / 6 0 0 0 RO (1-RO) / 6 0 (1-RO) / 6

6- 0 (1-RO) / 6 0 0 (1-RO) / 6 RO (1-RO) / 6 0

7- 0 0 (1-RO) / 6 0 0 (1-RO) / 6 RO (1-RO) / 6

8-0 0 0 (1-RO) / 6 (1-RO) / 6 0 (1-RO) / 6 RO

It is easy to show, even by inspection, that the eigenvector of B is composed of a single constant vector inputs (k, k, ..., k), k not equal to zero.

Moreover, if the eigenvalue of B is equal to ev1, the following equalities are satisfied:

the eigenvalue of B ^ 2 is ev1 ^ 2

the eigenvalue of B ^ 3 is ev1 ^ 3

. . . .

the eigenvalue of B ^ N is ev1 ^ N

The above axiom, we call it the Golden Rule 2.

As a numerical example, consider RO = 0.4

then the eigenvalue of B = .5 and that of B ^ 2 = .5 ^ 2 and so on.

Let us now move on to the transfer matrix E defined as,

E = B ^ 0 + B + B ^ 2 + ... B ^ N . . . . . . (2)

B^0=I

And for N large enough.

E = (I-B) ^ - 1 . . . . (3)

The axiomatic equations above 2,3, we call it the golden rule 3.

In fact it is not complicated to calculate the transfer matrix E for sufficiently large N,

E=1/(I-B)

For N tends to infinity,

And calculate the transfer matrix D for sufficiently large N,

D=E-I

either from Eq. 2, or by equivalent relationship (3).

Finally, we compute the matrix D defined by,

$D = E-I$ . . . . (4)

To justify the assertion in the article title, we construct Table 1 below:

Table I

RO . 0 0.1 0.2 0.3 0.4 0.5 .. . . 0.8 0.9 1.

evD .05 2/3 7/8 8/7 3/2 2.0. . . . . . . 6.5 14 infinity

evD is the eigenvalue of the transfer matrix D,

D=B+B^2+B^3+...+B^N

It is clear that Table I suggests the relationship:

evD = (1 + 2 RO) / (2-2 RO). . . . . (5)

where RO is an element of [0,1[

Combining equation 5. According to principle 1 (the golden rule),

$evB + evB^{2} + evB^{3} + \ldots + evB^{n} = evD$

Where evB^{n} = eigenvalue of the matrix $B^{n} = ev\square^{n}$

then the sum of the infinite power series,

$[(1 + 2R□)/3]^n$, is equal to $(1 + 2x)/(2 - 2x)$, $\forall x \in [0, 1[$ for all $x \in [0, 1[$.

The interrogative statement in the article's title is therefore proven.

Furthermore notice that equation 5 can be used to find theoretical and experimental values for the themal diffusivity of metals[9].

Finally, we introduce and explain the role of the B matrix chains in providing a general solution to the system of simultaneous linear algebraic equations:

a□□X□ + a□□X□ + ... + a□$_n$X$_n$ = b□

...

an□X□ + an□X□ + ... + an$_n$X$_n$ = b$_n$

Consider the rectangular domain with 9 equidistant free nodes, without loss of generality, illustrated in Figure 2.

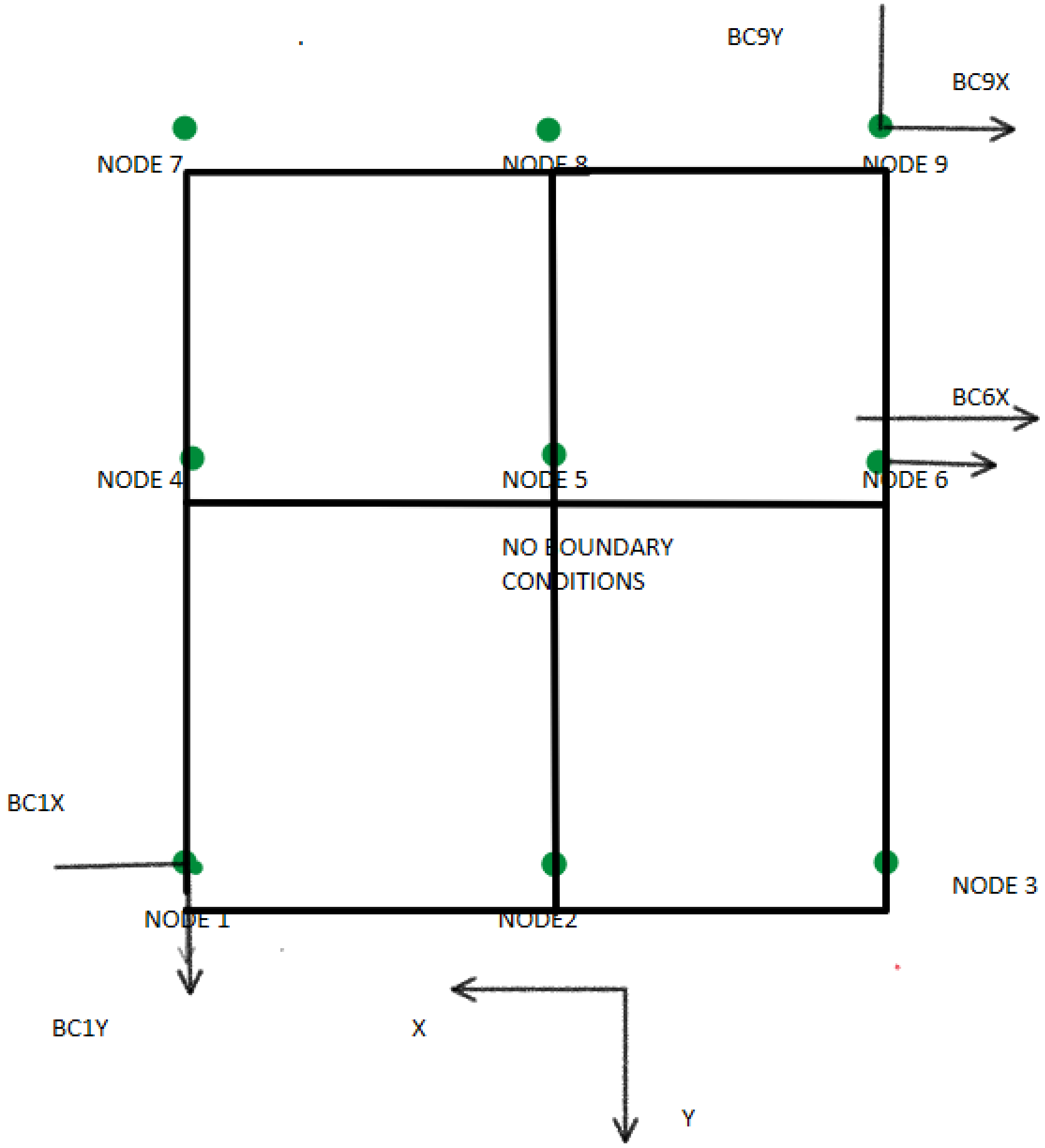

Fig. 1. A two-dimensional rectangular domain with 9 equidistant free nodes.

Consider the simple case of a rectangular domain with 9 equidistant free nodes, u□, u□, u□, ..., u□, and 12 modified Dirichlet boundary conditions BC□ to BC□□, reduced to 9 BC, as illustrated in Figure 1. Mathews [4] a The thermal system resulting from Figure 1 was classically solved using nine linear algebraic equations to determine the steady-state temperature distribution, employing the Gaussian elimination method. A more efficient method is proposed, extending the tridiagonal algorithm to the more sophisticated penta-diagonal algorithm for its arbitrarily chosen boundary conditions vector **b [12,13,14]**.

b=[100,20,20,80,0,0,260,180,180]T

The transfer matrix D for N tends to infinity and RO = 0 is given by,

**1.19545197 0.391880572 0.1240234380.391880572
0.248046875 0.106166303 0.1240234380.106166303
5.25949001E-02**

**0.391880572 1.31947541 0.3918805720.248046875
0.498046875 0.248046875 0.1061663030.176618338
0.106166303**

**0.1240234380.391880572 1.19545197 0.106166303
0.248046875 0.391880572 5.25949001E-02
0.106166303 0.124023438**

**0.3918805720.248046875 0.106166303 1.31947541
0.498046875 0.176618338 0.3918805720.248046875
0.106166303**

**0.2480468750.498046875 0.2480468750.498046875
1.49609375 0.498046875 0.2480468750.498046875
0.248046875**

**0.1061663030.248046875 0.3918805720.176618338
0.498046875 1.31947541 0.1061663030.248046875
0.391880572**

**0.1240234380.106166303 5.25949001E-02 0.391880572
0.248046875 0.106166303 1.19545197 0.391880572
0.124023438**

**0.1061663030.176618338 0.1061663030.248046875
0.498046875 0.248046875 0.391880572 1.31947541
0.391880572**

**5.25949001E-02 0.106166303 0.1240234380.106166303
0.248046875 0.391880572 0.1240234380.391880572
1.19545197**

It is easy to show that **D . b** gives exactly the same solution as Matews's.

Furthermore, we can show that the transfer matrix D is capable of solving the system of 9 linear simultaneous algebraic equations,

a□□X□ + a□□X□ + ... + a□$_{n}$X$_{n}$ = b□

...

an□X□ + an□X□ + ... + an$_{n}$X$_{n}$ = b$_{n}$

for any arbitrarily chosen vector **b**.:

1.19545197 0.391880572 0.1240234380.391880572
0.248046875 0.106166303 0.1240234380.106166303
5.25949001E-02

0.391880572 1.31947541 0.3918805720.248046875
0.498046875 0.248046875 0.1061663030.176618338
0.106166303

0.1240234380.391880572 1.19545197 0.106166303
0.248046875 0.391880572 5.25949001E-02
0.106166303 0.124023438

0.3918805720.248046875 0.106166303 1.31947541 0.498046875 0.176618338 0.3918805720.248046875 0.106166303

0.2480468750.498046875 0.2480468750.498046875 1.49609375 0.498046875 0.2480468750.498046875 0.248046875

0.1061663030.248046875 0.3918805720.176618338 0.498046875 1.31947541 0.1061663030.248046875 0.391880572

0.1240234380.106166303 5.25949001E-02 0.391880572 0.248046875 0.106166303 1.19545197 0.391880572 0.124023438

0.1061663030.176618338 0.1061663030.248046875 0.498046875 0.248046875 0.391880572 1.31947541 0.391880572

5.25949001E-02 0.106166303 0.1240234380.106166303 0.248046875 0.391880572 0.1240234380.391880572 1.19545197

X

a

b

c

d

e

f

g

h

k

=

(11954519700*a+3918805720*b+1240234380*c+3918805720*d+1061663030*f+1240234380*g+1061663030*h+525949001*k+2480468750*e)/10000000000

(391880572*a+1319475410*b+391880572*c+248046875*d+248046875*f+106166303*g+176618338*h+106166303*k+498046875*e)/1000000000

(1240234380*a+3918805720*b+11954519700*c+1061663030*d+3918805720*f+525949001*g+1061663030*h+1240234380*k+2480468750*e)/10000000000

(391880572*a+248046875*b+106166303*c+1319475410*d+176618338*f+391880572*g+248046875*h+106166303*k+498046875*e)/1000000000

(127*a+255*b+127*c+255*d+255*f+127*g+255*h+127*k+766*e)/512

(106166303*a+248046875*b+391880572*c+176618338*d+1319475410*f+106166303*g+248046875*h+391880572*k+498046875*e)/1000000000

(1240234380*a+1061663030*b+525949001*c+3918805720*d+1061663030*f+11954519700*g+3918805720*h+1240234380*k+2480468750*e)/10000000000

(106166303*a+176618338*b+106166303*c+248046875*d+248046875*f+391880572*g+1319475410*h+391880572*k+498046875*e)/1000000000

(525949001*a+1061663030*b+1240234380*c+1061663030*d+3918805720*f+1240234380*g+3918805720*h+11954519700*k+2480468750*e)/10000000000

Which means that,

X1=(11954519700*b1+3918805720*b2+1240234380*b3+3918805720*b4+1061663030*b5+1240234380*b6+1061663030*b7+525949001*b8+2480468750*b9)/10000000000

X2=(391880572*b1+1319475410*b2+391880572*b3+248046875*b4+248046875*b5+106166303*b6+176618338*b7+106166303*b8+498046875*b9)/1000000000

.. . .

Etc..etc.

Finally, let's ask ourselves a relevant question:

Did Einstein truly understand chess?

No.

We can assume that Einstein didn't truly understand chess.

Einstein used to discuss chess problems with his close friend, the mathematics professor and world champion Emanuel Lasker.

Einstein claimed that chess was not a fair game because White always had the advantage and therefore had to win by force.

Emanuel Lasker disagreed, arguing that Black always had additional information, thus compensating for White's advantage.

The question remained unanswered until 2000, when computer scientists built enormous computers equipped with powerful algorithms and tested this complex problem.

In every opening, the game between the two giant computers ended in a draw.

Einstein was wrong, and Lasker was right.

Note that the details inherent in this question concerns not only the importance of information in chess, but also its importance in all fields. The author believes that information theory is one of the most important living theories for all areas of research, but unfortunately, it is still in its infancy [1,14,15].

IV.CONCLUSION

We propose and examine an unconventional technique for finding the sum of symmetric physical matrices, based on 3 essential mathematical axioms, or rather on a physical principle.

These axioms or propositions lead to a powerful and unconventional technique for generating the sum of geometric series, which has the advantage of being unique, stable and quick to converge.

Furthermore, we show that the transfer matrix D is capable of solving the system of n=9 simultaneous linear algebraic equations, or any other number of equations n without loss of generality.

$$a_{\square\square}X_{\square} + a_{\square\square}X_{\square} + \ldots + a_{\square n}X_n = b_{\square}$$

...

$$a_{n\square}X_{\square} + a_{n\square}X_{\square} + \ldots + a_{nn}X_n = b_n$$

for any arbitrarily chosen vector b.

NB1. All experiments in this article were produced using the author's laboratory.

NB2. The author uses his own double precision algorithm, such as that of references 16,17,18,19.

No ready-to-use Python or MATLAB algorithms are needed.

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